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**MONETARY COORDINATION UNDER AN EXCHANGE RATE AGREEMENT
AND THE OPTIMAL MONETARY INSTRUMENT**

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This paper analyses the choice of the monetary instrument in a stochastic two-country setting, where exchange rate stability is maintained through the coordination of monetary policies. When shocks to the real exchange rate are significant, the optimal monetary instrument is sensitive to the importance attached to area-wide, as opposed to national, stabilisation objectives. An increase in the concern with area-wide objectives or in trade integration between the two countries is shown to entail a shift of the optimal monetary instrument towards interest rate pegging. The rise of significant phenomena of currency substitution leads to the adoption of an area-wide monetary instrument, which, instead, is invariant to the balance between area-wide and national stabilisation objectives.

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INTRODUCTION

This paper studies the optimal choice of monetary policy instrument under an exchange rate agreement. An extensive literature has been devoted to monetary policy coordination within currency areas and within the ERM in particular. Yet, analysis has relied on the implicit assumption that monetary conditions in the area always result from a money stock rule, either determined by the anchor country in the asymmetric framework described, for example, by Giovannini and Giavazzi (1989); or by an area-wide monetary planning as in McKinnon (1984) and Russo and Tullio (1989); or by the bargaining of monetary authorities, as in Klein (1991). In contrast, in a world of uncertainty, the choice of the monetary instrument is a crucial issue for the conduct of monetary policy. The superiority of the money stock or the interest rate as the operating target depends upon the sources of the random shocks hitting the economies and thus the appropriate choice of monetary instrument can greatly enhance the stabilising role of monetary policy.

The investigation of the optimal monetary instrument under an exchange rate agreement requires two important features in the framework of analysis. First, economic interdependence must be taken into account. Hence, the set-up traditionally employed to assess alternative monetary instruments, closed-economy or small open-economy models, is replaced by a two-country model along the lines of Turnovsky and d'Orey (1989) and Kawai (1992).

Secondly, the choice of the optimal monetary instrument has to be consistent with the maintenance of exchange rate stability. In the model of the paper, one of the two countries gives up monetary autonomy and adjusts its money supply to maintain exchange rate stability in the presence of shocks. This feature can be viewed as reflecting the evolution of the ERM as an asymmetric system, with Germany as the anchor-country and the other ERM participants importing the monetary stance through the exchange rate arrangement. However, it can also be given the more neutral interpretation of an efficient coordination agreement, which does not necessarily imply the subordination of the policy objectives of the non-anchor country. The stabilisation objectives of the two countries may in fact coincide. When they do not, it is likely that the increasing cooperation between monetary authorities and the prospects of EMU lead the anchor-country to be concerned (at least in part) with area-wide stabilisation objectives. To use the words of a Bundesbank official, "the Bundesbank does take account of its partners' interests both implicitly and explicitly" (Rieke, 1989, p. 291).

As the following analysis shows, the stabilisation objectives of the two countries might be in conflict only in so far as shocks to real exchange rate are significant. When such shocks are important, the analytical argument of the paper is cast in terms of investigating the changes in the optimal instrument induced by an increase in the concern attached to area-wide, as opposed to national, stabilisation objectives. However, this line of argument also

provides insights on the effects of growing economic integration. This can be intuitively appreciated by noticing that, under an exchange rate agreement, monetary policy can offset in one country the effects of shocks to the real exchange rate only by exporting instability to the other. The more the two economies are integrated the more the "exported" instability will be "reimported" through trade interlinkages. Thus, not only financial but also trade integration is likely to lead monetary authorities to strengthen coordination, as growing economic interdependence makes international feedbacks increasingly important.

The remainder of the paper is organised as follows: Section I develops the basic set-up which is then used in Section II to define the optimal monetary instrument and to investigate its properties. Section III extends the model to allow for the endogenous determination of the real exchange rate. Section IV studies the implications of the adoption of an area-wide monetary instrument. Section V draws the conclusions.

I. THE BASIC MODEL

This section presents a simple, two-country rational expectations model in which both economics are subject to real and nominal disturbances. In order to maximise analytical tractability, the model is log-linear; variables are expressed as deviations from their trend levels and both countries are assumed to be identical. Each country block contains three equations:

$$m_t^d - p_t = y_t - \alpha i_t + v_t \quad \alpha > 0 \quad (1)$$

$$y_t^d = \beta y_t^* - \phi [i_t - E_t p_{t+1} + p_t] - \delta [p_t - p_t^* - e_t] + u_t \quad 0 < \beta < 1 \quad \phi, \delta > 0 \quad (2)$$

$$y_t^s = \gamma [p_t - E_{t-1} p_t] + w_t \quad \gamma > 0 \quad (3)$$

The notation is standard, with m standing for the log of the nominal money shock; p of the price level; y of the real output; i the nominal interest rate and E the expectation operator. Variables of the country described by the above equations (for the later convenience, it shall be referred to as non-anchor, NA, country) are without stars; variables of the other (anchor, A, country) are starred. e_t stands for the log of the nominal exchange rate expressed in units of NA-country currency per unit of A-country currency. The A-country is described by three analogous equations, where unstarred variables are replaced by starred variables (elasticities are assumed to be equal in both countries). Equation (1) is the money demand; for simplicity income elasticity is set to one. Equation (2) represents output demand, which depends upon the other country's output, the real rate of interest and the real exchange rate. The third equation is a standard Lucas supply function where deviations in output from its natural rate are a positive function of unanticipated movements in the price level. The stochastic variables (money demand shocks v_t and v_t^* ; output demand shocks, u_t and u_t^* ; supply shocks, w_t and w_t^*) are assumed to be independently distributed with zero means and finite variances, denoted by $\sigma_u^2, \sigma_{u^*}^2, \sigma_v^2$ etc.

The two country blocks are connected by the following equations:

$$i_t = i_t^* + E_t e_{t+1} - e_t \quad (4)$$

$$p_t = p_t^* + e_t + \mu_t \quad (5)$$

Equation (4) is the interest parity condition which postulates the perfect substitutability between the bonds of the two countries. Equation (5) is a stochastic version of PPP which assumes perfect substitutability in the output market¹, except for a random disturbance which creates a wedge between the prices of the goods produced in the two countries (see for example Gros and Lane, 1991). Rather than the result of market imperfections which prevent the level of one price from holding, the random disturbance can be interpreted as an analytical short-cut to model the determination of the real exchange rate. This simplifying assumption is at first adopted to make clearer the exposition of the key insights of the paper. However, it is relaxed in Section III, where the real exchange rate is endogenously determined.

Next, the money supply has to be specified. In order to allow the analysis of the optimal choice of monetary instrument, the following general formulation is employed in the wake of the combination policy put forward in the classic paper by Poole (1970):

$$m_t^{**} = m_t^* + k^{**} i_t \quad k^{**} \geq 0 \quad (6)$$

This equation - which encompasses the pure policies of money targeting and interest rate pegging as special cases when $k^{**} = 0$ and $1/k^{**} = 0$, respectively - takes the money stock and the interest rate to be deterministically related even in the presence of stochastic disturbances. The authorities are in fact assumed to change the money stock in response to movements in the interest rate, as these embody information on the nature of the current shocks. The optimal monetary instrument is defined by the value of k^{**} which optimises the stabilisation objectives of the authorities. In general, it will not identify a pure policy, but will specify a feed-back rule from interest rate movements to money stock changes, defining the extent to which monetary authorities should "lean with the wind" according to the structure of both economies and the size of the variance of all the disturbances. Such feedback rules normally are quite complicated and, if they were to be implemented strictly, they would require a very precise knowledge of the structural parameters of the economy which monetary authorities can hardly be expected to attain. Rather, the properties of the optimal feedback rule supply qualitative insights on how strictly

¹ As a result of this assumption, the distinction of the aggregate demand equations for the two countries becomes redundant and thus equations (2) and its counterpart for the A-country collapse into:

$$(y_t^d + y_t^{d*}) = \beta(y_t + y_t^*) - \phi[i_t - E_t p_{t+1} + p_t] - \phi[i_t^* - E_t p_{t+1} + p_t^*] + (u_t + u_t^*)$$

pure policies should be applied in the face of changes in the environment in which monetary policy operates.

The money supply of both countries can be expressed by an equation similar to (6). However, an exchange rate agreement entails the loss of one degree of freedom in the management of national monetary policies and thus only one country can choose a combination policy as monetary instrument. In the light of the objectives of the present analysis, the combination policy formulation is therefore employed for the A-country, while the NA-country is assumed to use the money stock as monetary instrument. In analytical terms, this corresponds to leaving k^* unconstrained in equation (6), while setting $k' = 0$ in the corresponding equation for the NA-country.

The system comprising equations (1), (2), (3), (6); their counterparts for the A-country and equations (4) and (5) can now be solved. If the optimal policy is constant over time, given that the model is in deviation form, the following holds:

$$E_t p_{t+1} = E_t p_{t+1}^* = E_t e_{t+1} = 0$$

Making use of the above relation and applying the method of undetermined coefficient yield the solution for the price levels in terms of the stochastic disturbances:

$$p_t = \Pi_0(m_t - v_t) + \Pi_1(m_t^* - v_t^*) + \Pi_2 w_t + \Pi_3 w_t^* + \Pi_4(u_t + u_t^*) + \Pi_5 \mu_t \quad (7)$$

$$p_t^* = \Psi_0(m_t - v_t) + \Psi_1(m_t^* - v_t^*) + \Psi_2 w_t + \Psi_3 w_t^* + \Psi_4(u_t + u_t^*) + \Psi_5 \mu_t \quad (7')$$

where:

$$\begin{aligned} \Pi_0 &= [(2\varphi + \vartheta_1 \gamma)k^* + 2\varphi(1 + \gamma)]/\Delta > 0 & \Psi_0 &= -\vartheta_1 \gamma k^*/\Delta < 0 \\ \Pi_1 &= -\alpha \gamma \vartheta_1 / \Delta < 0 & \Psi_1 &= [2\varphi \vartheta_2 + \vartheta_1 \alpha \gamma] / \Delta > 0 \\ \Pi_2 &= -[(2\varphi + \vartheta_1(\alpha + \gamma))k^* + (1 + \gamma)(2\varphi + \alpha \vartheta_1)] / \Delta < 0 & \Psi_2 &= -(\alpha + 1)\vartheta_1 k^* / \Delta < 0 \\ \Pi_3 &= -\alpha \vartheta_1 (k^* + 1) / \Delta < 0 & \Psi_3 &= -[\vartheta_1 \vartheta_2 k^* + 2\varphi \vartheta_2 + \vartheta_1 \alpha \gamma] / \Delta < 0 \\ \Pi_4 &= \alpha(1 + \gamma + k^*) / \Delta > 0 & \Psi_4 &= \vartheta_2 k^* / \Delta > 0 \\ \Pi_5 &= \alpha[(\varphi + \vartheta_1 \gamma)k^* + (1 + \gamma)\varphi] / \Delta > 0 & \Psi_5 &= -k^*[\varphi \vartheta_2 + \vartheta_1 \alpha \gamma] / \Delta < 0 \\ \Delta &= [(2\varphi + \vartheta_1 \gamma)\vartheta_2 + \vartheta_1 \alpha \gamma]k^* + (1 + \gamma)[\vartheta_1 \alpha \gamma + 2\varphi \vartheta_2] > 0 \\ k^* &= k^* + \alpha > 0 & \vartheta_1 &= 1 - \beta & \vartheta_2 &= \alpha + 1 + \gamma \end{aligned}$$

Some observations on these reduced forms are worth making. Firstly, as a result of the simplifying assumption embodied in the stochastic version of PPP, shocks to output demand have the same influence on the price levels irrespective of the country where they occur. Thus, the parameter δ , denoting the elasticity of output demand to the real exchange rate, does not enter the solution for the price levels. Secondly, the impact of exogenous shocks on each of the two economies depends upon the choice of monetary instruments in both countries. Thirdly, even if the two economies are identical, the asymmetric choice of

monetary instruments is reflected in the asymmetry of the coefficients. In contrast, if both countries adopt the same instrument, the symmetry of the coefficients is restored, as can be easily appreciated by setting $k^* = \alpha$ (i.e., $k'^* = 0$) thereby reducing the combination policy of the A-country to a money stock rule.² Fourthly, the cross-country effect of monetary policy on the price levels is negative through the mechanism dating back to Mundell (1963). A monetary expansion abroad leads to an appreciation of the domestic currency and, in order to restore PPP, a decline in the domestic price level, associated with a fall in domestic output, is required. The solution for the exchange rate is given by:

$$e_t = (\Pi_0 - \Psi_0)(m_t - v_t) + (\Pi_1 - \Psi_1)(m_t^* - v_t^*) + (\Pi_2 - \Psi_2)w_t + (\Pi_3 - \Psi_3)w_t^* + (\Pi_4 - \Psi_4)(u_t + u_t^*) + (\Pi_5 - \Psi_5 - 1)\mu_t \quad (8)$$

As is apparent from the above solution, if monetary policies are left unconstrained, the exchange rate will fluctuate in response to the shocks hitting the economies. Conversely, if, for reasons not explored in this framework, the two countries choose to maintain exchange rate stability, monetary policies must be co-ordinated to ensure this outcome (for a discussion considering both the discipline and the cooperation argument for exchange rate stability in the ERM context, see Melitz, 1988). Setting the above solution to zero gives the equation which defines the relationship between money supplies under an exchange rate agreement. Here, it is assumed that the burden of monetary adjustment is entirely borne by the NA-country. Hence, m_t is endogenous to the model, as m_t^* is assumed to accommodate the random disturbances in order to maintain exchange rate stability. In contrast, A-country preserves the freedom of choosing its monetary instrument.

Such asymmetric characterisation of the adjustment duties can be viewed as reflecting the pivotal role of Germany in the EMS, as it has developed during the 1980s. On the other hand, this type of asymmetry can be given a more neutral interpretation of a coordination arrangement which possesses two desirable properties. Firstly, defining precisely the country entrusted with the task of maintaining exchange rate stability provides very clear "rules of the game" and thus avoids the concrete risk that a vague specification of the adjustment duties (or the uncertainty associated with the recurrent bargaining over them) results in an inefficient stabilisation performance. Secondly, by leaving the A-country unconstrained in the choice of the monetary instrument, the loss of the exchange rate as a stabilisation instrument is compensated by the freedom retained by the A-country in the use of a combination policy, whose benefits do not necessarily accrue only to the A-country. If countries share the adjustments in the money supplies to maintain the exchange rate constant, none of them will be able to maintain its money supply in a deterministic relation with the interest rate and both are thus "forced" to use a money stock instrument³.

² In fact, if $k^* = \alpha$, then: $\Pi_0 = \Psi_1$; $\Pi_1 = \Psi_0$; $\Pi_2 = \Psi_3$; $\Pi_3 = \Psi_2$; $\Pi_4 = \Psi_4$, and $\Pi_5 = -\Psi_5$

³ An appropriate use of the money stock instrument on the part of both countries can in principle replicate the stabilisation properties of a given combination policy followed by the A-country, when the NA-country pegs the exchange rate. Yet, this alternative appears very unsuitable to a practical implementation, given

In order to investigate the properties of the price levels when only the NA-country takes care to ensure exchange rate stability, equation (8) is set equal to zero, it then solved for m_t and the resulting expression is substituted in (7) and (7') to yield:

$$p_t = \Gamma_1(m_t^* - v_t^*) + \Gamma_2 w_t + \Gamma_3 w_t^* + \Gamma_4(u_t + u_t^*) + B\mu_t \quad (9)$$

$$p_t^* = \Gamma_1(m_t^* - v_t^*) + \Gamma_2 w_t + \Gamma_3 w_t^* + \Gamma_4(u_t + u_t^*) + B^*\mu_t \quad (9')$$

where: $B = \Gamma_5 + \Pi_0 / (\Pi_0 - \Psi_0) > 0$

$$B^* = \Gamma_5 + \Psi_0 / (\Pi_0 - \Psi_0) < 0$$

$$\Gamma_i = (\Pi_0 \Psi_i - \Pi_i \Psi_0) / (\Pi_0 - \Psi_0)$$

Solving out the algebra, the coefficients for the solution of the model under exchange rate stability are found to be:

$$\Gamma_1 = 2\varphi / \Delta_2 > 0$$

$$\Gamma_4 = k^* / \Delta_2 > 0$$

$$\Gamma_2 = -\vartheta_1 k^* / \Delta_2 < 0$$

$$\Gamma_5 = -\varphi k^* / \Delta_2 < 0$$

$$\Gamma_3 = (\Gamma_2 - \Gamma_1) < 0$$

$$\Delta_2 = 2[(\varphi + \vartheta_1 \gamma)k^* + \varphi(1 + \gamma)] > 0$$

The above equations provide several insights on the interaction between the two economies under an exchange rate agreement complemented with the understanding that only the NA-country ensures exchange rate stability. Firstly, financial shocks in the NA-country do not influence the price level in either country. Through the commitment to exchange rate stability, NA-country's monetary authorities automatically accommodate domestic monetary shocks, thereby preventing them from affecting real or nominal variables in either country. Thus, if monetary shocks in one country represent the major source of instability in the area, it is to the benefit of both countries if the country with an unstable money demand ensures exchange rate stability. By putting forward the stability of money demand as the criterion for the choice of the country to play the role of nominal anchor in an exchange rate arrangement, this result provides a rationale, besides the difference in anti-inflationary credibility, for the common agreement on the central role played by Germany in the ERM (see also Bini Smaghi - Vori (1991)).

Secondly, whereas the direction of the impact of output demand and supply shocks on the price level of both countries is the same as under flexible exchange rates, the negative cross-country effect of monetary policy on the price levels disappears under fixed exchange rates. By maintaining exchange rate stability, the NA-country prevents the A-country currency from appreciating in the case of a monetary expansion in the A-country and thus an increase in the money supply of the A-country leads to an inflationary impulse in both countries.

the difficulties in disentangling the effects on the exchange rate of shocks from those of movements in the other country's money stock.

Thirdly, inspection of equations (9) and (9') reveals that all shocks - except for the disturbance to PPP - affect the price level of both countries in the same direction and with the same intensity. Thus, when this source of instability is negligible, the stabilisation objectives of the monetary authorities of both countries coincide exactly and therefore, the asymmetric allocation of the burden of obtaining exchange rate stability does not provide any ground for contrasting interests between the two countries. Even if the A-country pursues purely domestic objectives in the choice of its monetary instrument, the NA-country will benefit from the actions of the A-country as if its own stability were given priority. This is no longer the case when shocks to PPP are important: consideration of the stabilisation objectives of NA-country on the part of the A-country has a bearing on the choice of the monetary instrument. The next section explores this interaction.

II. THE OPTIMAL MONETARY INSTRUMENT

Under an exchange rate agreement with asymmetric adjustment duties, only the A-country has the freedom of choosing the monetary instrument, i.e. of selecting the value of k^* which defines the optimal relationship between the money stock and the interest rate. The optimality of the monetary instrument is defined with respect to the stabilisation objectives, which are assumed to focus the price level.⁴ Thus, k^* will be chosen so as to maximise the following objective function:

$$\Phi = -[Var(p^*) + \theta Var(p)] \quad 0 \leq \theta \leq 1 \quad (10)$$

This objective function encompasses the whole spectrum of the degrees of symmetry that can characterise the stabilisation aims pursued by monetary policy under stable exchange rates. If $\theta = 0$, the system is totally asymmetric in the sense that the A-country only cares about domestic objectives and completely disregards the external implications of its choice of the monetary instrument. If $\theta = 1$, the system is perfectly symmetric as monetary policy aims at minimising the fluctuations of the area-wide price level.⁵ Between these two extremes, intermediate situations are possible, where the A-country places its domestic objectives before area-wide considerations, which are nonetheless taken into account. Before full EMU, there is no institutional mechanism ensuring that the A-country pays any attention to the implications for the other countries of its decisions about the monetary policy

⁴ Expressing the stabilisation objectives in terms of income stabilisation would produce exactly the same results.

⁵ Making use of the condition $B - B^* = 1$, simple algebra can in fact show that the optimal monetary policy is the same whether the area-wide objective function is defined as $-Var(p) - Var(p^*)$ or as $-Var(p + p^*)$.

instrument. However, it seems very likely that the increasingly closer interaction between monetary authorities in the transition to EMU leads the A-country to be concerned with the stabilisation objectives of its partners. The intensity of such concerns or their implications for the decision-making about monetary affairs in the system are issues about which the present analytical framework provides no insights. Insights can instead be gained on the interaction between the choice of the monetary instrument and the increasing symmetry of the objective function of monetary policy.

Making use of equations (9) and (9') and rearranging, Φ can be rewritten as:

$$\Phi = -(1+\theta)V - \theta\Omega \quad (11)$$

where

$$V = \Gamma_1^2 \sigma_{v^*}^2 + \Gamma_2^2 \sigma_w^2 + \Gamma_3^2 \sigma_{w^*}^2 + \Gamma_4^2 (\sigma_u^2 + \sigma_{u^*}^2) + B^{*2} \sigma_\mu^2$$

$$\Omega = [B^2 - B^{*2}] \sigma_\mu^2$$

The value of k^* which maximises Φ , denoted by \hat{k}^* , will be defined by:

$$\partial\Phi/\partial k^* = (1+\theta)\partial V/\partial k^* + \theta\Omega/\partial k^* = 0 \quad (12)$$

As shown in Appendix I, implicit differentiation can be used to investigate the properties of the optimal monetary instrument. In particular it can be shown that:

$$\text{sign } \partial k^*/\partial \theta = \text{sign } [-\partial\Omega/\partial k^*] > 0$$

since:

$$\partial\Omega/\partial k^* = \partial(B^2 - B^{*2})\sigma_\mu^2/\partial k^* = -[4\phi(1+\gamma)(\phi + \vartheta_1\gamma)]\sigma_\mu^2/\Delta_2^2 < 0$$

The higher the weight that the A-country attaches to the stabilisation objectives of the NA-country the more its optimal monetary instrument will depart from money stock targeting in the direction of interest rate pegging. The intuition behind this result can be appreciated by focusing on the way disturbances to PPP affect the two economies, given that, as noted above, under an exchange rate agreement only this kind of shock has an asymmetric impact on the price level of the two countries. Consider the case when the A-country pursues a pure policy of money targeting and therefore does not accommodate at all shocks to PPP. Their impact on the exchange rate will be most potent and will call for changes in the money supply of the NA-country to maintain exchange rate stability. Such variations in m_t will in turn feed back on the price levels of both countries although their impact on the price level of the NA-country will be comparatively stronger.⁶ If, however, the A-country moves towards interest

⁶ This can be appreciated by expressing the coefficient for μ_t in equations(9) and (9') in explicit form:

$$B = [(\phi + \vartheta_1\gamma)k^* + 2\phi(1+\gamma)]/2[(\phi + \vartheta_1\gamma)k^* + \phi(1+\gamma)]$$

rate pegging the smaller will be the response in the money supply of the NA-country required to maintain exchange rate stability, as a larger part of the shock is accommodated by changes in the money supply of the A-country. Therefore, the more the A-country cares for the stabilisation of the price level in the partner country the more it will be inclined, other things being equal, to accommodate part of the shocks to PPP by moving its monetary instrument towards interest pegging. If, in the limit case, the A-country pursues a pure policy of interest targeting, the money supply of both countries respond with the same intensity (but opposite directions) to shocks to PPP and therefore the price level in both countries is affected with equal intensity (but opposite direction) by shocks to PPP⁷.

We can now turn to the other properties of the optimal monetary instrument by investigating how it is affected by changes in the environment in which monetary policy operates. Making use of implicit differentiation of equation (12) allows to establish the following:

$$\begin{aligned} \partial \hat{k}^* / \partial \sigma_{v^*}^2 &> 0; && \text{(financial shocks in the A-country)} \\ \partial \hat{k}^* / \partial \sigma_u^2 = \partial \hat{k}^* / \partial \sigma_{u^*}^2 &< 0 && \text{(demand shocks in the A or in the NA-country)} \\ \partial \hat{k}^* / \partial \sigma_w^2 &< 0; && \text{(supply shocks in the NA-country)} \\ \partial \hat{k}^* / \partial \sigma_{w^*}^2 &> 0^8 && \text{(supply shocks in the A-country)} \\ \partial \hat{k}^* / \partial \sigma_\mu^2 &> 0 \text{ if } \theta < |B^*|/|B| && \text{(shocks to PPP)} \end{aligned}$$

The responses of the optimal monetary instrument to an increase in the intensity of demand shocks (wherever they occur) and of financial shocks in the A-country confirm the classic findings by Poole (1970). The predominance of real demand shocks calls for monetary targeting while financial shocks require interest rate pegging. The reaction of the monetary instrument to an increase in the variance of supply shocks instead depends on which country is affected. An increase in the intensity of supply disturbances in the NA-country unambiguously calls for a shift towards monetary targeting. Conversely, if supply-side shocks become more intense in the A-country, the optimal monetary instrument moves in the direction of interest rate pegging. These results stem from the application, in the framework of an exchange arrangement with asymmetric monetary duties, of the general principle that temporary supply shocks are to be partly accommodated by changes in the money supply of the country where they take place. In fact, when supply shocks hit the A-country, its monetary instrument moves in the direction of interest rate pegging, thereby allowing for

$$B^* = -(\varphi + \vartheta_1 \gamma) k^* / 2 [(\varphi + \vartheta_1 \gamma) k^* + \varphi(1 + \gamma)]$$

⁷ In fact, if $1/k^* = 0$, the coefficients for μ_i in the reduced forms for the price level become:

$$B = -B^* = 1/2$$

⁸ This result assumes that $2\varphi + \vartheta_1(1 - \gamma) > 0$, a condition very likely to be met in practice.

some accommodation of the shocks. When supply shocks hit the NA-country, the A-country instrument moves towards money targeting, thereby leading to greater accommodation on the part of the NA-country, as the latter is in charge of maintaining exchange rate stability.

An increase in the variance of the shocks to PPP leads the A-country to move its optimal monetary instrument in a direction which depends on the importance attached to area-wide objectives. The reason is that, as discussed above, the choice of the monetary instrument affects the allocation of disturbances to PPP between the price level of the two countries. Therefore, the response of the monetary instrument to this kind of shocks can only reduce the instability in the price level of one country at the expense of increasing the instability in the price level of the other. This, however, is not a one-to-one trade-off, but is in relation to the relative strength of the effects of shocks to PPP in the two countries, as specified by the ratio of the absolute values of the relevant coefficients in the solution for the price levels ($|B^*/B|$). Since this ratio is strictly smaller than one for any value of k^* (except in the limiting case when $1/k^* = 0$), a change in the monetary instrument to reduce the instability in the price level of the A-country will induce a more than proportional increase in the instability of the price level of the NA-country. As a result, if the A-country pursues purely domestic objectives (i.e. if $\theta = 0$), the response of the monetary instrument to an increase in the intensity of the disturbances to PPP is in the direction of money targeting, while the stabilisation of the area-wide price level (i.e. $\theta = 1$) calls for a shift in the direction of interest rate pegging. In the special case when θ is exactly equal to the relative impact of the shocks to PPP in the two countries, the optimal monetary instrument will be invariant to the intensity of this type of shocks.

III. IMPERFECT SUBSTITUTABILITY IN THE GOODS MARKET

For analytical convenience, the model of the previous sections has relied on a strong simplifying assumption on the determination of the real exchange rate. Equation (5) in fact postulates that PPP holds continuously except for a stochastic exogenous shock, which can be viewed as an analytical short-cut to model the real exchange rate. In this section, the simplifying assumption is relaxed. The analysis is carried out within a framework which explicitly recognises the difference in the goods produced in the two countries and thus allows the derivation of the real exchange rate as a function of the shocks hitting the two economies.

Of the three equations which define each country block, two have to be amended to take into account that the price index relevant to demand decisions is a weighted average of the prices of domestic and foreign goods. Thus:

$$m_t^d - q_t = y_t - \alpha i_t + v_t \quad (1')$$

$$y_t^d = \beta y_t^* - \phi [i_t - E_t q_{t+1} + q_t] - \delta [p_t - p_t^* - e_t] + u_t \quad (2')$$

$$q_t = \lambda p_t + (1-\lambda)(p_t^* + e_t) \quad \frac{1}{2} < \lambda < 1 \quad (13)$$

Equation (13), which defines the general price index of the NA-country, postulates that a constant proportion of income λ is spent on the domestic good; the condition $\lambda > \frac{1}{2}$ expresses the preference of residents for the home good.

The model allowing for the endogenous determination of the real exchange rate is then defined by equations (1), (2'), (3), (6), (13) for the NA-country; by their counterparts for the A-country and by the interest parity condition given by equation (4), as the assumption of perfect substitutability between the bonds of the two countries is maintained. In analogy with the analysis of the previous sections, the model is solved at first for the price levels and the nominal exchange rate. Setting the nominal exchange rate equation to zero defines the money supply rule which the NA-country has to follow in order to preserve exchange rate stability. The resulting expression for m_t is then substituted in the equations for the price levels so as to obtain explicit solutions under an exchange rate agreement which entrusts the NA-country with the maintenance of exchange rate stability.

$$p_t = \Gamma_1(m^* - v_t^*) + \Gamma_2 w_t + \Gamma_3 w_t + \Gamma_4(u_t + u_t^*) + B' \mu_t \quad (14)$$

$$p_t^* = \Gamma_1(m^* - v_t^*) + \Gamma_2 w_t + \Gamma_3 w_t^* + \Gamma_4(u_t + u_t^*) + B^* \mu_t \quad (14)$$

$$\mu_t = \left[(1+\beta)(w_t^* - w_t) - (u_t^* - u_t) \right] / \Delta_3 \quad (15)$$

where

$$\Delta_3 = 2\delta + \varphi(2\lambda - 1) + (1+\beta)\gamma$$

$$B' = \Pi_0 / (\Pi_0 - \Psi_0) + \Gamma_5 - (1-\lambda)\Gamma_1 = B - (1-\lambda)\Gamma_1 > 0$$

$$B^* = \Psi_0 / (\Pi_0 - \Psi_0) + \Gamma_5 - (1-\lambda)\Gamma_1 = B^* - (1-\lambda)\Gamma_1 < 0$$

Comparing the above equations with the solutions for the simplified model given by equations (9) and (9') suggests the following comments. Firstly, the real exchange rate is now endogenous and equation (15) provides insights on its determinants. With rational expectations and the exchange rate agreement which prevents monetary shocks from affecting the nominal exchange rate, the real exchange rate is independent of monetary shocks. It responds only to real shocks hitting the two economies in an asymmetric way, as shown by the fact that demand and supply shocks enter the solution for μ_t in difference form. Asymmetric supply shocks are more powerful than demand ones, since they also feed on prices through the open economy multiplier, as shown by the $(1+\beta)$ factor in equation (15).

Secondly, even when the real exchange rate is endogenously determined under an exchange rate agreement, real and financial shocks affect the price level of both countries in the same direction and with the same intensity except for their impact on the real exchange rate. This result, thus, confirms the intuition provided by the simple model that the asymmetric allocation of adjustment duties entails contrasts of interests between the participating countries only insofar as disturbances affect the real exchange rate. In fact, as

equations (14) and (14') show, the effect of disturbances on the price level of the two countries can be decomposed into two distinct parts: one which affects both countries in the same way and one which induces changes to the real exchange rate and thus have a different impact on the price level of the A and NA countries.

Thirdly, the coefficient for the effect of financial shocks in the A-country is unchanged. Under fixed exchange rates, monetary policy has no effect on the real exchange rate, as a monetary expansion increases both price levels with the same intensity. Thus, the transmission of monetary impulses from the A-country is impervious to the specification of the model with respect to the endogeneity of the real exchange rate. An analogous invariance holds for the effects of monetary shocks in the NA-country, which still do not affect the price level in either country as they are automatically accommodated through the commitment of the NA-country to maintain exchange rate stability.

Fourthly, the coefficients for the impact of the real exchange rate on the output price of both countries (i.e. both B' and B^{*}) differ from their counterparts in the simplified model as to each of them a further part is added: $(1-\lambda)\Gamma_1$, i.e. the coefficient for the effects of financial shocks in the A-country multiplied by the proportion of income which is spent on the foreign good. This result points to a further channel of interaction between the two economies. The demand for real money depends on the general price index, which is a weighted average of the price of domestic and foreign goods. By modifying their relative price, variations in the real exchange rate have a direct effect on the demand for money which in turn feeds on the output prices in the same way as financial shocks do. When the price of the foreign good increases, the demand for money increases proportionally to the share of the foreign good in the consumer basket and this affects the price of domestic output as if liquidity preference in the A-country had increased.

The difference in the impact of changes in the real exchange rate on the output prices of the two economies - which mirrors the difference in the transmissions of shocks to PPP in the simplified model - provides the rationale for the interaction between the choice of the monetary instrument on the part of the A-country and the extent to which the NA-country stabilisation objectives are taken into account.

To facilitate the comparison with the previous section and to simplify the algebra, two assumptions are made. First, the objective function is expressed in terms of the variance of output prices and not in terms of the variance of the general price indexes of the two countries. This hypothesis, however, does not entail any loss of generality, since the qualitative result on the optimal monetary instrument is not affected, as shown in Appendix II. Secondly, the variances of demand and supply shocks in the two countries are respectively assumed to be equal (i.e. $\sigma_w^2 = \sigma_{w^*}^2$ and $\sigma_u^2 = \sigma_{u^*}^2$). This assumption does not seem very strong and is in line with the symmetry of the models where the two countries have equal size and economic structure.

Under these assumptions, making use of equations (14) and (14'), the objective function can be written as:

$$\Phi' = -(1+\theta)W - \theta\Omega' \quad (16)$$

where

$$W = \Gamma_2^1 \sigma_{v^*}^2 + \Gamma_2^2 \sigma_w^2 + \Gamma_3^2 \sigma_{w^*}^2 + \Gamma_4^2 (\sigma_u^2 + \sigma_{u^*}^2) + B^{*2} T$$

$$\Omega' = (B'^2 - B^{*2}) T$$

$$T = \left[(1+\beta)^2 (\sigma_w^2 + \sigma_{w^*}^2) + \sigma_u^2 + \sigma_{u^*}^2 \right] / \Delta_3^2$$

The optimal monetary instrument \hat{k}^* is defined by:

$$\partial\Phi' / \partial k^* = (1+\theta) \partial W / \partial k^* + \theta \partial \Omega' / \partial k^* = 0$$

Implicit differentiation can be again used to analyse the response of the optimal monetary instrument to an increase in the concern of the A-country to the stabilisation objectives of the NA-country:

$$\text{sign} \left[\partial \hat{k}^* / \partial \theta \right] = \text{sign} \left[-\partial \Omega' / \partial k^* \right]$$

and

$$\partial \Omega' / \partial k^* = \partial (B'^2 - B^{*2}) T / \partial k^* = -[4\varphi(\gamma + 2\lambda - 1)(\varphi + \vartheta_1 \gamma)] / \Delta_2^2 T < 0$$

The above expression mirrors the result obtained in the simplified model, confirming the validity of the insight it provides. As the shocks in the real exchange rate impinge on the two countries in a different way, the choice of the monetary instrument affects the allocation of the disturbances between the two countries. By moving the monetary instrument towards interest pegging, the A-country accommodates part of the shocks in the real exchange rate and by doing so alleviates the instability burden to be borne by the NA-country. Thus, an increase in the concern with area-wide stabilisation objectives entails a tendency to shift towards a less strict control of the money supply.

Moving to the analysis of the response of the optimal monetary instrument to changes in the intensity of demand and supply shocks, the results of the previous section (which replicated the classic findings by Poole, 1970) are confirmed. The only exception is for the response of \hat{k}^* to an increase in the variance of supply shocks, which has an ambiguous direction. The ambiguity is not surprising, given that equation (16) is valid under the maintained assumption that the variance of demand and supply shocks is equal across countries and that, as discussed in page 9, $\partial \hat{k}^* / \partial \sigma_w^2$ and $\partial \hat{k}^* / \partial \sigma_{w^*}^2$ have opposite sign.

In a model where consumers spend their income on both home and foreign goods, the above arguments can be given an interpretation also in terms of economic integration. So far, the shift of the optimal monetary instrument towards interest rate pegging has been

discussed with reference to a possible increase in the attention to area-wide considerations on the part of the A-country. However, the same effect will hold, through the same channels, as an implication of greater trade integration: when the proportion of country-A income which is spent on the foreign good increases, its price has a larger weight in the price index of the A-country. Therefore, if monetary authorities aim at stabilising the consumer price level, they will be more concerned with the stability of the prices of the goods produced in the NA-country and thus will change the optimal monetary instrument in the same direction as if they increased their concern with the stabilisation objectives of the NA-country. The analytical argument follows the previous lines. Let the objective function be⁹:

$$\Phi'' = -\lambda^2 \text{Var}(p^*) - (1-\lambda)^2 \text{Var}(p) \quad (18)$$

Substituting the expressions for the variances and using the notation of equation (16), the above can be rewritten as:

$$\Phi'' = -[\lambda^2 + (1-\lambda)^2] \mathcal{W} - (1-\lambda)^2 \Omega'$$

As shown in Appendix II, neglecting the second order effects, the direction of the response of the optimal monetary instrument to a change in λ , the parameter denoting trade integration, will be given by:

$$\text{sign} \left[\frac{\partial \hat{k}^*}{\partial \lambda} \right] = \text{sign} \left[\frac{\partial \Omega'}{\partial k^*} \right]$$

An increase in trade integration - i.e. a decrease in λ , in the proportion of income spent on domestic goods - has the same qualitative effects on the choice of monetary instrument as an increase in the attention paid to area-wide stabilisation objective. This finding can be viewed as an extension of the traditional result that international financial integration tends to undermine the effectiveness of money stock targeting, as is normally associated with an increase in the variance of monetary shocks. Thus, international integration, both in the financial and economic sphere, move the optimal monetary instrument in the direction of interest rate pegging.

IV. AN AREA-WIDE MONETARY INSTRUMENT

So far, the analysis has focused on a national monetary instrument, i.e. on the class of rules which may regulate the money supply in the A-country. This section, instead, investigates the implications of shifting monetary control at the area-level, that is, of adopting a monetary instrument which is defined in terms of the money supply or the interest rate (or a

⁹ Making use of the condition $B' - B^{*'} = 1$, simple but tedious algebra can show that the argument is unchanged if the objective function is defined as $-\text{Var}(q^*)$.

combination of the two) for the area as a whole.¹⁰ The move to an area-wide instrument, however, should not be misinterpreted as being equivalent for a monetary union between the two countries. Even if exchange rates are fixed and the money supply rule is defined for the area as a whole, agents might still have distinct preferences for the two currencies. The difference in the transaction services offered by the two monies, perhaps due to legal provisions which force the settlement in national currency,¹¹ might be a plausible motivation underlying the existence of separate demand for money in the two countries. As a result, for any given monetary instrument at the area level, the composition of the area-wide money supply has to be suitably adjusted, if the stability of the exchange rate is to be maintained.

For analytical simplicity the basic, one-good framework developed in Sections I and II is also adopted here. No loss of generality is however entailed because, as it will become apparent, the same line of arguments holds for the model which explicitly acknowledges the imperfect substitutability in the goods market. Therefore, let us consider a model composed of equations (1), (2) and (3), of their counterparts for the A-country, and of equations (4) (interest parity) and (5) (stochastic version of PPP). The money supply is defined by the following area-wide combination policy, which encompasses a monetary stock target for the whole area and an interest rate pegging strategy at the area level as special cases when $\bar{k}' = 0$ and $1/\bar{k}' = 0$, respectively:

$$(m_t + m_t^*)^s = m_t + m_t^* + \bar{k}'(i_t + i_t^*) \quad \bar{k}' \geq 0 \quad (6')$$

The supply of money of the NA-country is again determined so as to maintain exchange rate stability through the accommodation of the shifts in the desired composition of the area-wide money stock which stem from the random disturbances hitting the two economies.

The solution for the price level of the two countries for the model defined above is given by:

$$p_t = \Xi_1(m_t^* + m_t - v_t^* - v_t) + \Xi_2 w_t + \Xi_3 w_t^* + \Xi_4(u_t + u_t^*) + B'' \mu_t \quad (19)$$

$$p_t^* = \Xi_1(m_t^* + m_t - v_t^* - v_t) + \Xi_2 w_t + \Xi_3 w_t^* + \Xi_4(u_t + u_t^*) + B^{*''} \mu_t \quad (19')$$

where¹²:

$$\begin{aligned} \Xi_1 &= \Gamma_1 & \Xi_4 &= \Gamma_4 \\ \Xi_2 &= \Xi_3 = \Gamma_3 & B &= -B^{*''} = 1/2 \end{aligned}$$

¹⁰ A discussion of the institutional arrangements required to implement the adoption of an area-wide monetary instrument lies beyond the scope of this paper.

¹¹ Legal provisions forcing the use of domestic currencies, e.g. for tax payments, are put forward as factor limiting currency substitution for example in Giovannini (1991).

¹² Naturally, in the Ξ_i 's $\bar{k} = \bar{k}' + \alpha$ replaces k^* , which instead enters the Γ_i 's.

These equations have a number of noteworthy implications. Firstly, shocks in the money demand of the NA-country enter the solution for both price levels. As the monetary instrument is defined at the area level, the endogeneity of m_t , although necessary to ensure exchange rate stability, is no longer sufficient to absorb the financial shocks hitting the NA-country fully. In fact, m_t will always accommodate random changes in the desired composition of the area-wide money stock but will not necessarily prevent these changes from having a spillover effect on the total demand for money in the area.

Secondly, financial shocks affecting the two economies enter the price solution in additive form. Consequently, if the assumption that financial shocks are independent is maintained, an area-wide monetary instrument will always be inferior to a national monetary instrument,¹³ as the effects of the financial shocks are compounded. In fact, an area-wide monetary instrument is preferable to a national monetary instrument if:

$$Var(v_t + v_t^*) < Min[\sigma_v^2, \sigma_{v^*}^2]$$

To be satisfied, this condition requires the presence of a (sufficiently large) negative correlation between the financial shocks which affect the two countries, as is the case if currency substitution is an important feature of the economic environment.

In the simplified framework of the present model, the negative correlation between shocks to national money demands is an essential ingredient to motivate the resort to an area-wide monetary instrument. As argued by Kremens and Lane (1992), also other factors - such as the reduction of the specification bias affecting econometric estimates - can lead to an area-wide money demand function which is more stable than its national counterparts, providing the rationale for the adoption of a monetary instrument at the area level. The initial attempts to estimate an area-wide money demand equation for the ERM countries have provided encouraging results, surveyed in detail by Van Riet (1993).

Thirdly, the coefficients for w_t and w_t^* are equal. Thus, the difference in the impact of supply shocks according to the country they hit which characterised the previous models now disappears.¹⁴ The adoption of an area-wide monetary instrument in fact implies that supply shocks feed through the interest rate elasticity of the aggregate demand for goods irrespective of the country where they occur. Instead, under an exchange rate agreement, this channel of transmission of supply shocks is usually effective only when such disturbances

¹³ The only exception occurs when financial shocks are the only source of disturbance for the two economies and, therefore, interest pegging is the optimal monetary strategy. In this case, under an exchange rate agreement, the national and the area-wide monetary instrument coincide, as $\Xi_1 = \Gamma_1 = 0$, since $1/\bar{k} = 1/k^* = 0$.

¹⁴ If the real exchange rate is endogenously determined, as in the model of section III, this consideration only applies to the "symmetrical" impact of supply shocks on price and income levels, i.e. to the component which abstracts from the change in μ_t induced by asymmetric supply shocks.

affect the A-country, i.e. the country whose monetary instrument is not employed to maintain exchange rate stability. Only in this case, in fact, the impact on the demand for money stemming from supply shocks can have an additional effect on the interest rate (unless this is pegged, when $1/k^* = 0$) which in turn affects aggregate demand.

Fourthly, the coefficient for the disturbances to the real exchange rate is the same in the two equations and is independent of the choice of the monetary instrument. With an area-wide monetary instrument, monetary policy is unable, under an exchange rate agreement, to offset shocks to the real exchange rate in either of the two countries and hence it cannot influence the variability of the price levels due to asymmetric shocks, whatever their source. The impotence of monetary policy to cope with asymmetric shocks leads to a perfect coincidence of the stabilisation objectives of the two countries (unless they attach different weights to output and price stabilisation) and thus the choice of the monetary instrument is invariant for the intensity of the concern of the anchor-country with area-wide stabilisation objectives. In the formal terms of the previous Sections: $\frac{\partial \hat{k}}{\partial \theta} = 0$.

This conclusion offers a different point of view to conjecture about the possible future developments in the conduct of monetary policy in the ERM. If financial integration within the Community were to accelerate significantly and to make an area-wide money demand significantly more stable than the money demand of the anchor country, then it would be optimal for all the participants to the exchange rate agreement to move from a national to an area-wide monetary instrument, since the stabilisation performance of monetary policy would be enhanced for every country.¹⁵ In this scenario, if the objectives of monetary policy are confined to price stability, no conflict between national stabilisations objective could possibly arise and the asymmetric features of the ERM would *ipso facto* disappear as a result of developments in financial markets. This scenario is also implicitly assuming that shocks to the real exchange rate are of relatively minor importance. If this were not the case, then it would not be so much the selection of an area-wide versus a national monetary instrument to be affected, but rather the choice to maintain an exchange rate agreement. As the analysis has shown, under an exchange rate agreement monetary policy cannot offset the impact of shocks to the real exchange rate at all (with an area-wide monetary instrument) or in either country without exporting instability to the other (with a national monetary instrument).

As a final remark, it may be worth noting that, as can be expected, the traditional results on the response of the optimal monetary instrument to a change in the intensity of the different types of disturbances are confirmed for the area-wide instrument. Financial shocks (which are not offset within the area) call for a shift towards interest rate pegging, while

¹⁵ The adoption of interest rate pegging could insulate the anchor country from financial shocks. Yet, this choice of monetary instrument would exacerbate the impact of real shocks which, although assumed to be less important in this scenario, might well be non-negligible.

shocks in the demand and supply¹⁶ of goods require a move in the direction of interest rate targeting.

V. CONCLUSIONS

The main conclusions of the paper can be summarised as follows:

1. If shocks to the real exchange rate are negligible, the stabilisation objectives of the countries participating to an exchange rate agreement coincide exactly. In this case, it is equally beneficial to all participants if the anchor country is the one less subject to financial shocks. By pegging their exchange rate vis-à-vis the anchor, the other countries automatically accommodate money demand disturbances, thereby preventing them from destabilising the whole area. Turning to the response of the optimal monetary instrument to changes in the origin of the shocks, the traditional results of Poole (1970) are confirmed in the two-country framework developed in the paper.

2. If shocks to the real exchange rate represent an important feature of the economic environment, the choice of the monetary instrument on the part of the anchor country affects the allocation of the impact of the shocks between the countries. As a result, the choice will depend on the importance attached to area-wide stabilisation objectives by the anchor country. In particular, an increase in the concern with price stability at the area level entails a shift of the monetary instrument in the direction of interest rate pegging.

3. An increase in economic integration between countries is shown to have the same qualitative implications on the choice of the monetary instrument as an increase in the importance attached to area-wide considerations. Hence, international integration, both in the financial and in the trade sphere, change the optimal monetary instruments in the direction of interest rate pegging.

4. When financial shocks are negatively correlated across the two economies (as is the case in the presence of significant phenomena of currency substitution), it is in the interest of both countries to adopt an area-wide monetary instrument, i.e. to regulate the sum of the supply of money in the two countries, leaving endogenous its currency composition to maintain exchange rate stability. Monetary policy however loses the power to offset shocks to the real exchange rate, which then affect the two countries with equal intensity, irrespectively of their origin. As a result, the stabilisation objectives of the two countries coincide and hence the importance attached to area-wide stabilisation objectives of the anchor country becomes immaterial to the choice of the optimal area-wide monetary instrument.

The derivation of policy prescriptions from the theoretical model of the paper is, as usual, conditional on the simplifying assumptions entertained. Bearing this general caveat

¹⁶ Again the condition $2\varphi + \vartheta_1(1 - \gamma) > 0$ is needed for this result.

in mind, the present analysis suggests that the anchor country of the ERM should interpret monetary targets more flexibly (i.e. move its monetary instrument in the direction of interest rate pegging) as a result of the current increase in the disturbances affecting its money demand as well of the prospective increase in trade integration within the Community and of the greater emphasis on Community-wide considerations which is possibly associated with the progress towards EMU. Developments in the financial sphere, especially if they entail significant phenomena of currency substitution, might however push towards the adoption of monetary control at the area level.

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APPENDIX I

1. Proof that $\partial \hat{k}^*/\partial \theta > 0$

Implicit differentiation of equation (12) in the text yields:

$$\partial \hat{k}^*/\partial \theta = -\left[\frac{\partial^2 \Phi / \partial k^* \partial \theta}{\partial^2 \Phi / \partial k^{*2}}\right]$$

As equation (12) identifies the maximum of the objective function $\partial^2 \Phi / \partial k^{*2} < 0$ and hence

$$\text{sign} \left[\partial \hat{k}^*/\partial \theta \right] = \text{sign} \left[\partial^2 \Phi / \partial k^* \partial \theta \right]$$

By making use of equation (12), it can be easily shown that:

$$\partial^2 \Phi / \partial k^* \partial \theta = -\left[1/(1+\theta)\right] \partial \Omega / \partial k^*$$

where Ω is defined by equation (11) in the text. Performing the differentiation and rearranging yields:

$$\partial \Omega / \partial k^* = -\left[4\varphi(1+\gamma)(\varphi + \vartheta_1 \gamma)\right] \sigma_\mu^2 / \Delta_2^2 < 0$$

which proves the claim.

The same procedure can be applied to investigate the response of the optimal monetary instrument to changes in the variances of shocks.

APPENDIX II

1. Proof that defining the objective function in terms of general prices indexes does not change the qualitative result

Define the objective function as:

$$\Phi''' = -\text{Var}(q^*) - \theta \text{Var}(q)$$

Substituting the expressions for q and q^* and making use of the notation of equation (16) in the text, the objective function can be rewritten as:

$$\Phi''' = -(1+\theta)W' - \left[\theta\lambda^2 + (1-\lambda)^2\right] \Omega'$$

$$\text{where } W' = W + [2\lambda(1-\lambda)B^*]T \text{ (use is made of } B' - B^{*'} = 1).$$

Taking the cross-derivative of the above function yields:

$$\partial^2 \Phi''' / \partial k^* \partial \theta = (1+\theta) \partial W' / \partial k^* + \left[\theta\lambda^2 + (1-\lambda)^2\right] \partial \Omega' / \partial k^*$$

Substituting in the above equation the expression for $\partial W' / \partial k^*$ obtained from the first order condition $\partial \Phi''' / \partial k^* = 0$ yields

$$\partial^2 \Phi''' / \partial k^* \partial \theta = [1/(1+\theta)] [\lambda^2 - (1-\lambda)^2] \partial \Omega' / \partial k^*$$

Hence, as long as $\lambda > 1/2$

$$\text{sign} [\partial^2 \Phi''' / \partial k^* \partial \theta] = \text{sign} [\partial \Omega' / \partial k^*]$$

2. Proof that, if second order effects are neglected, $\text{sign} [\partial \hat{k}^* / \partial \theta] = \text{sign} [\partial \hat{k}^* / \partial \lambda]$

The cross derivative of the objective function, rearranged as $\Phi'' = -[\lambda^2 + (1-\lambda)^2] \mathcal{W} - (1-\lambda)^2 \Omega'$, is given by:

$$\begin{aligned} \partial^2 \Phi'' / \partial k^* \partial \lambda &= 2[(2\lambda - 1) \partial \mathcal{W} / \partial k^* - (1-\lambda) \partial \Omega' / \partial k^*] + \\ &\left\{ [\lambda^2 + (1-\lambda)^2] \partial^2 \mathcal{W} / \partial k^* \partial \lambda + (1-\lambda)^2 \partial^2 \Omega' / \partial k^* \partial \lambda \right\} \end{aligned}$$

If the second order terms in curly brackets are neglected and the expression for $\partial \mathcal{W} / \partial k^*$ obtained from the first order condition $\partial \Phi''' / \partial k^* = 0$, the above expression becomes:

$$\partial^2 \Phi'' / \partial k^* \partial \lambda = -2 \left\{ [(2\lambda - 1)(1-\lambda)^2] / [\lambda^2 + (1-\lambda)^2] + (1-\lambda) \right\} \partial \Omega' / \partial k^*$$

Given that the expression in curly brackets is positive as long as $\lambda > 1/2$, resorting to manipulations presented in Appendix I can easily establish the claim.