THE NEW KEYNESIAN CLIMATE MODEL

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ABSTRACT. As climate change accelerates, central banks face two emerging challenges in their price stability mandates. The first is *climateflation*, which captures the inflationary impacts of a warming planet. The second is *greenflation*, which refers to inflationary pressures stemming from the implementation of climate mitigation policies to reach a low-carbon economy. We explore these phenomena and their implications for central bank policy making. To this end, this paper develops and estimates a nonlinear four-equation New Keynesian Climate (NKC) model for the world economy featuring climate change damage and mitigation policy. We use this model to analyze various transition scenarios and their implications for economic activity, inflation, and monetary policy.

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1 Introduction

As climate change accelerates, central banks worldwide will be increasingly confronted with unprecedented challenges to their price stability mandates. This paper explores two emerging phenomena, *climateflation* and *greenflation*, and their implications for central bank policy-making. Climateflation denotes broader inflationary impacts resulting from climate-related events, such as extreme weather events, natural disasters, and supply chain disruptions. Conversely, greenflation refers to inflationary pressure stemming from the transition to a low-carbon economy. While the first phenomenon can be compared with a negative supply shock, the second is a combination of demand and supply shocks. Thus, because of climateflation and greenflation, central banks face a delicate balancing act in maintaining price stability while supporting economic resilience.

This paper explores these two phenomena and their implications for central bank policy-making. To this end, we develop and estimate a tractable nonlinear New Keynesian Climate (NKC) model for the world economy featuring climate change damage and mitigation policies. As illustrated in Galí (2015)'s textbook, the New Keynesian framework captures the interplay between aggregate demand and aggregate supply, highlighting the role of inflation, output, and monetary policy in shaping the overall economic landscape. By augmenting the traditional 3-equation model with elements that capture climate externality and abatement costs, we aim to enrich our understanding of how climate change affects the economy. The tractability of this framework makes it possible to analytically decompose the effects of various economic mechanisms, including climateflation and greenflation.

Our first contribution is therefore to bridge the gap between integrated assessment models (IAMs), developed to study carbon mitigation policies from a long-term perspective, and complex dynamic stochastic general equilibrium models, more focused on economic fluctuations. Our New Keynesian Climate (NKC) model keeps the elegance and tractability of the textbook model by incorporating *(i)* a single additional endogenous variable (the stock of carbon), *(ii)* four exogenous trends (population, carbon intensity, abatement efficiency and technological) and, *(iii)* one exogenous carbon tax. Consequently, the model boils down to four equations. The IS curve incorporates green investment spending to reduce carbon emission. The Phillips curve considers the economic damage from rising carbon stocks and the production cost from increasing the carbon tax. The monetary policy rule links the nominal interest rate to the deviation of inflation from its target and the output gap. Finally, the last equation is the law of motion that governs the accumulation of carbon dioxide emissions, which makes it depend on the current flow of production adjusted by abatement efforts. These equations form the basis for analyzing the impact of climate change on key macroeconomic variables and policy responses.

Climate change and its associated mitigation policies have non-linear and long-lasting effects on both the supply and demand sides of the economy and the natural real interest rate. Therefore, the usual practice in the monetary policy literature of analyzing the propagation of small shocks around a balanced growth path is not appropriate. Our second contribution is to use a solution method that accounts for both (i) the structural change from rising carbon emissions and (ii) stochastic fluctuations due to exogenous shocks around the evolving economy. To this end, we first use the extended path solution method from Fair and Taylor (1983) to numerically solve for the stochastic path, which is consistent with our model. In summary, the extended path approach uses a perfect foresight solver to obtain endogenous variables that are path-consistent with model equations. In each period, agents are surprised by the realization of shocks but still expect that in the future, shocks are zero on average, consistent with rational expectations. Second, an inversion filter is used to calculate the likelihood function. By extracting the sequence of innovations recursively through the inversion of observation equations for a given set of initial conditions, this filter has recently emerged as a computationally efficient alternative (Guerrieri and Iacoviello, 2017). Finally, using Bayesian techniques as in Smets and Wouters (2003), we estimate the structural parameters using four World's macroeconomic and climate-related time series from 1985Q1 to 2023Q2. Together, these methods allow us to estimate a highly nonlinear model and simulate it under various climate change scenarios.

Our third contribution is to use this estimated model to assess the importance of climateflation and greenflation and the associated monetary policy responses in different climate-change and transition scenarios. The first scenario is a "laissez-faire" economy characterized by an increasing stock of carbon that warms the planet and makes resources scarcer. The increasing damage to total factor productivity acts as a permanent negative supply shock that fuels inflation and drives output below its technological trend. This phenomenon has been referred to as *climateflation* (see e.g., Schnabel, 2022). The second scenario captures the "Paris-Agreement," which requires world governments to implement mitigation policies to reach net-zero carbon emissions by 2050. In our framework, this scenario takes the form of a linear increase in the carbon tax such that full abatement is reached in 2050. The rise in carbon tax forces firms to internalize the effects of their carbon emissions on aggregate productivity. In response, they reduce their emissions by increasing abatement expenditures, creating a demand-driven boom. The resulting impact of inflation from higher marginal costs and increased demand is usually referred to as *greenflation*. We use these two scenarios (i) to explore the trade-offs between current abatement efforts and future damages, and (ii) to study their implications for natural output and real interest rate, inflation, and monetary policy responses.

Our paper contributes to the burgeoning literature that focuses on climate issues using microfounded structural models. Fischer and Springborn (2011), Heutel (2012), and Angelopoulos et al. (2013) are among the first contributions to introduce carbon emissions in real business cycle models. They assume that emissions stem from production and adversely impact utility or have a negative impact on productivity and production. More recent contributions have extended these models in several directions, including *(i)* multisector aspects (Golosov et al., 2014a; Dissou and Karnizova, 2016), *(ii)* labor market frictions (Gibson and Heutel, 2020; Finkelstein Shapiro and Metcalf, 2023), *(iii)* distortionary fiscal policy (Barrage, 2020), *(iv)* endogenous entry (Annicchiarico et al., 2018; Finkelstein Shapiro and Metcalf, 2023), or *(v)* nominal rigidities and monetary policy (Annicchiarico and Di Dio, 2015, 2017; Carattini et al., 2021; Diluiso et al., 2021; Ferrari and Landi, 2022; Coenen et al., 2023; Nakov and Thomas, 2023; Del Negro et al., 2023; Ferrari and Nispi Landi, 2024). While these studies provide interesting insights into the role of transition locally, they do not explicitly deal with the nonlinear nature of carbon accumulation and its permanent effects on the economy. In contrast, we consider long-run trends in carbon emissions and macroeconomic variables in the spirit of Jondeau et al. (2023), which makes our framework well-suited for studying the effects of environmental policies.

The remainder of this paper is organized as follows. Section 2 describes the features of the model. Section 3 reports on the data, estimation methodology, parameter estimates, and assesses the fit of the model. Section 4 presents the climate transition scenarios and the anatomy of climateflation and greenflation. Section 5 concludes.

2 The 4 equation New Keynesian climate (NKC) model

Our starting point is the textbook three equation New Keynesian model (Woodford, 2003, Galí, 2015), which includes an IS curve, a Phillips curve, and a Taylor-type rule. To this standard framework, we add a climate dynamics mixing Golosov et al. (2014a) and Nordhaus (1992).

2.1 Household sector The economy is populated by a mass l_t of *ex ante* atomistic, identical, and infinitely lived households. This mass is time-varying and captures the upward trend of population observed over the last sixty years. Formally, it is assumed that the population asymptotically converges to a long-run level $l_T > 0$, such as $l_t = l_{t-1} \left(l_T / l_{t-1} \right)^{\ell_g}$, with $\ell_g \in [0,1]$ being the geometric rate of convergence to l_T. Each household indexed by $i \in [0, l_t]$ maximizes its sequence of present and future utility flows that depend positively on consumption $c_{i,t}$ and negatively on labor $n_{i,t}$:

$$\mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \tilde{\beta}_{t,t+s} \varepsilon_{b,t+s} \left(\frac{c_{i,t+s}^{1-\sigma_c} - 1}{1-\sigma_c} - \psi_{t+s} \frac{n_{i,t+s}^{1+\sigma_n}}{1+\sigma_n} \right) \right\}, \tag{1}$$

where $\mathbb{E}_t$ denotes the expectation conditional upon information available at t, $\tilde{\beta}_{t,t+s}$ is the technological-neutral discount factor,[1] $\sigma_c > 0$ is the inverse of the intertemporal elasticity of substitution in consumption, $\sigma_n > 0$ is the inverse of the Frisch labor supply elasticity, and ψ_t is a scale variable pinning down hours worked in balanced growth path.[2] In addition, $\varepsilon_{b,t}$ is a preference shock that captures unexpected changes in aggregate demand. It follows an AR(1) process: $\varepsilon_{b,t} = (1-\rho_b) + \rho_c \varepsilon_{b,t-1} + \eta_{b,t}$, with $\eta_{b,t} \sim \mathcal{N}(0, \sigma_b^2)$.

As in McKay et al. (2017), households are endowed with stochastic idiosyncratic employment status $\varsigma_{i,t} \in \{0,1\}$, with 0 meaning low productivity (denominated "type L" worker) and 1 high productivity (denominated "type H" worker). The level of productivity is drawn i.i.d. with probabilities $\Pr(\varsigma_{i,t} = 0) = \omega$ and $\Pr(\varsigma_{i,t} = 1) = 1 - \omega$. The sequence of real

[1] The presence of a permanent increase in technology affects the Euler equation, and consequently the neutral equilibrium rate and the monetary policy rule. To keep the framework tractable, we mute the effect of technology on long-run equilibrium rate by imposing: $\tilde{\beta}_{t,t+s} = \beta(z_{t+s}/z_t)^{\sigma_c}$ with $\beta \in (0,1)$. Note that this assumption is standard in models featuring recursive utility functions such as Epstein-Zin for instance.

[2] Note that ψ_t must grow proportionally to the flow of current consumption. Thus, if z_t denotes the trend in per capita consumption, then $\psi_t = \psi z_t^{1-\sigma_c}$, with ψ as a scale parameter.

budget constraints for each type of households reads as:

$$c_{i,t} + b_{i,t} + T^s = \frac{r_{t-1}}{\pi_t} b_{i,t-1} + \Pi_{i,t} + w_t n_{i,t} + T^e_{i,t}, \text{ if } \varsigma_{i,t} = 1, \tag{2}$$

$$c_{i,t} + b_{i,t} = \frac{r_{t-1}}{\pi_t} b_{i,t-1} + d, \text{ if } \varsigma_{i,t} = 0, \tag{3}$$

where variable $b_{i,t}$ is the one-period riskless bond, r_t is the gross nominal interest rate on bonds, $\pi_t = p_t/p_{t-1}$ is gross inflation with p_t is the price index, $\Pi_{i,t}$ are real dividend payments received from holding shares of firms, w_t is the aggregate real wage, and $T^e_{i,t}$ represents the revenues of the carbon tax redistributed through lum-sum transfers. Low-productivity households receive d units of the consumption good as a transfer, and high-productivity households pay a tax of $\omega z_t d/(1-\omega)$ to finance the transfer. This transfer is assumed to grow proportionally to productivity z_t.

The Euler equation associated with the problem of the household's i of productivity type $q \in \{H, L\}$ is thus given by:

$$\varepsilon_{b,t} c_{i,q,t}^{-\sigma_c} \geq \mathbb{E}_t \left\{ \frac{\tilde{\beta}_{t,t+1} \varepsilon_{b,t+1} r_t}{\pi_{t+1}} \left((1-\omega) \, c_{i,H,t+1}^{-\sigma_c} + \omega c_{i,L,t+1}^{-\sigma_c} \right) \right\}, \tag{4}$$

where $c_{i,H,t}$ and $c_{i,L,t}$ denote consumptions for high and low productive households, respectively.

2.2 Business sector The business sector is characterized by final good producers who sell a homogeneous final good to households and the government. To produce, they buy and pack differentiated varieties produced by atomistic and infinitely lived intermediate good firms that operate in a monopolistically competitive market. Intermediate good firms contribute to climate change by emitting CO_2 as an unintended result of their production process.

2.2.1 Final good sector At every point in time t, a perfectly competitive sector produces a final good Y_t by combining a continuum of intermediate goods $y_{j,t}$, $j \in [0, l_t]$, according to the technology $y_t = \left[l_t^{-1/\zeta} \int_0^{l_t} y_{j,t}^{\frac{\zeta-1}{\zeta}} \mathrm{d}j \right]^{\frac{\zeta}{\zeta-1}}$. The number of intermediate good firms owned by households is equal to the size of the population L_t. Parameter $\zeta > 1$ measures the substitutability across differentiated intermediate goods. Final good producing firms take their output price, p_t, and their input prices, $p_{i,t}$, as given and beyond their control. Profit maximization implies the demand curve $y_{j,t} = l_t^{-1} \left(p_{j,t}/p_t \right)^{-\zeta} y_t$, from which we deduce the relationship between the price of the final good and the prices of intermediate goods $p_t \equiv \left[l_t^{-1} \int_0^{L_t} p_{j,t}{}^{1-\zeta} \mathrm{d}j \right]^{\frac{1}{1-\zeta}}$.

2.2.2 *Intermediate goods sector*

Intermediate good j is produced by a monopolistic firm using the following production function:

$$y_{j,t} = \Gamma_t \left(n_{j,t}^d \right)^\alpha , \tag{5}$$

where Γ_t is the total factor productivity (TFP) that affects the labor demand $n_{j,t}^d$, with intensity $\alpha \in [0, 1]$.

The TFP is actually determined by two components:

$$\Gamma_t = z_t \Phi \left(m_t \right) , \tag{6}$$

where z_t is the deterministic component of productivity and $\Phi(m_t)$ is a damage function that represents the impact of climate change on the production process. The deterministic component of TFP follows the process $z_t = z_{t-1}(1 + g_{z,t})$, where $g_{z,t} = g_{z,t-1} \left(1 - \delta_z\right)$ is the productivity growth rate, δ_z is the rate of decline in productivity. This formulation indicates that productivity growth decreases over time by a factor δ_z to match the observed slowdown in economic growth over the last sixty years.

Finally, following Golosov et al. (2014a), we assume an exponential damage function:[3]

$$\Phi(m_t) = \exp(-\gamma(m_t - m_{1750}))$$

where $m_t - m_{1750}$ is the excess carbon in the atmosphere net of its (natural) removal, with m_{1750} representing the stock of carbon in the preindustrial era, i.e., the steady-state level in the absence of anthropogenic emissions. The law of motion of the atmospheric loading of CO_2 (in gigatons of CO_2) is given by:

$$m_t - m_{1750} = (1 - \delta_m)(m_{t-1} - m_{1750}) + \xi_m e_t, \tag{7}$$

where e_t denotes the anthropogenic carbon emissions in t, $\delta_M \in [0, 1]$ represents the rate of transfer of atmospheric carbon to the deep ocean, and $\xi_m \geq 0$ is the atmospheric retention ratio.

[3] This function approximates the damage function generally used by the DICE literature and that depends on the atmospheric temperature.

A firm's CO_2 emissions stemming from the production process are denoted by $e_{i,t}$. As they are subject to a carbon tax $\tau_{e,t}$, which aims at internalizing the social cost of carbon emissions, the firm is incentivized to reduce its impact by investing in an emission abatement technology. The abatement effort by the firm yields a reduction by $\mu_{i,t}$ (in %) in its CO_2 emissions. A firm's emissions take the following form:

$$e_{j,t} = \sigma_t \left(1 - \mu_{j,t}\right) y_{j,t}\varepsilon_{e,t},$$

where σ_t denotes the aggregate carbon intensity of the production sector. Its law of motion is $\sigma_t = \sigma_{t-1}(1 - g_{\sigma,t})$, where $g_{\sigma,t}$ captures possible changes in the decrease of carbon decoupling rate. This changes follows $g_{\sigma,t} = \left(1 - \delta_\sigma\right) g_{\sigma,t-1}$, where $\delta_\sigma \in [0,1]$ is the rate of decline of the trend. This trend is set to match the decline in the emissions-to-GDP ratio observed over the last sixty years. Last, the firm's carbon intensity can be temporarily affected by an aggregate exogenous emissions shock, $\varepsilon_{e,t} = (1 - \rho_e) + \rho_e \varepsilon_{e,t-1} + \eta_{e,t}$, with $\eta_{e,t} \sim N(0, \sigma_e^2)$, which captures the cyclical changes in the emissions-to-output ratio. A rise in $\varepsilon_{e,t}$ induces a cyclical increase in the carbon intensity of the production sector.

Firms may substitute carbon-intensive technologies with low-carbon technologies, but this change in the existing lines of production is costly. We assume that the cost of abatement technology (in proportion to output) is given by:

$$\mathcal{C}^a_{j,t} = \theta_{1,t}\mu_{j,t}^{\theta_2} y_{j,t}, \tag{8}$$

where $\theta_{1,t} = (p_b/\theta_2)(1 - \delta_{pb})^{t-t_0}\sigma_t$ is the time-varying level of the cost of abatement, $p_b > 0$ is a parameter determining the initial cost of abatement and $0 < \delta_{pb} < 1$ captures technological progress, which lowers the cost of abatement by a factor δ_{pb} each year. Finally, $\theta_2 > 0$ represents the curvature of the abatement cost function, which typically exhibits increasing returns in IAM's literature.

Intermediate goods producers solve an usual two-stage problem. In the first stage, taken the input price w_t as given, firms seek to maximize their one-period profits:

$$\max_{\{y_{j,t},\mu_{j,t}\}} mc_{j,t} y_{j,t} - w_t \left(\frac{y_{j,t}}{\Gamma_t}\right)^{1/\alpha} - \mathcal{C}^a_{j,t} - \tau_{e,t}\sigma_t \left(1 - \mu_{j,t}\right) y_{j,t}\varepsilon_{e,t} \tag{9}$$

where $mc_{i,t}$ denotes the real marginal cost of producing one additional good.

In the second stage, firms decide their selling price under a Rotemberg price setting. The Rotemberg price adjustment cost is given by:

$$\mathcal{C}_{j,t}^{p} = \frac{\kappa}{2}\left(\frac{p_{j,t}}{p_{j,t-1}} - \pi_t^{\star}\right)^2 \frac{y_t}{l_t} \tag{10}$$

where $\kappa > 0$ is the price stickiness parameter, y_t/l_t is the average market share per firm, and $\pi_t^{\star}$ is the gross inflation target, which follows the following deterministic process (Ireland, 2007, Fève et al., 2010, Del Negro et al., 2015):

$$\pi_t^{\star} = (1-\rho_{\pi^{\star}})\,\pi + \rho_{\pi^{\star}}\pi_{t-1}^{\star}, \tag{11}$$

where $\rho_{\pi^{\star}}$ is the autocorrelation coefficient that reflects the slow pace at which monetary authorities allegedly adjusted its inflation target, and π is the steady-state gross inflation.

To attenuate the expectation channel of inflation, an exogenous exit shock is introduced consistently with empirical evidence on the survival rate of firms across time (OECD, 2017). As in Bilbiie et al. (2012), we assume a "death" shock, which occurs with probability $\vartheta \in (0,1)$ in every period. This means that the profit of each firm is subject to an idiosyncratic shock $\omega_{j,t}$ that takes the value 0 for the fraction of firms exiting the market. Thus, the problem faced by firms can be written as follows:

$$\max_{\{p_{j,t}\}} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \omega_{j,t+s} \left(y_{j,t+s}\frac{p_{j,t+s}}{p_{t+s}} - \varepsilon_{p,t+s} mc_{t+s} y_{j,t+s} - \mathcal{C}_{j,t+s}^{p} \right) \right\}, \tag{12}$$

subject to demand $y_{j,t} = l_t^{-1}\left(p_{j,t}/p_t\right)^{-\zeta} y_t$. $\varepsilon_{p,t}$ is a cost-push shock that follows an AR(1) process: $\varepsilon_{p,t} = (1-\rho_p) + \rho_p \varepsilon_{p,t-1} + \eta_{p,t}$, with $\eta_{p,t} \sim \mathcal{N}(0,\sigma_p^2)$.

Because all intermediate goods firms face an identical profit maximization problem, they chose the same price $p_{j,t} = p_t$. In a symmetric equilibrium, where $\int_0^{l_t} = \omega_{j,t+1}/\omega_{j,t} \mathrm{d}f(\omega_{j,t}) \simeq 1-\vartheta$, the optimal pricing rule implies:

$$\kappa\left(\pi_t - \pi_t^{\star}\right)\pi_t = (1-\vartheta)\beta\kappa\mathbb{E}_t\left\{\left(\pi_{t+1} - \pi_{t+1}^{\star}\right)\pi_{t+1}\frac{y_{t+1}}{y_t}\frac{l_t}{l_{t+1}}\right\} + \zeta\varepsilon_{p,t} mc_t + (1-\zeta). \tag{13}$$

The above equation is the New Keynesian Phillips curve, relating current inflation to the discounted sum of marginal costs.

2.3 Public sector The government issues short-term bonds, collects revenues from the carbon tax and redistributes them entirely to households on a lump-sum basis:

$$\int_0^{l_t} b_{i,t}\mathrm{d}i + +\tau_{e,t}\int_0^{l_t} e_{j,t}\mathrm{d}j = \frac{r_{t-1}}{\pi_t}\int_0^{l_t} b_{i,t-1}\mathrm{d}i + \int_0^{l_t} \Pr\left(z_{i,t}=1\right) T^e_{i,t}\mathrm{d}i. \tag{14}$$

The monetary policy authority follows a Taylor-type rule by gradually adjusting the nominal interest rate in response to *(i)* the inflation gap and *(ii)* the output gap:

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\rho_r}\left[\left(\frac{\pi_t^\star}{\pi}\right)\left(\frac{\pi_t}{\pi_t^\star}\right)^{\phi_\pi}\left(\frac{y_t}{y_t^n}\right)^{\phi_y}\right]^{1-\rho_r}\left(\frac{\pi_t^\star}{\pi_{t-1}^\star}\right)^{\phi_{\pi^\star}}\varepsilon_{r,t}, \tag{15}$$

where r is the long-run nominal interest rate, y_t^n is the natural output and the parameters ρ_r, ϕ_π, ϕ_y captures the degree of interest-rate smoothing, and the responsiveness of the policy rate to the inflation and output gaps, respectively. In addition, we allow the nominal interest rate to react to changes in the inflation target, with sensitivity parameter $\phi_{\pi^\star}$. This allows us to separate the consequences of monetary policy inertia from those of gradual disinflation. Finally, $\varepsilon_{r,t}$ is a monetary policy shock that follows the following process: $\varepsilon_{r,t} = (1-\rho_r) + \rho_r\varepsilon_{r,t-1} + \eta_{r,t}$, with $\eta_{r,t} \sim N(0,\sigma_r^2)$,

2.4 Aggregation Let us first aggregate consumption over the two types of households:

$$l_t c_t = \int_0^{l_t} \Pr\left(z_{i,t}=0\right) c_{L,t}\mathrm{d}i + \int_0^{l_t} \Pr\left(z_{i,t}=1\right) c_{H,t}\mathrm{d}i, \tag{16}$$

It leads to:

$$c_t = \omega c_{L,t} + (1-\omega)\, c_{H,t}. \tag{17}$$

It is assumed that bonds net supply is zero:

$$\int_0^{l_t} b_{i,t}\mathrm{d}i = 0. \tag{18}$$

As discussed by McKay et al. (2017), as long as $c_{L,t} < c_{H,t}$, then the Euler equation for the low productive worker does not bind to equality as right hand side will always be lower than left hand side. Therefore, let $\lambda_t = \varepsilon_{b,t} c_{H,t}^{-\sigma_c} = \varepsilon_{b,t}\left(\frac{c_t-\omega d}{1-\omega}\right)^{-\sigma_c}$ denote the marginal utility of consumption of high productive households, the *aggregate* Euler equation reads as follows:

$$\lambda_t = \mathbb{E}_t\left\{\frac{\tilde{\beta}_{t,t+1} r_t}{\pi_{t+1}}\left((1-\omega)\lambda_{t+1} + \omega\varepsilon_{b,t+1} d^{-\sigma_c}\right)\right\}. \tag{19}$$

In contrast, the general equilibrium for hours worked reads as:

$$(1-\omega)\, n_t = n_t^d. \tag{20}$$

Finally, the resource constraint is given by:

$$y_t = l_t c_t + \frac{\kappa}{2}\left(\pi_t - \pi_t^\star\right)^2 y_t + \theta_{1,t}\mu_t^{\theta_2} y_t + \vartheta \Pi_t. \tag{21}$$

2.5 Final System The system can be summarized by the following set of four core equations that determine four endogenous variables $\{\tilde{y}_t, r_t, \pi_t, \tilde{m}_t\}$. These variables are respectively the detrended GDP, nominal interest rate, the inflation rate and the atmospheric carbon stock.

- IS curve:

$$\epsilon_{b,t}\left(\frac{x_t\tilde{y}_t - \omega d}{1-\omega}\right)^{-\sigma_c} = \beta\frac{\varepsilon_{b,t+1} r_t}{\pi_{t+1}}\left((1-\omega)\left(\frac{x_{t+1}\tilde{y}_{t+1} - \omega d}{1-\omega}\right)^{-\sigma_c} + \omega d^{-\sigma_c}\right), \tag{22}$$

where $x_t = 1 - (1-\vartheta)\frac{\kappa}{2}\left(\pi_t - \pi_t^\star\right)^2 - \theta_{1,t}\tilde{\tau}_{e,t}^{\theta_2/(\theta_2-1)} - \vartheta(1-\epsilon_{p,t} mc_t)$.

- Phillips curve:

$$\left(\pi_t - \pi_t^\star\right)\pi_t = (1-\vartheta)\beta\mathbb{E}_t\left\{(1+g_{z,t+1})\frac{\tilde{y}_{t+1}}{\tilde{y}_t}\left(\pi_{t+1} - \pi_{t+1}^\star\right)\pi_{t+1}\right\} + \frac{\zeta}{\kappa}\varepsilon_{p,t} mc_t + \frac{1-\zeta}{\kappa} \tag{23}$$

where $mc_t = \frac{\psi}{\epsilon_{b,t}(1-\omega)^{\sigma_c+\sigma_n}}\frac{(x_t\tilde{y}_t - \omega d)^{\sigma_c}\tilde{y}_t^{\sigma_n}}{\Phi(\tilde{m}_t)^{1+\sigma_n}} + \tilde{\tau}_{e,t}\theta_{1,t}\left[\theta_2 + \tilde{\tau}_{e,t}^{\frac{1}{\theta_2-1}}(1-\theta_2)\right]$

- Monetary policy rule:

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\rho_r}\left[\left(\frac{\pi_t^\star}{\pi}\right)\left(\frac{\pi_t}{\pi_t^\star}\right)^{\phi_\pi}\left(\frac{y_t}{y_t^n}\right)^{\phi_y}\right]^{1-\rho_r}\left(\frac{\pi_t^\star}{\pi_{t-1}^\star}\right)^{\phi_{\pi^\star}}\varepsilon_{r,t}, \tag{24}$$

- Pollution stock:

$$\tilde{m}_t = (1-\delta_m)\tilde{m}_{t-1} + \xi_m\sigma_t\left(1 - \tilde{\tau}_{e,t}^{\frac{1}{\theta_2-1}}\right)z_t l_t \tilde{y}_t \varepsilon_{e,t} \tag{25}$$

3 Bayesian inference and model evaluation

In this section, we estimate the model using Bayesian methods (see An and Schorfheide, 2007, for an overview). The posterior distribution associated with the vector of observable variables is computed numerically using a Monte Carlo Markov Chain sampling approach. Specifically, we rely on the Metropolis-Hastings algorithm to obtain a random draw of size

20,000 from the posterior distribution of the parameters (8 parallel chains simultaneously draw 2,500 iterations, with a common jump scale parameter to match an acceptance rate of approximately 30%). We first describe how the non-linear model with trends is solved. We then discuss the data retained and our choice of priors and comment on the posterior distribution of the structural parameters.

3.1 Numerical solution method with stochastic growth We consider the *extended path solution method* from Fair and Taylor (1983) and Adjemian and Juillard (2014) to accurately measure non-linear effects of the environmental constraint on growth. In a nutshell, the extended path approach uses a perfect foresight solver to obtain endogenous variables that are path consistent with the model's equations. Each period, agents are surprised by the realization of shocks, but still expect that in the future, shocks are zero on average (consistently with rational expectations). The advantage of this method is that it provides an accurate and fast solution while taking into account all the non-linearities of the model. The drawback of the approach is that the Jensen's inequality binds to equality, which means that the non-linear uncertainty stemming from future shocks is neglected. Note that this drawback also applies to usual linearized DSGE models, such as Smets and Wouters (2007).

Taking non-linear models to the data is a challenge as non-linear filters, which are required to form the likelihood function, are computationally expensive. An inversion filter has recently emerged as a computationally-cheap alternative (e.g., Guerrieri and Iacoviello, 2017, Atkinson et al., 2020). Initially pioneered by Fair and Taylor (1983), this filter extracts the sequence of innovations recursively by inverting the observation equation for a given set of initial conditions. Unlike other filters (e.g., Kalman or particle filters), the inversion filter relies on an analytic characterization of the likelihood function.[4]

The inversion takes place using the perfect foresight solution proposed by Juillard et al. (1996). The standard approach is to compute the dynamics of the variables given current and future shocks. In the extended path context, the inversion filter *(i)* substitutes current shocks and some endogenous variables when applying the perfect foresight solution and *(ii)* computes current shocks and non-observable variables paths given the set of observable variables. Finally, we use the Metropolis-Hastings algorithm as a sampler to draw from the parameter uncertainty.

[4]For a presentation of alternative filters to calculate the likelihood function, see Fernández-Villaverde et al. (2016). See also Cuba-Borda et al. (2019) and Atkinson et al. (2020) for details on the relative gains of the inversion filter.

A perfect foresight algorithm typically requires *(i)* a finite number of periods *(ii)* a terminal period to compute each endogenous variable for realization of economic surprises. To fix notation, this general representation in the presence of extended path takes the form:

$$\tilde{y}_t = g_\Theta\left(y_0, y, 0\right) \tag{26}$$

$$y_t = \mathbb{E}_{t,t+S}\left\{g_\Theta\left(y_{t-1}, \tilde{y}_{t+S+1}, \varepsilon_t\right)\right\} \tag{27}$$

$$\mathcal{Y}_t = h_\Theta\left(y_t\right) \tag{28}$$

$$\varepsilon_t \sim \mathcal{N}\left(0, \Sigma_\varepsilon\right) \tag{29}$$

The first equation determines the deterministic evolution of the endogenous variables in absence of shocks summarized in the vector $\tilde{y}_t$ with initial conditions y_0 and terminal (asymptotic) state y for a given set of nonlinear equations $g_\Theta(\cdot)$. The second equation determines the path of endogenous variables y_t with economic surprise, ε_t is a vector of exogenous stochastic innovations that are normally distributed with mean zero and covariance Σ_ε; Θ is the vector a structural parameters; $h_\Theta(\cdot)$ and $g_\Theta(\cdot)$ are the set of nonlinear equations. $\mathbb{E}_{t,t+S}\{\cdot\}$ is the extended path-consistent expectation operator, that updates expectations over a specific time horizon of size S, and takes as given $\tilde{y}_{t+S+1}$ the terminal period of the expectation. Therefore, the size of the expectation window S must be sufficiently large to ensure that the value of $\tilde{y}_{t+S+1}$ does not affect the outcome.[5] The third equation relates the observations summarized in the vector $\mathcal{Y}_t$ to the endogenous variables in y_t. The last equation concerns the distribution of exogenous innovations.

For each evaluation of the sample likelihood, we first compute the deterministic path providing the transition between the initial period $\{\tilde{y}_t\}_{t=1}^{T}$ and the terminal period. We select a value of $T = 1,000$ to allow a convergence some the terminal state. Formally, we use Equation 26 assuming (*i*) no shock with sequence $\{\varepsilon_t\}_{t=1}^{T}$ is all of zeros, (*ii*) a terminal condition that is the steady state of the model $\tilde{y}_{t+S+1} = y$, which can written as $\tilde{y}_t = g_\Theta\left(y_{t_0}, y, 0\right)$. Next, we use the inversion filter to find the sequence of $\{\varepsilon_t\}_{t=1}^{T^*}$ that matches sample $\{\mathcal{Y}_t\}_{t=1}^{T^*}$ with T^* observations using $\{\tilde{y}_t\}_{t=1}^{T}$ as the terminal value of the expectation window. This implicitly assumes that agents expect the economy to return to its deterministic path $\tilde{y}$ after

[5]One must strike a balance between the length of the expectation window to mimic infinite horizon rational expectations, and the computation burden from updating the expectations. We select an expectation horizon of 40 years ($S = 160$). This length is large enough to ensure that terminal conditions are not quantitatively affecting the numerical value of the likelihood function,but exhibits a moderate computational burden.

S periods. Based on smoothed sequence $\{\varepsilon_t\}_{t=1}^{T^*}$, the likelihood function $\mathcal{L}(\theta, \mathcal{Y}_{1:T^*})$ of the model is obtained, conditional on the matrix of observations through time T^*.

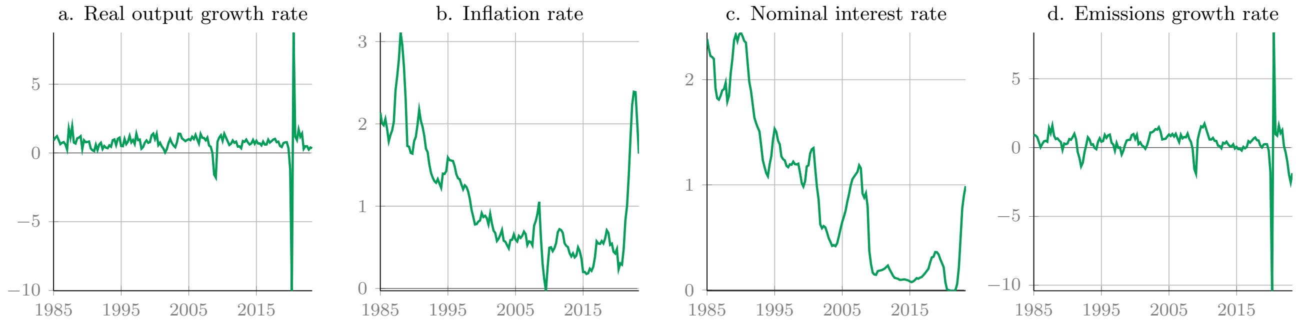
3.2 Data description The model is estimated using worldwide quarterly data from 1985Q1 to 2023Q2. As time series are not available on a quarterly basis for the world, some transformations are necessary. First, the annual GDP in constant 2015 US$ is taken from the *World Bank* (https://data.worldbank.org/indicator/NY.GDP.MKTP.KD), and is converted on a quarterly basis using time disaggregation method of Chow and Lin (1971) using real quarterly GDP for total OECD countries from the *OECD Economic Outlook* database (https://data.oecd.org/gdp/quarterly-gdp.htm).[6] Quarterly headline inflation (https://db.nomics.world/OECD/EO?q=OECD%2FEO%2FOTO.CPI_YTYPCT.Q) and the nominal interest rates (https://db.nomics.world/OECD/EO?dimensions=%7B%22VARIABLE%22%3A%5B%22IRS%22%5D%7D&q=OECD%20economic%20outlook%20interest%20rate) are taken from the *OECD Economic Outlook* database. The aggregate interest rate is a weighted average of the rates over OECD countries. Annual CO_2 emissions, which correspond to the emissions from the burning of fossil fuels for energy and cement production, are from *Our World In Data* (https://ourworldindata.org/explorers/co2?facet=none&country=~OWID_WRL&Gas+or+Warming=CO2&Accounting=Territorial&Fuel+or+Land+Use+Change=All+fossil+emissions&Count=Per+country). We convert annual data into quarterly data using the same disaggregation approach as for GDP.

Our solution method explicitly deals with trends and thus does not impose that variables must return to steady state.[7] Consequently, we simply use the growth rate (i.e., the first difference of the logarithm) for GDP and CO_2 emissions and maintain the level of inflation and the interest rates. Figure 1 displays the temporal evolution of all the observable variables of the model.

[6]This temporal disaggregation technique uses a statistical relationship between low-frequency data and higher-frequency indicator variables. First, regressions are done at the low-frequency level, at this level the target time series and the indicator time series are both available. Second, the resulting estimates are used to obtain the high-frequency target series.

[7]Linearization methods impose to approximate any model's decisions rules around some fixed point, and therefore impose that the model is stationary in the neighbourhood of the fixed point. As a result, inference must be assessed based on stationary data, the latter implies a set of transformations (e.g., dividing by the population, business cycle filters, etc.).

FIGURE 1. Observable variables



The measurement equations mapping our model to the four observable macroeconomic and climate-related time series are given by:

$$\begin{bmatrix} \text{Real output growth rate} \\ \text{Inflation rate} \\ \text{Short-term interest rate} \\ \text{CO}_2 \text{ emissions growth rate} \end{bmatrix} = \begin{bmatrix} \Delta \log (y_t) \\ \pi_t - 1 \\ r_t - 1 \\ \Delta \log (e_t) \end{bmatrix}. \tag{30}$$

3.3 Calibration A first set of parameters are calibrated. These parameters can be divided into two groups: the structural parameters and the initial conditions. We start by discussing the calibration of the structural parameters reported in Table 1.

These parameters are categorized into three panels, the first panel is related to climate dynamics. The parameters are related to carbon law of motion in Equation 7, we assume that carbon lifetime δ_m is set to 150 years (600 quarters). First prototypes of DICE and IAM (e.g. Nordhaus 1992) as well as E-DSGE (e.g. Heutel 2012), typically assumed a short living cycle lying between 80 to 100 years. Recent advances in climate science, summarized in Dietz and Venmans (2019), highlights the presence of a much longer carbon lifetime than previously measured. Because this paper is about mitigation policy in a cost-efficiency perspective, the calibration of this parameter is not critically driving the quantitative outcome. Our assumption of a carbon lifetime of 600 quarters appears rather conservative with respect to the usual economic literature with climate dynamics. The two next parameters are also taken from DICE. Parameters ξ_m simply converts CO2 units into carbon units as follows GtC = ξ_m GtCO2 (as damages are typically measured by the radiative forcing from carbon), while m_{1750} is the natural stock of carbon in the atmosphere back in 1750. The last parameter γ maps carbon stock to the economic damage. Because temperatures and carbon stock are co-integrated

TABLE 1. Calibrated parameter values (quarterly basis)

PARAMETER	NAME	VALUE
Panel A: Climate Parameters		
CO_2 rate of transfer to deep oceans	δ_m	0.00125
Marginal atmospheric retention ratio	ξ_m	0.27273
Pre-industrial stock of carbon (GtC)	m_{1750}	545
Climate damage elasticity	γ	2.379e-05
Initial stock of carbon (GtC)	$m_{1984:4}$	736.98
Panel B: Socio-economic Parameters		
Firm exit shock	ν	0.025
Low productivity worker payoff-to-consumption	d/c	0.85
Initial population (billions)	$l_{1984:4}$	4.85
Terminal population (billions)	l_T	11.42107
Population growth growth	l_g	0.0055
Goods elasticity	ς	6
Decay rate of TFP	$\delta_z \times 400$	0.3
Initial hours worked	$h_{1984:4}$	1
Labor intensity	α	0.7
Initial emissions ($GtCO_2$)	$e_{1984:4}$	5.0825
Initial GDP (trillions USD PPP)	$y_{1984:4}$	11.25
Panel C: Abatement Sector Parameters		
Initial abatement cost	$\theta_{1,1984:4}$	0.30604
Abatement cost	θ_2	2.6
Decay rate of abatement cost	δ_{pb}	0.004277
Initial abatement	$\mu_{1984:4}$	0.0001
Abatement in 2020	$\mu_{2020:1}$	0.05

variables, we follow Golosov et al. (2014b) and assume that damages directly emerges from atmospheric carbon concentration. Parameter γ is set to 2.379e-5 to entail a permanent 3% output loss under the business-as-usual scenario as in DICE.

The next panel concerns the calibration of socio economic parameters. For economic parameters, the exit rate ν is taken from the firm entry literature and actually assumes a 10% which are in line with OECD data document the death rates in manufacturing and services sectors. The inflation target is set to replicate the sample mean. The substitutability of intermediate goods provides a 20% markup that lies usual calibration of macroeconomic model with imperfect competition such as Smets and Wouters (2007). The low productive payoff to consumption is set to 0.85, a value relatively larger than McKay et al. (2017) but is necessary in order to account for the COVID outbreak fall in consumption that would make otherwise the Euler in Equation 4 for the low productive worker to erroneously bind. Regarding socio parameters, these are taken directly from DICE in Nordhaus (2018). In particular, some parameters related to demography are consistent with United Nation population forecasts while the two remaining trend-related parameters are just meant to get a finite terminal state, but play a modest role on the quantitative analysis.

The last set of parameters concerns the abatement sector, and are mainly taken from latest version of DICE in Barrage and Nordhaus (2023). The abatement cost is meant to reach 10.9% of GDP in 2020 in presence of full abatement ($\mu = 1$). As our first date of simulation is much earlier, we retropolate this value back in $t_0 = 1984Q4$ and find a value close to 0.18%. Technological progress through cost-efficient technologies makes this abatement cost to decrease by 1.7% per year.

TABLE 2. Initial conditions

NAME	PARAMETER	VALUE	SOURCE
Initial period	t_0	1984Q4	Authors' calculations
Abatement effort	μ_{t_0}	0.03	Nordhaus (2018)
Hours demand	$h^d_{t_0}$	1	RBC literature
Population (billion)	l_{t_0}	4.85	Data
Real Output (U.S. dollar trillion)	y_{t_0}	45/4	Data
Emissions (GtCO2)	e_{t_0}	20.33/4	Data
Stock of carbon (GtC)	m_{t_0}	346×2.13	Authors' calculations
Exogenous shocks	ε_{j,t_0}	1	Authors' calculations

Initial conditions are described in Table 2. Because world's time series exhibit a downward trend in inflation and interest rate, we start our sample in 1985Q1 and state variables are initialized for 1984Q4. In particular, we fix the population based on data provided by *Our World In Data*, while hours worked is normalized to one. Emissions as well as real GDP are set to match their observed value in t_0 based on the data source described in the previous subsection. The stock of carbon is set to the 346 ppm concentration recorded in 1984 and converted in GtC.

The last variable that needs to be discussed is the expected path of the carbon tax $\tilde{\tau}_{e,t}$. The transition scenario that will occur out-of-sample affect expectations and therefore the representation of the data provided by the estimated model. Instead of imposing arbitrary a mitigation scenario (such as Paris-agreement), we let the data be informative about the expected path of inflation. Let $\{\tilde{\tau}_{e,t}\}_{1984Q1}^{T}$ denote the path of the carbon tax under temperature stabilization by 2050, we impose the following expectation scheme $\mathbb{E}_{t,t+S}\{\tilde{\tau}_{e,t}\} = \varphi\tilde{\tau}_{e,t}$ where $\varphi \in [0,1]$ is the market belief about the realization of the Paris-Agreement. Parameter φ has multiple interpretations, it can be interpreted as the prior over the probability of realization of mitigation policy, or the fraction of agents believing on a complete mitigation policy, or the belief about the stringency of the policy.

3.4 Prior and posterior distributions The remaining parameters are estimated. Their prior distributions are reported in Table 3. For exogenous disturbances, the standard deviations are imposed an inverse gamma "type 2" as Christiano et al. (2014) with prior mean 0.001 and standard error 1. The AR of shocks follow a Beta distribution with prior mean 0.5 and standard deviation 0.15 which is in line with Smets and Wouters (2007). Regarding structural parameters, we estimate the annualized slope of growth of TFP and carbon decoupling. In DICE models, these parameters typically lie between 1% and 1.5%. We therefore impose a diffuse Gamma distribution with mean 0.5 and 0.15 standard deviation in order to allow only positive value close to 1%. In contrast, we also estimate the initial interest rate and set a diffuse prior 5 and standard deviation 1.5. Concerning utility parameters, our prior are inspired by Smets and Wouters (2007), but with tighter priors to avoid unrealistically low values for these two parameters. We also impose a Gamma distribution to force a positive support. Concerning the Rotemberg adjustment cost, this parameter typically lies between 20 to 200 in the literature. We therefore impose a Gamma distribution with prior mean 25 and standard deviation 7.5 to allow a possibly intense price stickiness. The share of low productive worker, driving the discounting within the Euler equation, follows a Beta distribution to limit its support to [0,1] with prior mean 0.05 and standard deviation 0.01. This prior let the data be informative about a possibly larger Euler discounting rate for the world economy, than reported in McKay et al. (2017) for the US. As the world include a large panel of developing economies with limited asset market participation, ω is expected to be much larger than for the US. We next turn to comment parameters related to monetary policy smoothing. We mostly consider the prior distributions of Smets and Wouters (2007), but consider Gamma shape for ϕ_π and ϕ_y. The discount rate $100(\beta-1)^{-1}$ is set with prior mean 1 and standard deviation 0.2. The mean is much larger than in Smets and Wouters (2007), but is an artefact of the Euler discount that requires lower discount factor to match the observed steady state real rate. Finally, the mitigation belief probability φ is completely uninformative, with uniform distribution that simply limits the support to [0,1].

We next turn to the posterior distribution generated by the Metropolis-Hasting sampler expressed in 90% confidence intervals in 3. Regarding shocks, one can note as Smets and

TABLE 3. Prior and posterior distributions of structural parameters

		PRIOR DISTRIBUTION			POSTERIOR DISTRIBUTION		
		Shape	Mean	Std	Mode	Mean [5%:95%]	
Panel A: Shock processes							
Std demand	σ_b	$\mathcal{IG}_2$	0.001	1	0.0245	0.0253	*[0.0231:0.0275]*
Std price	σ_p	$\mathcal{IG}_2$	0.001	1	0.0046	0.0051	*[0.0045:0.0056]*
Std MPR	σ_r	$\mathcal{IG}_2$	0.001	1	0.0009	0.0009	*[0.0008:0.0011]*
Std emissions	σ_e	$\mathcal{IG}_2$	0.001	1	0.0049	0.0047	*[0.0042:0.005]*
AR demand	ρ_b	$\mathcal{B}$	0.5	0.15	0.5974	0.6115	*[0.5965:0.6322]*
AR price	ρ_b	$\mathcal{B}$	0.5	0.15	0.9839	0.9839	*[0.9839:0.9839]*
AR MPR	ρ_r	$\mathcal{B}$	0.5	0.15	0.5407	0.5341	*[0.4711:0.5932]*
AR emissions	ρ_e	$\mathcal{B}$	0.5	0.15	0.9686	0.9707	*[0.9592:0.9823]*
Panel B: Structural parameters							
Initial TFP growth	$g_{z,t_0} \times 400$	$\mathcal{G}$	1.5	0.5	1.735	1.735	*[1.735:1.735]*
Decoupling rate	g_{σ,t_0}	$\mathcal{G}$	1.5	0.5	1.262	1.2254	*[1.1465:1.323]*
Decay TFP	$\delta_z \times 400$	$\mathcal{G}$	0.5	0.35	0.0464	0.0519	*[0.0411:0.0732]*
Risk aversion	σ_c	$\mathcal{G}$	2	0.15	1.1394	1.2608	*[1.1371:1.3715]*
Labor disutility	σ_h	$\mathcal{G}$	2	0.5	0.1708	0.1799	*[0.1649:0.2094]*
Rotemberg Cost	κ	$\mathcal{G}$	25	7.5	117.9202	117.9202	*[117.9194:117.9205]*
Initial inflation trend	$\pi_{*,t_0} \times 400$	$\mathcal{G}$	12	1	12.8434	12.6125	*[11.3687:13.6154]*
Initial inflation trend	$g_\pi \times 400$	$\mathcal{N}$	8	2	9.2703	9.2905	*[9.1232:9.4962]*
Initial interest rate	$r_{t_0} \times 400$	$\mathcal{N}$	12	2	8.7509	8.7746	*[8.3039:9.362]*
Share Low prod.	ω	$\mathcal{B}$	0.05	0.01	0.0512	0.0496	*[0.0422:0.0556]*
Inflation stance	ϕ_π	$\mathcal{G}$	0.75	0.05	0.5883	0.6367	*[0.5702:0.6943]*
MPR GDP stance	ϕ_y	$\mathcal{G}$	0.5	0.1	0.5265	0.5342	*[0.4712:0.6058]*
Discount rate	$(\beta^{-1}-1) \times 100$	$\mathcal{G}$	1	0.5	0.8055	0.8282	*[0.7976:0.8617]*
Mitigation policy belief	φ	$\mathcal{U}$	0.5	0.2887	0.5264	0.5235	*[0.5047:0.552]*
MPR smoothing	ρ	$\mathcal{B}$	0.5	0.075	0.9127	0.9127	*[0.9127:0.9128]*
Trend stance	ϕ_*	$\mathcal{N}$	0.5	0.5	-0.4482	-0.4434	*[-0.4625:-0.4185]*
Log marginal data density						-3104.54	

Note: $\mathcal{B}$ denotes the Beta, $\mathcal{G}$ the Gamma, and $\mathcal{IG}_2$ the Inverse Gamma (type 2) distributions. 8,000 draws are used to compute the posterior mean and 90% confidence interval.

Wouters (2007) the main source of persistence is the nominal-related disturbances. As Jondeau et al. (2023), the pollution shock exhibits also important persistence. The productivity initial growth are on average above values found in DICE, but this is not surprising as our initial period of simulation (namely 1985) exhibits more TFP Growth than in 2015-2020 of DICE. Similar patterns are also observed for decoupling rate. Risk aversion parameter is remarkably the same as the value reported in Smets and Wouters (2007), while we get a much lower value for the labor disutility. The Rotemberg price stickiness exhibit a large value, suggesting large nominal rigidities that are consistent with the Great Moderation period characterized by stable inflation rate. The long term inflation rate close to 4% annually. Low productive workers represents more than 6% households, thus offering a large attenuation of forward forces in the Euler equation. The smoothing of interest rate is much larger than reported in Smets and Wouters (2007). Conversely, the stance on inflation aligns closely with their findings. However, a marked contrast emerges in relation to the stance on the output gap parameter, where our estimates indicate a significantly larger weight for output stabilization. Lastly, the model's estimations suggest that achieving full decarbonation policy is conceivable with a confidence level of approximately 12%.

3.5 Impulse response functions This section discusses the dynamic properties of the model through the impulse response functions (IRF) of a number of variables of interest to various shocks. IRF are useful in assessing how shocks to economic variables reverberate through economic and climate systems. Figure 2 reports the generalized impulse response function of the estimated model taking parameters are their posterior mode among metropolis-hasting draws.

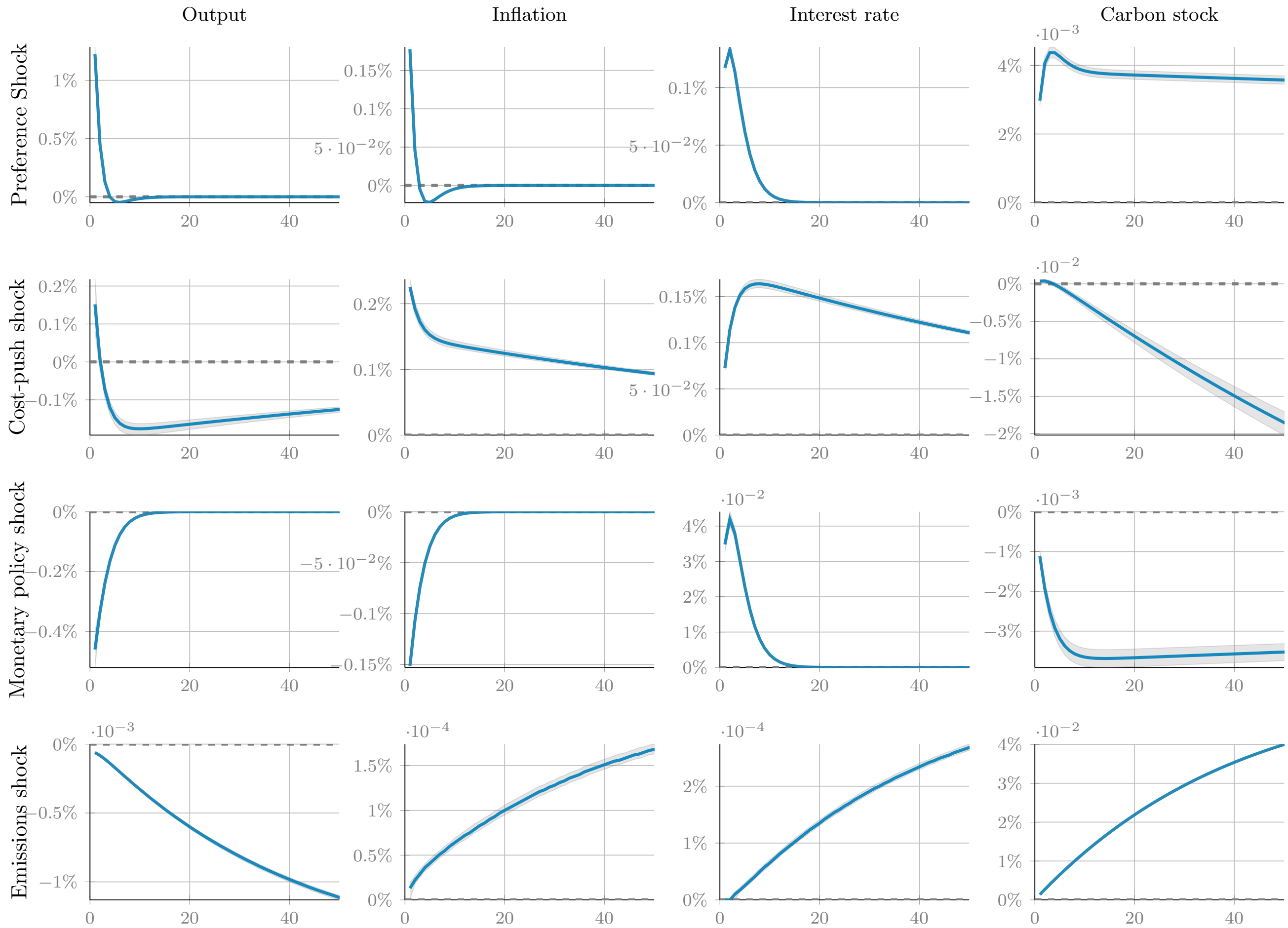
Consider the preference shock in first row of Figure 2, that captures changes in household preferences boosting consumption of households as in Smets and Wouters (2003). In response, aggregate output experiences a rise, reflecting the positive impact of increased wealth on overall economic activity. The increase in output driven by the preference shock leads to a corresponding rise in both inflation rate and carbon stock. Simultaneously, the interest rate aggressively responds to this boom, thus creating a modest recession when the shock process has decayed enough. This adjustment of the interest rate serves as a mechanism to reduce inflation by adjusting aggregate demand.

The second shock, reported in the second row of Figure 2, is the cost-push shock that has similar interpretation as the markup shock of Smets and Wouters (2007). This exogenous shock increases the selling price of firms and is typically detrimental for the rest of the economy. The central bank must strike a balance between price and quantity stabilization, as the interest rate cannot achieve stabilize when these two variables are going in opposite directions. The real interest rate slightly increases following the realization of the shock which reduces in turn aggregate demand on impact. Notably, the reduction in aggregate demand also has a positive consequence in terms of emissions: there is a corresponding decrease in production and economic activity, resulting in reduced emissions.

The third shock, reported in Figure 2, is the monetary policy shock. This shock is interpreted as a temporary deviation of the nominal rate from its policy rule. By boosting the return of safe assets, this shock reduces the willingness to consume, and depresses in turn aggregate demand. This decline in aggregate demand forces firms to reduces their demand for hours. The equilibrium wage clearing the labor market declines, thus creating a joint decline in the marginal cost and the selling price of goods. This decline in quantities also reduces emissions and makes the stock of carbon to be lower than expected.

The last shock is the emission intensity shock, that materializes as an exogenous increase in carbon emissions. This shock rise the stock of carbon in the atmosphere, and creates some modest economic damages. Its effects on inflation and interest rates are however too small to be measured.

FIGURE 2. Generalized impulse response functions of the estimated model



Note: The figure displays the generalized impulse response functions (GIRFs) of several variables to five shocks: preference, cost-push, monetary policy, and emission shocks in Lines 1 to 4, respectively. GIRFs are computed using the value of state variables in end-of-sample in 2021, and each GIRF is expressed in percentage deviations from its initial value in 2021. GIRFs are averaged based on 500 exogenous draws.

3.6 Moments Table 4 provides the empirical second moments of our four observable variables and the 95% confidence interval, as obtained with our model. This comparison between sample and model-implied moments allows to whether the model does a good job in capturing salient features of the data.

In assessing the first and second order moments generated by the model, we find that the model performs well overall, with the moments falling within the confidence bands of the observed variables. These results indicate that the model successfully captures the statistical properties of the variables of interest.

When examining the second order moments, we observe that the model's performance is relatively good compared to the usual standards in the inference of real business cycle models. Specifically, the model accurately replicates the volatility of inflation and the interest rate. However, it tends to underestimate the volatility of output and emissions, suggesting a potential area for improvement.

In terms of persistence, the model slightly underestimates the negative autocorrelation, indicating that the model's ability to capture the persistence of certain variables could be further refined. Additionally, the model tends to overestimate the persistence of inflation, indicating a potential area where adjustments could enhance the model's performance.

On the other hand, the model successfully matches the moments for the nominal interest rate and emissions, indicating that the model's ability to replicate these particular aspects is well done.

Regarding the correlation with output, the model does an excellent job, accurately capturing the relationship between output and most variables, except for emission growth. This suggests that further refinement or adjustments may be needed to better align the model's representation of emission growth with observed patterns.

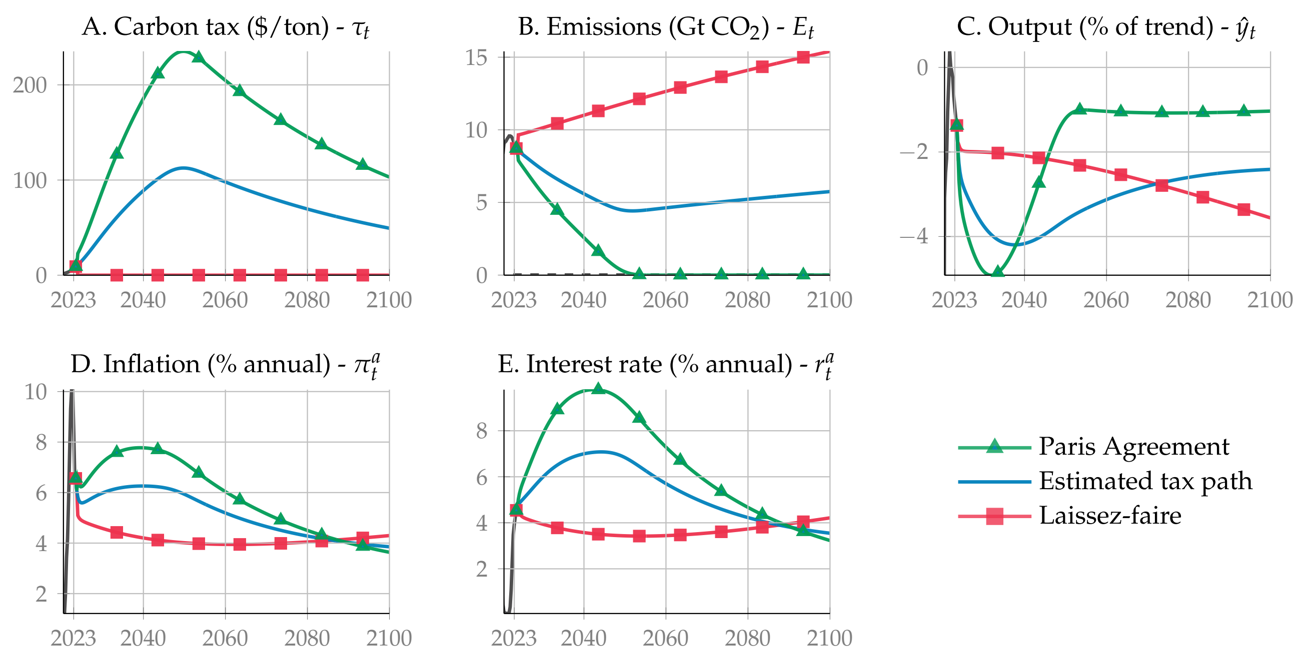
TABLE 4. Empirical and model-implied moments

	DATA	Baseline model [5%;95%]	DATA	Baseline model [5%;95%]
	Mean		**Standard deviations**	
Output growth	0.007	[0.007;0.008]	0.012	[0.008;0.010]
Inflation rate	0.010	[0.006;0.031]	0.007	[0.005;0.011]
Nominal rate	0.009	[0.010;0.049]	0.007	[0.004;0.015]
Emission growth	0.004	[0.004;0.005]	0.013	[0.009;0.011]
	Autocorrelation		**Correlation w/ output**	
Output growth	-0.221	[-0.193;0.121]	1.000	[1.000;1.000]
Inflation rate	0.979	[0.464;0.875]	-0.028	[-0.159;0.262]
Nominal rate	0.994	[0.968;0.998]	-0.038	[-0.187;0.234]
Emission growth	-0.131	[-0.206;0.116]	0.911	[0.927;0.968]

Note: Model-implied moments are computed across 1,000 random artificial series, each with the same size as the data sample.

4.1 A baseline transition to net-zero We now present long-term projections derived from the dynamic version of our model, which are based on two alternative scenarios. The first scenario matches the so-called SSP1-1.9 pathway of the IPCC (2021) in terms of carbon emissions (Paris Agreement). It assumes that carbon neutrality is reached in 2050 thanks to the introduction of a carbon tax, with net emissions close 0 by 2050. The second scenario is equivalent to IPCC (2021)'s SSP3-7.0 pathway, i.e., it assumes that there are no environmental policies, resulting in a continuous increase in carbon emissions (laissez-faire). In our simulations, the value of the carbon tax is determined to match the desired control rate of emissions for each scenario, and the model endogenously generates out-of-sample forecasts based on the posterior distribution of both parameters and shocks. The future path of the carbon tax rate was announced in 2023Q3, and expectations adjusted in response to this new environment. It is important to note that our analysis focuses on climate change mitigation, not on an optimal tax per se.

FIGURE 3. Model-implied projections based on alternative control rates of emissions



Note: This figure displays the projections of the main variables of the macro-climate model under two scenarios, corresponding to no carbon tax implementation (laissez-faire) and linear path to net zero between 2023Q3 to 2050Q1 (Paris Agreement).

Figure 3 shows the results of these simulations. The red line corresponds to the laissez-faire trajectory, which would result in a 4°C increase in temperatures. The green line is associated with the carbon trajectory that would be consistent with temperatures below 2°C above pre-industrial levels. Under the laissez-faire scenario, where no emission control measures are implemented, our simulations reveal a significant increase in climate damages over time. As a consequence, total factor productivity (TFP) experiences a downward trajectory, reflecting the detrimental impact of climate change on productive resources. Additionally, the reduction in demand because of damages reduces in turn inflation. In response monetary policy reduces interest rates to revive inflation.

Under the Paris-agreement, our simulations demonstrate a markedly different economic landscape. In this scenario, a carbon tax is implemented, gradually increasing and eventually reaching a level of \$210 per tonne of emissions, which proves to be sufficient for achieving the transition to a net-zero carbon economy. This significant policy intervention boosts aggregate demand. However, the investment-led expansion also brings about certain challenges. One notable consequence is the emergence of heightened inflationary pressures. The joint combination of the rising carbon tax and the output expansion contributes to a substantial surge in inflation, which we term as "greenflation." This phenomenon reflects the price increases stemming from the cost of transitioning. to a sustainable and low-carbon economy.

In response to the elevated inflationary environment, monetary policy takes a more restrictive stance. The central bank implements measures to tighten monetary conditions, aiming to temper the investment boom and alleviates the inflationary pressures associated with the transition process. By stabilizing the climate, economic damages are stabilized about 1% below the technological trend, while they continue to grow in the laissez-faire economy.

4.2 A decomposition of Aggregate Demand (AD) We next examine the various forces that influence aggregate demand. First the Aggregate demand schedule can be written as follows:

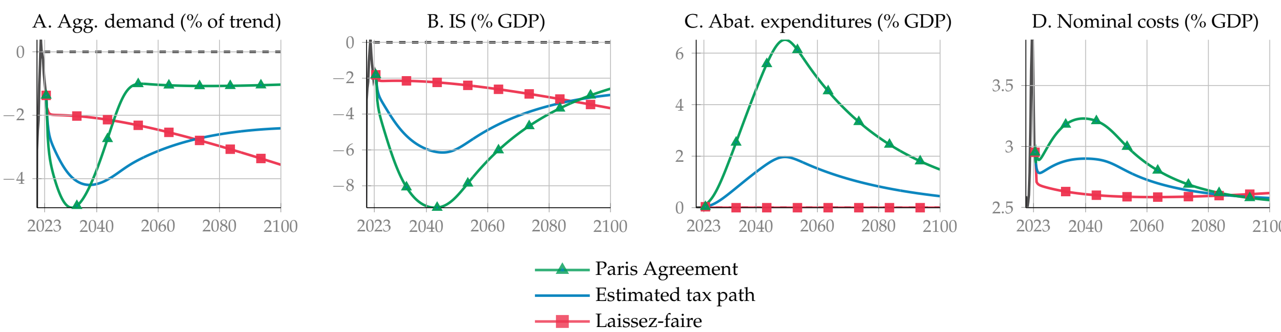
$$\hat{y}_t = \underbrace{\widehat{IS}_t}_{\text{forward real interest rate}} + \underbrace{(1-\vartheta)\frac{\kappa}{2}(\pi_t - \pi_t^\star)^2 + \vartheta(1 - mc_t)}_{\text{nominal wedge}} + \underbrace{\theta_{1,t}\tilde{\tau}_{e,t}^{\theta_2/(\theta_2-1)}}_{\text{green investment}}. \tag{31}$$

In this expression, one can distinguish three main forces. The **forward real interest rate** term, $\widehat{IS}_t$, captures the role of monetary policy such that the future product of interest rates drives current spending of consumption. This is the main channel channel of transmission of monetary policy, that is attenuated by the discounting within the Euler equation. The second term, called **nominal wedge** captures how changes in menu cost/price rigidity diverts a fraction of resources by reducing current consumption. In the literature of first best allocations in

New Keynesian models, monetary policy is typically committed to reduce this wedge. Lastly, the term **green investment** captures the extra spending that are necessary to reach net zero.

To provide a comprehensive view, Figure 4 reports the decomposition of aggregate demand into three complementary components: (i) the consumption/IS effects, (ii) the abatement expenditures, and (iii) the cost of nominal rigidities arising from Rotemberg. This decomposition is further explained in subsection 2.5.

FIGURE 4. Decomposition of aggregate demand during transition



Note: This figure displays the projections of the main variables of the macro-climate model under two scenarios, corresponding to no carbon tax implementation (laissez-faire) and linear path to net zero between 2023Q3 to 2050Q1 (Paris Agreement).

Figure 4 reports the behavior of these forces in green when the carbon tax is adjusted to achieve a net-zero emissions target by 2050, and compared with the laissez-faire in red. The rise in the carbon tax, represented by the green line in the figure (Panel A), creates a recession leading to output levels approximately 2% below the technological-neutral average. This recessionary effect is primarily driven by the dominant force of monetary policy, that tightens strongly (Panel B). This outcome is attributable to inflation pressures stemming from the implementation of the carbon tax, leading to greenflation. In response to these inflationary pressures, the central bank, adhering to the Taylor principle, overreacts to inflation by reducing aggregate demand. As a result, the consumption/IS effects partially counteracts the expansionary impact of the abatement expenditures.

In contrast, the path to net zero implies a transformation of production lines through strong abatement expenditures (Panel C). These expenditures increase proportionally to the carbon tax, eventually reaching a level equivalent to 4.2% of output at the peak of the transition, before gradually decreasing as a result of technological efficiency. In contrast, the contributions of nominal rigidities arising from Rotemberg pricing and exit shock (Panel D) are relatively similar across scenarios. The influence of these rigidities on the overall economic dynamics is relatively small compared to the dominant forces of abatement expenditures and the consumption/IS effects.

The path under laissez-faire is characterized by a decreasing GDP, that is driven by the economic damages from a warming planet. Much of this decline is explained by the conduct of monetary policy, that through an IS effect penalizes climate damages by imposing a positive real interest rate. Because no carbon tax is implemented, firms do not abate carbon emissions, making the abatement expenditure to be zero over the century. We discuss this aspect in the next subsection.

4.3 A decomposition of Aggregate Supply (AS) We next examine the various forces that influence aggregate supply. To do so, we provide a decomposition of inflation into four complementary forces. To provide this accounting, one needs first to decompose the marginal cost as follows:

$$\widehat{mc}_t \simeq \underbrace{\widehat{mc}_t^s}_{\text{standard term}} + \underbrace{\widehat{mc}_t^c}_{\text{climateflation}} + \underbrace{\widehat{mc}_t^g}_{\text{greenflation}}$$

where $\widehat{mc}_t$ is the percentage deviation from average $\overline{mc}$.[8] The New Keynesian Phillips is a highly nonlinear equation. In order to keep track of most nonlinearities in the decomposition, we propose a semi-linearization approach. the sum of the New Keynesian Phillips curve in order to express each marginal costs into current inflation terms:

$$\hat{\pi}_t^j = \frac{\xi - 1}{\kappa}\widehat{mc}_t^a + E_t\beta_{t+1}^{\pi}\hat{\pi}_{t+1}^j \text{ with } j = \{s, c, g\}.$$
$$\hat{\pi}_t^x = \frac{\xi - 1}{\kappa}\left(\varepsilon_{p,t} - 1\right) mc_t + E_t\beta_{t+1}^{\pi}\hat{\pi}_{t+1}^x$$

where the discount factor is adjusted by GDP growth as follows: $\beta_{t+1}^{\pi} = (1 + g_{z,t+1})\,\tilde{y}_{t+1}/\tilde{y}_t$.

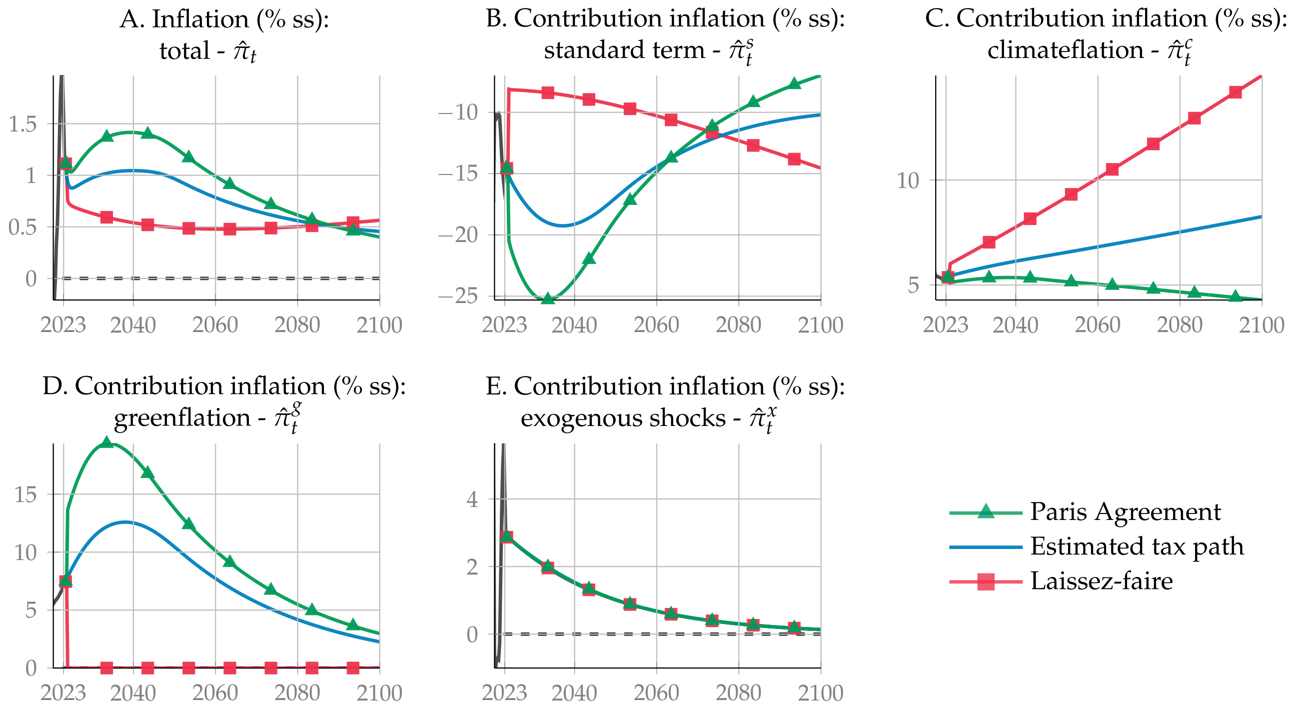
One can express the New Keynesian Phillips Curve as follows:

$$\hat{\pi}_t \simeq \underbrace{\hat{\pi}_t^s}_{\text{standard term}} + \underbrace{\hat{\pi}_t^c}_{\text{climateflation}} + \underbrace{\hat{\pi}_t^g}_{\text{greenflation}} + \underbrace{\hat{\pi}_t^x}_{\text{exogenous shocks}},$$

[8]Formally, note that the standard term reads as: $\widehat{mc}_t^s = (x_t\tilde{y}_t - \omega D)^{\sigma_c}\,\tilde{y}_t^{\sigma_h/\alpha} v^{-1} - 1$, while the climat term reads as $\widehat{mc}_t^c = (x_t\tilde{y}_t - \omega D)^{\sigma_c}\,\tilde{y}_t^{\sigma_h/\alpha}\left(\Phi\left(m_t\right)^{-1-\sigma_h/\alpha} - 1\right) v^{-1}$ and the green term is given by $\widehat{mc}_t^g = \tilde{\tau}_{e,t}\theta_{1,t}(\theta_2 + \tilde{\tau}_{e,t}^{\frac{1}{\theta_2-1}}\,(1-\theta_2))\overline{mc}^{-1}$. Note that constant term is given by $v = (\bar{y} - \omega D)^{\sigma_c}\,\bar{y}^{\sigma_h/\alpha}$.

In Figure 5, we examine the different forces operating on aggregate supply and their impact on inflation. The first row of the figure represents the approximate effects of the standard term, climateflation, greenflation, and exogenous shocks on total inflation.

FIGURE 5. Decomposition of aggregate supply during transition



Note: This figure displays the projections of the main variables of the macro-climate model under two scenarios, corresponding to no carbon tax implementation (laissez-faire) and linear path to net zero between 2023Q3 to 2050Q1 (Paris Agreement).

Under the laissez-faire scenario (red line), the economy experiences below-average inflation, estimated to be around -0.5%. This lower inflation rate is primarily driven by the standard term of the New Keynesian Phillips curve $\hat{\pi}_t^s$. The recent surge in inflation in the last observations and its recessionary forces has push strong downward pressures on the real wage and the marginal cost. In contrast, the presence of climate change-related factors also significantly affects inflation dynamics. The climateflation term, $\hat{\pi}_t^c$, representing the impact of climate change on inflation through resources scarcity, pushes inflation up to 4% in 2023, and is expected to rise further in the future as resources become more and more scarce. This scarcity emerges from the damage function, which leads to a decline in total factor productivity (TFP) over time as carbon stock increases. Note a downward trend on the standard term, $\hat{\pi}_t^s$, the indirect effect of climate change on the wealth effect of labor supply causes the marginal cost and, consequently, inflation to mechanically decrease over time.[9] Note also that the contribution of exogenous markup shocks has remained small in out-of-sample.

In contrast, under the net-zero scenario, inflation is relatively higher compared to the laissez-faire scenario. This surge in inflation is primarily driven by the greenflation term, which represents the effect of the carbon tax on firms' pricing and increases their production costs. In particular, aggressive carbon taxing makes inflation to be 4 pp above its average. The implementation of the carbon tax leads to a relatively larger cost of production for firms, contributing to the overall increase in inflation. The gain of such policy is to stabilize the climate, and it turn it stabilize the climateflation term to remain about 3%.

5 Monetary policy analysis

In progress.

6 Conclusion

By extending the traditional 3-equation New Keynesian framework to include carbon abatement costs, climate externalities, and carbon stock dynamics, this paper provides a comprehensive framework for understanding the interactions between environmental policies and macroeconomic outcomes. By using Bayesian techniques and fully nonlinear methods, we are able to provide a data grounded quantitative analysis of the transition to a net-zero carbon economy. Our framework is able to rationalize either *climateflation* or *greenflation* mechanisms by adjusting the carbon tax. The tractability of our framework allows us to analytically decompose the relative forces by which climate change and climate mitigation policies affect the demand and supply sides of the economy. In a "laissez-faire" scenario, the increasing damage to TFP creates a long and lasting recession similar to a permanent supply shock that fuels inflation and makes output below its technological trend. In contrast in the wake of a net zero transition, the rise in the production cost of firms boosts inflation, which combined with an increase in green investment, creates an economic expansion.

[9] Our decomposition exercise relies on a nonlinear model in which cross-products are not washed out by the linearization phase. Therefore, note that factors are not orthogonal with $cov\left(\hat{\pi}_t^g, \hat{\pi}_t^s\right) \neq 0$.

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INTERNET APPENDIX

(not for publication)

A FULL MODEL

Our model includes four core equations and variables $\{\tilde{y}_t, \pi_t, r_t, \tilde{m}_t\}$:

$$\varepsilon_{b,t}\left(\frac{x_t\tilde{y}_t-\omega\tilde{D}_t}{1-\omega}\right)^{-\sigma_c}=\beta\mathbb{E}_t\left\{\frac{\varepsilon_{b,t+1}r_t}{\pi_{t+1}}\left((1-\omega)\left(\frac{x_{t+1}\tilde{y}_{t+1}-\omega\tilde{D}_{t+1}}{1-\omega}\right)^{-\sigma_c}+\omega\tilde{D}_{t+1}^{-\sigma_c}\right)\right\} \tag{A.1}$$

$$\left(\pi_t-\pi_t^{\star}\right)\pi_t=(1-\vartheta)\beta\mathbb{E}_t\left\{(1+g_{z,t+1})\frac{\tilde{y}_{t+1}}{\tilde{y}_t}\left(\pi_{t+1}-\pi_{t+1}^{\star}\right)\pi_{t+1}\right\}+\frac{\zeta}{\kappa}\varepsilon_{p,t}mc_t+\frac{(1-\zeta)}{\kappa} \tag{A.2}$$

$$\frac{r_t}{r}=\left(\frac{r_{t-1}}{r}\right)^{\rho}\left[\left(\frac{\pi_t}{\pi_t^{\star}}\right)^{\phi_\pi}\tilde{y}_t^{\phi_y}\right]^{1-\rho}\varepsilon_t^r \tag{A.3}$$

$$\tilde{m}_t=(1-\delta_m)\tilde{m}_{t-1}+\xi_m\sigma_t\left(1-\tilde{\tau}_{e,t}^{\frac{1}{\theta_2-1}}\right)z_tl_t\tilde{y}_t\varepsilon_{e,t} \tag{A.4}$$

Our model also includes auxiliary variables:

$$x_t=1-(1-\vartheta)\frac{\kappa}{2}\left(\pi_t-\pi_t^{\star}\right)^2-\theta_{1,t}\tilde{\tau}_{e,t}^{\theta_2/(\theta_2-1)}-\vartheta(1-\epsilon_{p,t}mc_t) \tag{A.5}$$

$$mc_t=\frac{\psi}{\epsilon_{b,t}\left(1-\omega\right)^{\sigma_c+\sigma_n}}\frac{\left(x_t\tilde{y}_t-\omega\tilde{D}_t\right)^{\sigma_c}\tilde{y}_t^{\sigma_n}}{\Phi\left(\tilde{m}_t\right)^{1+\sigma_n}}+\tilde{\tau}_{e,t}\theta_{1,t}\left[\theta_2+\tilde{\tau}_{e,t}^{\frac{1}{\theta_2-1}}\left(1-\theta_2\right)\right] \tag{A.6}$$

where $\tilde{y}_t=y_t/(z_tl_t)$, $\tilde{D}_t=d+\tilde{T}_t^e$, $\tilde{T}_t^e=T_t^e/z_t$, $\tilde{\tau}_{e,t}=\tau_{e,t}\sigma_t\varepsilon_{e,t}/(\theta_2\theta_{1,t})$, and $\tilde{m}_t=m_t-m_{1750}$.

It also comprises five trend related deterministic processes:

$$\sigma_t=\sigma_{t-1}(1-g_{\sigma,t}) \tag{A.7}$$

$$g_{\sigma,t}=\left(1-\delta_\sigma\right)g_{\sigma,t-1} \tag{A.8}$$

$$z_t=z_{t-1}(1+g_{z,t}) \tag{A.9}$$

$$g_{z,t}=g_{z,t-1}\left(1-\delta_z\right) \tag{A.10}$$

$$\theta_{1,t}=(p_b/\theta_2)(1-\delta_{pb})^{t-t_0}\sigma_t \tag{A.11}$$

$$l_t=l_{t-1}\left(l_T/l_{t-1}\right)^{\ell_g} \tag{A.12}$$

And four stochastic processes:

$$\begin{aligned}\varepsilon_{b,t}&=(1-\rho_b)+\rho_b\varepsilon_{b,t-1}+\eta_{b,t}\\ \varepsilon_{p,t}&=(1-\rho_p)+\rho_p\varepsilon_{p,t-1}+\eta_{p,t}\\ \varepsilon_{e,t}&=(1-\rho_e)+\rho_e\varepsilon_{e,t-1}+\eta_{e,t}\\ \pi_t^{\star}&=\left(1-\rho_{\pi^\star}\right)\pi+\rho_{\pi^\star}\pi_{t-1}^{\star}+\eta_{\pi^\star,t}\end{aligned}$$

Demand part. Detrended Euler equation reads as:

$$\tilde{\lambda}_t = \mathbb{E}_t \left\{ \beta \frac{r_t}{\pi_{t+1}} \left((1-\omega)\tilde{\lambda}_{t+1} + \omega \varepsilon_{b,t+1} \tilde{D}_{t+1}^{-\sigma_c} \right) \right\}. \tag{B.13}$$

It can be rewritten as:

$$\begin{aligned} \lambda_t &= \mathbb{E}_t \left\{ R_t \left((1-\omega)\lambda_{t+1} + \omega \varepsilon_{b,t+1} \tilde{D}_{t+1}^{-\sigma_c} \right) \right\} \\ &= \omega \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} (1-\omega)^s \varepsilon_{b,t+s} \tilde{D}_{t+s}^{-\sigma_c} \prod_{j=0}^{s} R_{t+j} \right\}, \end{aligned}$$

where $R_t = \beta r_t / \pi_{t+1}$.

Recall that: $\tilde{\lambda}_t = \varepsilon_{b,t} \left(\frac{\tilde{c}_t - \omega \tilde{D}_t}{1-\omega} \right)^{-\sigma_c}$, the Euler equation becomes:

$$\left(\frac{c_t/z_t - \omega \tilde{D}_t}{(1-\omega)} \right)^{-\sigma_c} = \omega \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} (1-\omega)^s \varepsilon_{b,t+s} \tilde{D}_{t+s}^{-\sigma_c} \prod_{j=0}^{s} R_{t+j} \right\}$$

which can be rewritten as:

$$c_t/z_t = IS_t,$$

where $IS_t = \omega \tilde{D}_t + (1-\omega) \left[\omega \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta (1-\omega)^s \varepsilon_{b,t+s} \tilde{D}_{t+s}^{-\sigma_c} \prod_{j=0}^{s} \frac{r_{t+j}}{\pi_{t+1+j}} \right\} \right]^{-1/\sigma_c}$.

In addition, we know that:

$$IS_t = c_t/z_t = x_t y_t / (z_t l_t),$$

where $x_t = 1 - (1-\vartheta)\frac{\kappa}{2} (\pi_t - \pi_t^\star)^2 - \theta_{1,t} \tilde{\tau}_{e,t}^{\theta_2/(\theta_2-1)} - \vartheta(1 - \epsilon_{p,t} mc_t)$, with $\mu_t = \tilde{\tau}_{e,t}^{1/(\theta_2-1)}$.

As $c_t = x_t y_t / l_t$, it comes:

$$IS_t = x_t \tilde{y}_t.$$

Applying the lagarithm therefore yields:

$$\hat{y}_t \simeq \widehat{IS}_t + (1-\vartheta)\frac{\kappa}{2} (\pi_t - \pi_t^*)^2 + \theta_{1,t} \tilde{\tau}_{e,t}^{\theta_2/(\theta_2-1)} + \vartheta(1 - \epsilon_{p,t} mc_t),$$

with $\hat{y}_t = \log(\tilde{y}_t/\tilde{y})$ and $\widehat{IS}_t = \log(IS_t/IS)$.

Marginal Cost. The marginal cost is given by:

$$mc_t = \frac{w_t}{\Gamma_t} + \theta_{1,t} \mu_t^{\theta_2} + \tau_{e,t} \sigma_t (1-\mu_t) \varepsilon_{e,t}$$

Let us consider the real wage of the high productive worker $w_t = \psi_t n_t^{\sigma_n} / \lambda_t$, the general equilibrium condition $(1-\omega) n_t = n_t^d = N_t$ and the production function $y_t = l_t \Gamma_t N_t^\alpha$, we get:

$$mc_t = \frac{1}{\lambda_t \Gamma_t} \psi_t \left(\frac{y_t}{l_t \Gamma_t (1-\omega)} \right)^{\sigma_n} + \theta_{1,t} \mu_t^{\theta_2} + \tau_{e,t} \sigma_t (1-\mu_t) \varepsilon_{e,t}$$

Recall that $\Gamma_t = \Phi(\tilde{m}_t) z_t$ and $\tilde{y}_t = y_t / (l_t z_t)$, thus:

$$mc_t = \frac{1}{\lambda_t \Phi(\tilde{m}_t)} \psi z_t^{-\sigma_c} \left(\frac{\tilde{y}_t}{\Phi(\tilde{m}_t)(1-\omega)} \right)^{\sigma_n} + \theta_{1,t} \mu_t^{\theta_2} + \tau_{e,t} \sigma_t (1-\mu_t) \varepsilon_{e,t}$$

Next, replacing λ_t by its expression in function of $\tilde{c}_t$ gives:

$$mc_t = \frac{\psi}{\varepsilon_{b,t}\left(\frac{\tilde{c}_t-\omega\tilde{D}_t}{1-\omega}\right)^{-\sigma_c}\Phi\left(\tilde{m}_t\right)}\left(\frac{\tilde{y}_t}{\Phi\left(\tilde{m}_t\right)\left(1-\omega\right)}\right)^{\sigma_n} + \tilde{\tau}_{e,t}\theta_{1,t}\left(\theta_2 + \tilde{\tau}_{e,t}^{\frac{1}{\theta_2-1}}\left(1-\theta_2\right)\right).$$

Finally,

$$mc_t = \frac{\psi}{\left(1-\omega\right)^{\sigma_c+\sigma_n}}\frac{\left(x_t\tilde{y}_t-\omega\tilde{D}_t\right)^{\sigma_c}\tilde{y}_t^{\sigma_n}}{\varepsilon_{b,t}\Phi\left(\tilde{m}_t\right)^{1+\sigma_n}} + \tilde{\tau}_{e,t}\theta_{1,t}\left(\theta_2 + \tilde{\tau}_{e,t}^{\frac{1}{\theta_2-1}}\left(1-\theta_2\right)\right).$$

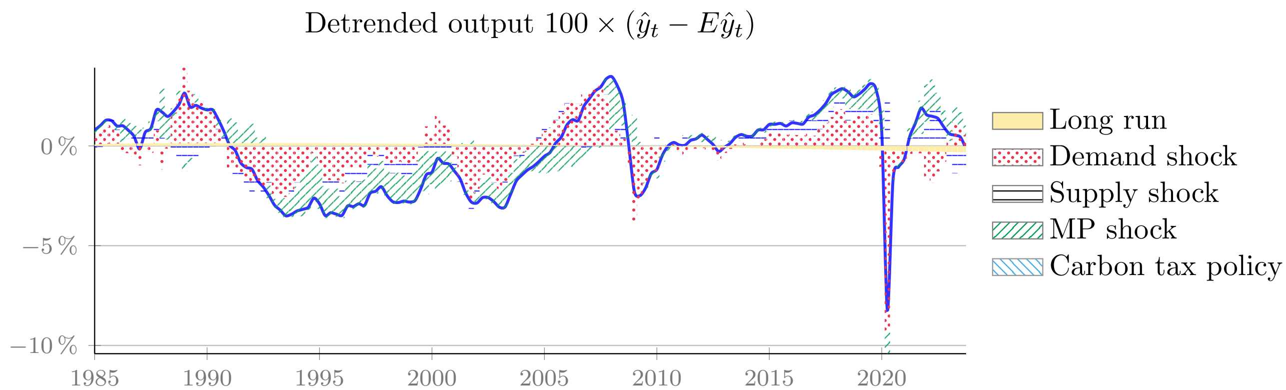
Phillips curve. The Phillips curve is a discounted sum of future marginal costs

$$\pi_t = \frac{\zeta}{\kappa}\mathbb{E}_t\sum_{s=0}^{\infty}\hat{\beta}_{t,t+s}\left[\varepsilon_{p,t+s}mc_{t+s} + \frac{1-\zeta}{\zeta}\right],$$

with $\hat{\beta}_{t,t+s} = \beta^s\frac{y_{t+s}l_t}{y_tl_{t+s}}\frac{1}{\pi_t-\ \pi_t^{\star}}$.

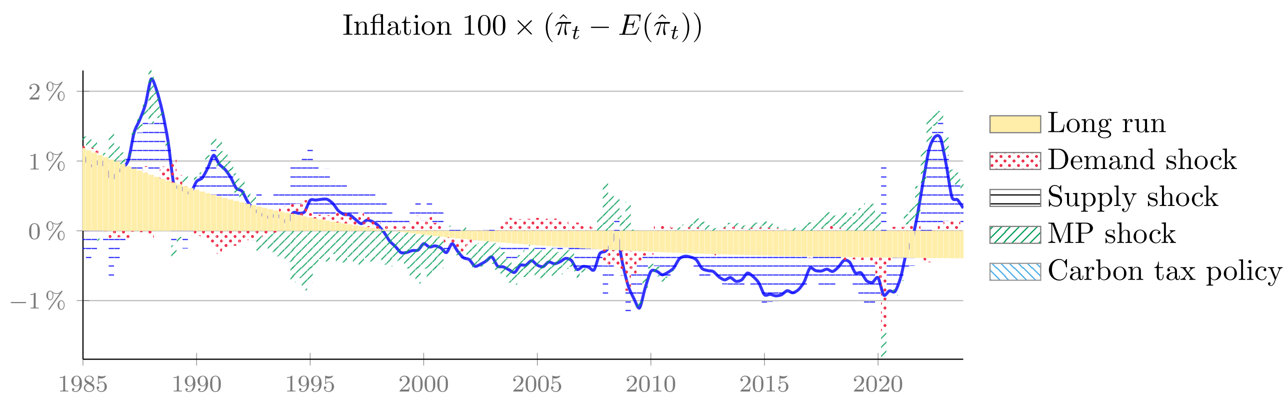
C Historical decomposition

FIGURE 6. Historical decomposition of output on the sample period



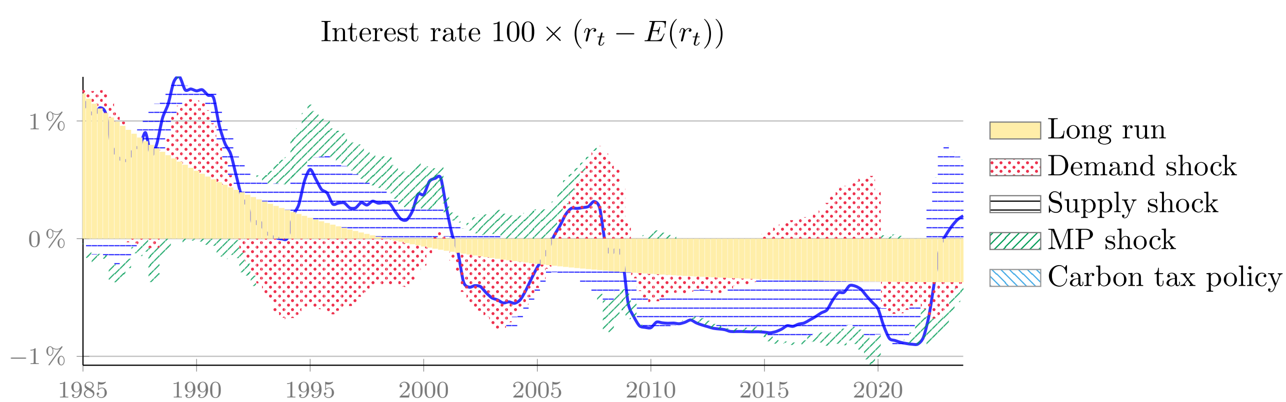
Note: This figure displays the approximated contribution of each shock to the determination of the variable of interest. Cross-products across contribution of shocks are neglected.

FIGURE 7. Historical decomposition of inflation on the sample period



Note: This figure displays the approximated contribution of each shock to the determination of the variable of interest. Cross-products across contribution of shocks are neglected.

FIGURE 8. Historical decomposition of interest rate on the sample period



Note: This figure displays the approximated contribution of each shock to the determination of the variable of interest. Cross-products across contribution of shocks are neglected.