DISCUSSION PAPER SERIES

No. 10594

NON-NEUTRALITY OF OPEN-MARKET OPERATIONS

Pierpaolo Benigno and Salvatore Nisticò

FINANCIAL ECONOMICS and INTERNATIONAL MACROECONOMICS

Centre for Economic Policy Research

NON-NEUTRALITY OF OPEN-MARKET OPERATIONS

Pierpaolo Benigno and Salvatore Nisticò

NON-NEUTRALITY OF OPEN-MARKET OPERATIONS[†]

Abstract

Unconventional monetary policy can have consequences for inflation and output because of income losses on central-bank balance sheet. A proposition of neutrality holds under some special monetary and fiscal policy regimes in which the treasury is ready to back central bank's losses through appropriate transfers levied as taxes on the private sector. In absence of fiscal backing, large and recurrent central bank's losses can undermine its long-run solvency and should be resolved through a prolonged increase in inflation. Small and infrequent losses are backed by future profits without any further consequences. A central bank averse to declining net worth commits to a more inflationary stance and delayed exit strategy from a liquidity trap. If fiscal policy is active, it is also desirable to reduce the duration of central bank's losses through higher inflation.

Pierpaolo Benigno   pbenigno@luiss.it
LUISS Guido Carli, EIEF and CEPR

Salvatore Nisticò   salvatore.nistico@uniroma1.it
Sapienza University of Rome

1 Introduction

The recent financial crisis has shown an unprecedented intervention of central banks around the world in an attempt to mitigate the adverse effects on the economy through the purchases of long-term risky securities. The Bank of England, the Bank of Japan, the European Central Bank, the Federal Reserve System and the Swedish Central Bank have all enlarged at various stages, and with different speed and composition, their asset holdings to include long-term private securities and government debt of different maturities and credit worthiness.

All these policies have raised worries about the possible stress that the central bank's balance sheet could suffer in terms of income losses and declining net worth.[1] In this paper, we take a general-equilibrium perspective on the concerns that could follow the implementation of unconventional monetary policies. The main focus is to understand under which conditions equilibrium prices and output vary because of income losses on central-bank balance sheets.

To this end, we have to challenge an important proposition, discussed first by Wallace (1981), which states the neutrality of standard open-market operations for the equilibrium prices and quantities and can be extended to any unconventional composition of central bank's assets. Even in this more general form, the intuition behind it is simple. If the central bank bears some risk that was before in the hands of the private sector, the equilibrium allocation of prices and quantities does not change simply because the possible realization of that risk is ultimately borne by the private sector, through appropriate lump-sum taxes, which are collected by the treasury and transferred back to the central bank to cover income losses.

This neutrality result goes straight to the heart of a long-lasting debate on how central banks should control the value of money in connection with the assets that they hold in their balance sheet. Indeed, under unconventional asset holdings, it is not gold, nor reserves, nor "real bills" that help to back the value of money.[2] Taxpayers do. Relying on this support, the central bank can control the value of money by just setting the policy rate or money supply regardless of what it holds in its asset portfolio.[3]

A result we emphasize in this work is that the validity of the proposition of neutrality relies on particular specifications of the transfer policies.[4] Two channels are at work identified

---

[1]A recent literature has evaluated these risks for the U.S. economy based on some projection analysis and conclude that they can be in general of minor importance (see Carpenter et al., 2015, Christensen et al., 2015, Greenlaw et al., 2013).

[2]Before unconventional monetary policy took place, what seemed to be the prevailing view shares common traits with the "real bills" doctrine (Smith, 1809), according to which central banks should issue money backed by short-term securities free of risk. In a system of this kind, it is understood that the central bank can control the value of money by setting the interest rate on the safe assets held in its portfolio.

[3]Sargent (2011) discusses Wallace's irrelevance result in light of the "real bills" doctrine.

[4]Sargent and Smith (1987, p.91), who provide a neutrality result in a model where money is dominated

by the possibility that symmetric and appropriate state-contingent transfers can occur 1) between the treasury and the central bank, 2) between the private sector and the treasury. To question the proposition of neutrality some risk should stay in the hands of the central bank.

First, we break channel 1) by assuming that there is no treasury's support, meaning that the treasury does not transfer resources to the central bank in case of losses leading to a decline in central bank's net worth. Interestingly enough, results of neutrality still emerge in this case relying on the possibility that the central bank can absorb losses by covering them with retained future profits at the expenses of lower remittances to the treasury. At the end, for neutrality to hold, any loss and associated lower remittance should be entirely paid by higher taxes levied on the private sector.

However, for the validity of this result, key is that losses are limited in size and time. In particular, a limited size is necessary because otherwise large losses can irrevocably impair the central bank's profitability leaving no possibility to back current deficit with future retained earnings. On the other hand, the limited duration ensures that there is a return to normal conditions in which central bank's income is not overburden by further losses and leaves room to recover previous ones without changing the monetary policy stance.

On the contrary, if losses are large and frequent without treasury's support the proposition of neutrality does not hold and equilibrium prices and output change together with the monetary policy stance.[5] Indeed, in the case of large losses, the value of money should change – i.e. inflation rises – up to the point in which private agents are forced to hold more currency, so that the seigniorage earnings of the central bank can increase and profitability be restored.

We then challenge the proposition of neutrality by breaking channel 2) and assuming that the treasury is unable or unwilling to tax the private sector to cover losses made by the central bank. In this case, the risk remains in the hands of the central bank and, when it materializes, has direct fiscal consequences with spillovers on equilibrium prices and output through a mechanism similar to that of the fiscal theory of the price level.[6] Whoever in the

in return, also argue that "irrelevance requires that fiscal policy be held constant in a precise sense". In our work we give a thorough understanding on what "held constant" means. See also the discussion of Sargent (2011).

[5]However even if losses are limited in size and time, a central bank feeling political pressure or a possible loss of independence may sometimes decide to change interest-rate policies in order to avoid territories of negative profits altogether or at least shorten their occurrence. In early 2002, the market questioned the willingness of the Bank of Japan to pursue quantitative easing policies on the ground of the possible losses that could result after an eventual rise in interest rates (see Stella, 2005, for an insightful discussion).

[6]Stella (2005) discusses the case of the Central Bank of Chile in the early 2000s as an outlier for the general rule that central-bank financial weakness impacts the monetary policy. He labels the Chile case as one of "benevolent fiscal dominance". This would correspond to a regime of passive fiscal policy in our analysis.

private sector unloaded the risky securities to the central bank experiences a positive blip in financial wealth. Demand will surge and so will inflation. The value of money will fall.

We apply this theoretical framework to economies in which the long-term assets held by the central bank are subject to either credit risk or interest-rate risk, in the latter case as a consequence of exit strategies from a liquidity trap. Three results arise from our numerical examples: 1) in the absence of treasury's support large losses, mainly due to credit events, should be resolved by a prolonged increase in inflation in order to restore the long-run profitability of the central bank; 2) a central bank which is averse to income losses and declining net worth commits to a more inflationary monetary policy stance and delayed exit strategies from a liquidity trap; 3) if fiscal policy is unable or unwilling to tax the private sector to cover central bank's losses, it is also desirable to shorten their duration through higher inflation.

Quantitative easing can be inflationary not because the central bank "prints money" or increases the size of its balance sheet, but because a higher inflation is either the desirable response to sizeable income losses in order to regain central bank's profitability or the outcome of a policy geared toward avoiding losses altogether.

Our work aims at providing a comprehensive understanding of the monetary and fiscal policy regimes supporting or not the neutrality of unconventional open-market operations by using a framework in which balance sheets of central bank and treasury are kept separate and by paying particular attention to the current institutional settings characterizing transfer mechanisms between central bank and treasury in advanced economies.[7]

Our analysis is inspired by the seminal works of Sims (2000, 2005) who has first emphasized in theoretical models the importance for policy analysis of separating the budget constraint of the treasury from that of the central bank.[8] Sims (2000, 2005) and Del Negro and Sims (2015) have already provided some non-neutrality results under certain specifications of monetary/fiscal policy regimes. We instead develop a more general framework. In particular Del Negro and Sims (2015) have discussed the importance of treasury's support for the control of inflation and studied the feasibility of certain monetary policy rules for the U.S. economy given its current and prospect balance-sheet composition.[9] Reis (2013, 2015) and

---

[7]Eggertsson and Woodford (2003) have already provided a generalization of Wallace's neutrality result to encompass unconventional open-market operations maintaining however a consolidated budget constraint between treasury and central bank.

[8]Drawing from the experience of several central banks, Stella (1997, 2005) has also provided evidence for the relationship between central-bank financial strength and monetary policy. See also the recent work of Adler et al. (2012).

[9]Berriel and Bhattarai (2009) analyze a particular case of non-neutrality in which the transfers between the central bank and the treasury are assumed always constant over time and the central bank holds only short-term securities. Zhu (2004) also distinguishes between the budget constraint of the central bank and that of the treasury but he focuses on how the properties of equilibrium determinacy change when the interest-rate rule followed by the central bank reacts also to variations in its net worth with respect to a target. Jeanne and Svensson (2007) discuss the importance of balance-sheet considerations as a credible device to exit from

Hall and Reis (2015) have instead investigated the consequences of income losses for central bank's solvency by analyzing several realistic transfer regimes between treasury and central bank. However, their partial equilibrium analysis allows only to speculate on the equilibrium implications, for prices and other variables, which are instead naturally uncovered by our general equilibrium analysis. Similarly for Bassetto and Messer (2013) who mainly focus on the fiscal consequences of alternative compositions of central-bank assets emphasizing in an interesting way the different accounting procedures and remittance policies between treasury and central bank.[10]

Our work is also related to a more extensive literature which has studied the monetary policy consequences of alternative assumptions on fiscal policy (see among others Sargent and Wallace, 1981, Sargent, 1982, Leeper, 1991, Sims, 1994, 2013, and Woodford, 1994, 2001) but which, on the contrary, has disregarded the distinction between central bank and treasury balance sheets. This separation is key in our analysis.

The plan of this work is the following. Section 2 presents an overview of the main results and the intuition behind them. Section 3 provides the details of the model. Section 4 discusses alternative assumptions on monetary and fiscal policy regimes and studies the consequences of alternative compositions of the central bank's balance sheet for monetary policy. Section 5 evaluates through some simple numerical examples how the equilibrium allocation can change under the different regimes underlined in Section 4. Section 6 discusses the robustness of the results and concludes.

2 Overview of main results and intuition

In this section we discuss our main results – and the related intuition – through a stylized model of price and output determination. Consider a money demand equation of the form

$$\frac{M_t}{P_t} \geq L(Y_t, i_t, z_t) \tag{1}$$

where M_t is the money held by households which in equilibrium is equal to the money supplied by the central bank, P_t is the price level, i_t is the risk-free nominal interest rate and Y_t is real output, while z_t is a vector of exogenous disturbances. The function $L(\cdot)$ has standard properties and in particular a satiation point when the nominal interest rate reaches zero.

---

a liquidity trap; however, their focus is on the balance-sheet losses possibly arising because of the effect of exchange-rate movements on the value of reserves.

[10]There is a substantial literature which has analyzed the different central-bank accounting procedures and remittance policies as, among others, Stella (1997, 2005) and Archer and Moser-Boehm (2013).

Consider also an aggregate demand function of the form

$$Y_t = E_t \Gamma(Y_{t+1}, i_t, \Pi_{t+1}, z_t) \tag{2}$$

where E_t is the conditional expectation operator, $\Pi_t = P_t/P_{t-1}$ is the gross inflation rate and the function $\Gamma(\cdot)$ has the standard properties of relating current output negatively to the real interest rate and positively to future output. An aggregate supply equation completes the model economy

$$\Pi_t = E_t \Upsilon(Y_t, \Pi_{t+1}, z_t) \tag{3}$$

for some general function $\Upsilon(\cdot)$ relating in a positive way current inflation with output and future inflation.

It is important to note that there is nothing special about the above model which is going to drive the results except that the equations can be implied by optimizing behavior of rational-expectations agents in a dynamic general equilibrium model.[11] The set of equations (1)-(3) involves four endogenous variables: inflation, output, nominal interest rate and money supply. This leaves the possibility to specify one policy instrument, between nominal interest rate and money, possibly as function of the other variables or exogenous states. Later we name this specification *conventional monetary policy*. A "candidate" equilibrium is a set of stochastic processes $\{Y_t, \Pi_t, i_t, M_t\}$ satisfying equations (1)-(3) consistently with the specification of *conventional monetary policy* and subject to the zero-lower bound on the nominal interest rate. Standard no-arbitrage conditions can be added to price long-term bonds or state-contingent securities given the allocation $\{Y_t, \Pi_t, i_t, M_t\}$. We denote with Q_t and r_t, respectively, the time-t price and return of a long-term bond and with $\tilde{R}_{t,T}$ the state-contingent time-t price of one unit of nominal income at time T.

We can now move to understand our Proposition of Neutrality using this simple model. Consider the "candidate" equilibrium $\left\{Y_t, \Pi_t, i_t, M_t, Q_t, r_t, \tilde{R}_{t,T}\right\}$ for a given *conventional monetary policy* and let the central bank change the maturity structure of its assets or even the size of the overall balance sheet. These alternative policies, that later we label *balance-sheet policies*, are said to be neutral if $\left\{Y_t, \Pi_t, i_t, M_t, Q_t, r_t, \tilde{R}_{t,T}\right\}$ is still an equilibrium. How could it not be? Nothing has changed in the above equilibrium conditions nor in the policy rule to imply a change in the equilibrium allocation. However, this is not enough to check neutrality, something else could have instead changed that should be considered as relevant for the determination of the equilibrium. In our analysis, we show that the intertemporal

---

[11]Indeed, the model presented in the next section has slightly different functional forms than the ones of equations (1)-(3).

budget constraint of the central bank can be an equilibrium restriction to take into account:

$$\frac{X_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} - \frac{B_{t-1}^C}{P_t} - (1+r_t)\frac{Q_{t-1}D_{t-1}^C}{P_t} = E_t \sum_{T=t}^{\infty} \tilde{R}_{t,T} \left[\frac{i_T}{1+i_T}\frac{M_T}{P_T} - \frac{T_T^C}{P_T}\right], \qquad (4)$$

where X_t denotes central bank's reserves, B_t^C and D_t^C are central bank's holdings of short-term and long-term securities, respectively, and T_t^C are transfers between the central bank and the treasury. Solvency of the central bank requires the outstanding real market value of its net liabilities to be equal to the present discounted value of the seignorage revenues net of the transfers made to treasury. Therefore $\left\{Y_t, \Pi_t, i_t, M_t, Q_t, r_t, \tilde{R}_{t,T}\right\}$ is an equilibrium if it is also consistent with (4) given some balance-sheet policy. *A fortiori* the Proposition of Neutrality requires consistency for *any* balance-sheet policy.

We show in our analysis that one of the key features for the validity of the Proposition of Neutrality is the specification of the transfer policy between the central bank and the treasury. To simply see this point, consider a constant real transfer policy of the form $T_t^C/P_t = \bar{T}^C$ for some $\bar{T}^C$. It is only by coincidence that a "candidate" allocation $\left\{Y_t, \Pi_t, i_t, M_t, Q_t, r_t, \tilde{R}_{t,T}\right\}$ can satisfy the above intertemporal budget constraint at each time t and in each contingency and for some composition of the central-bank balance sheet. It should be clear that it can never satisfy (4) for all possible compositions. We have therefore a theory of price (and quantity) determination which operates also through the solvency condition of the central bank.

We provide an example of a transfer rule that can instead make *any* allocation – satisfying equations (1)-(3) for a *conventional monetary policy* – consistent with the central-bank intertemporal budget constraint regardless of the specification of the balance-sheet policy:

$$\frac{T_t^C}{P_t} = \bar{T}^C + \frac{\Psi_t^C}{P_t} + \psi_t\frac{N_{t-1}}{P_t}, \qquad (5)$$

where $1 - \Pi_t < \psi_t \leq 1$ at each t. The above rule belongs to a class of "passive remittances' policies", according to a definition that later we give in the text. In (5), Ψ_t^C are central-bank profits while N_t is its net worth. The key feature of (5) is the one-to-one relationship between central bank's profits and remittances to the treasury. In particular, negative profits imply lower, or even negative, remittances. This observation suggests that there can be direct fiscal consequences of central bank's losses. Therefore, to verify the validity of the Proposition of Neutrality, we need also to check the consistency of the "candidate" equilibrium

$\left\{ Y_t, \Pi_t, i_t, M_t, Q_t, r_t, \tilde{R}_{t,T} \right\}$ with the solvency condition of the treasury:

$$\frac{B^G_{t-1}}{P_t} + (1 + r_t) \frac{Q_{t-1} D^G_{t-1}}{P_t} = E_t \sum_{T=t}^{\infty} \tilde{R}_{t,T} \left[\frac{A_T}{P_T} + \frac{T^C_T}{P_T} \right],$$

in which B^G_t and D^G_t are the treasury's short-term and long-term debt, respectively, and A_t is the primary surplus. The real market value of outstanding treasury's obligations should be equal to the expected present discounted value of primary surpluses and central-bank remittances. Lower remittances should be offset by a higher primary surplus otherwise some endogenous variable, like prices, should adjust in the above equation. Passive fiscal policies ensure that the above intertemporal budget constraint is satisfied for any allocation $\left\{ Y_t, \Pi_t, i_t, M_t, Q_t, r_t, \tilde{R}_{t,T} \right\}$ consistent with equations (1)-(3) given a *conventional monetary policy* and for any composition of the liabilities of the treasury. A rule in this class is:

$$\frac{A_t}{P_t} = \bar{a} - \frac{T^C_t}{P_t} + \phi \left[\frac{(1 + r_t) Q_{t-1} D^G_{t-1} + B^G_{t-1}}{P_t} \right] \tag{6}$$

for some $\bar{a}$ and ϕ, with $0 < \phi < 1$.

Our first result, discussed in Section 4.1, shows that the Proposition of Neutrality holds under a combination of passive fiscal policy and passive remittances' policy. The intuition is simple. Suppose that the central bank bears risk that was before in the hands of the private sector by purchasing long-term bonds. In the case this risk materializes in losses, rule (5) ensures that they are backed by transfers from the treasury while (6) ensures that the treasury covers them through an increase in the primary surplus and therefore in the lump-sum taxes levied on the private sector. The risk at the end remains in the hands of the private sector. There is no reason for equilibrium output and prices to change following a different composition of the central-bank balance sheet.

Rule (5) is certainly special in order for the Proposition of Neutrality to hold, but it is even more special if we look at common central bank's practices. One interesting example that gets close is that of a *full treasury's backing regime*, where transfers from central bank to treasury are always equal to profits, $T^C_t = \Psi^C_t$.[12] It is indeed a common practice for central banks to transfer all remittances to the treasury, in particular if profits are positive. However, a *full treasury's backing regime* involves a symmetric relationship since it requires

---

[12]Our definition of "full treasury's backing regime" should not be confused with the full-fiscal-backing terminology used in the literature on the fiscal theory of the price level where fiscal backing means a fiscal policy that supports the equilibrium prices chosen by monetary policy, as in our definition of passive fiscal policy. Here, instead, "treasury's backing" or "treasury's support" is meant to capture the possibility that the treasury directly transfers resources to the central bank to avoid a fall in net worth.

the treasury to transfer resources to the central bank in the case of negative profits.[13]

Our second result, discussed at the end of Section 4.1, shows that a *full treasury's backing regime* together with a passive fiscal policy still ensures that the Proposition of Neutrality holds, although a *full treasury's backing regime* does not belong to the class of passive remittances' policy.

We then move to understand the consequences of absence of treasury's support, i.e. $T_t^C \geq 0$, which implies a fall in central bank's net worth in the case of income losses. We analyze a regime in which the central bank can pay remittances that are less than their profits in order to recover from periods of declining net worth. This is in line with the experience of the U.S. Federal Reserve which, in the case of negative profits, can issue a "deferred asset" that can be paid back by future positive profits. Only once the deferred asset is paid in full, remittances are rebated back to the treasury.

Our third result, presented in Section 4.2.1, refers to a combination of deferred-asset regime and passive fiscal policy showing that the Proposition of Neutrality does not hold unless central bank's losses are limited in time and size. In time because otherwise the central bank does not have enough room to back previous losses with future profits and therefore is forced to change its *conventional monetary policy* in order to ensure that the solvency condition (5) is always satisfied. The size dimension is also relevant because otherwise central bank's profitability can be permanently impaired leaving no possibility to back current losses with future profits. In this respect, our analysis shows that this profitability is related to the overall level of non-interest bearing liabilities of the central bank, $N_t + M_t$. Accordingly, the overall amount of such liabilities should remain positive in the long run, implicitly imposing a (possibly negative) lower bound on the level that net worth can reach. If losses are sizeable to the extent that net worth falls below this lower bound, the central bank should restore profitability by changing its monetary policy stance to increase the price level in a way that agents hold more money balances for their transaction.

This last result supports the view often expressed in the policy debate that quantitative-easing policies can be inflationary. Note, however, that this is not much because they directly expand the central-bank balance sheet, but rather because the possible losses on long-term bonds, if sizeable, can only be absorbed if the central bank raises at some point in time prices and therefore the money balances held by the private sector, to the end of restoring its own profitability.

---

[13]An example is that of the Bank of England which in January 2009 established a wholly-owned subsidiary called Bank of England Asset Purchase Facility Fund Limited with the responsibility of buying private and public long-term securities through funds of the same Bank of England raised through increases in reserves (see Bank of England, 2013). The created company is fully indemnified by the Treasury since any financial losses as a result of the asset purchases are borne by the Treasury and any gains are owed to the Treasury.

An interesting case, analyzed in Section 4.2.2, considers a central bank not backed by the treasury but completely averse to income losses. In this respect, our fourth conclusion reiterates again a non-neutrality result. More interestingly, a numerical example discussed in Section 5 shows that, to avoid any income loss, the central bank should commit to a more inflationary monetary policy stance and eventually a delayed exit strategy from a liquidity trap.

Finally, we investigate the implications of assuming an active fiscal policy. In this case, any central bank's loss due to purchases of long-term securities is not passed to the private sector implying an increase in private wealth which raises aggregate demand and lowers the value money, as in the fiscal theory of the price level. In this direction, our fifth result confirms in this case the non-neutrality of unconventional open-market operations. Moreover, a numerical exercise in Section 5 shows that under a deferred asset regime, and active fiscal policy, the central bank endogenously reduces the duration of income losses through higher inflation to the end of backing the overall liabilities of the whole government.

In our analysis, we also evaluate the welfare consequences of unconventional monetary policies. Results can be intuited using the simple model of this section. Consider now an allocation $\left\{Y_t, \Pi_t, i_t, M_t, Q_t, r_t, \tilde{R}_{t,T}\right\}$ consistent with (1)-(3) and maximizing a well-defined welfare criterion. Later we call it *benchmark* optimal policy. This is clearly the best equilibrium since it is not constrained by the solvency conditions of treasury and central bank but can be consistent with them if supported by appropriate transfer policies coherent with the Proposition of Neutrality. Under these policies, unconventional monetary policy is clearly not welfare improving. For other transfer policies, the feasible optimal policy can deviate from the *benchmark* optimal policy and be constrained by either one or both the solvency conditions (5) and (6). Starting from a second-best allocation, it may happen that unconventional monetary policy can improve welfare, but never enough to exceed the level implied by the *benchmark* optimal policy.

All these non-neutrality results point towards saying that unconventional monetary policy can be an additional dimension along which to specify the overall monetary policy stance. In the case in which the economy is stuck in a sub-optimal allocation, quantitative-easing policies can signal a more inflationary monetary policy stance for the two main reasons outlined in the discussion above and supported by the numerical examples of Section 5: inflation is either the desirable response to central bank's losses in order to restore central-bank profitability or the means to prevent any loss altogether. Clearly, the first case would put the central bank in unchartered waters with the risk of leaving the currency completely unbacked, an issue that we do not discuss in this work.

3 Model

We present our analysis in an infinite-horizon monetary economy in which three main sectors interact in exchanging goods and assets: the private sector comprising households and firms, the treasury and the central bank. A key feature of our analysis, as mentioned in the introduction, is the separation between the balance sheets of treasury and central bank. There are three frictions in the model which are, however, not essential for the main results. The first is a financial friction for which money has a liquidity premium over other assets since it is the only security used in exchange of goods. This friction is not important at all for our results. It is only useful to capture features of current economies in which money has a relevant role for transactions and for financing central-bank assets.[14] Our main results are robust to alternative monetary models with different transaction frictions, or models in which money has only a pecuniary value or even cashless economies. The second and third frictions affect the goods market, since firms operate under monopolistic competition and prices are subject to a staggered adjustment mechanism. These two additional features are only relevant for our numerical examples when we compare the implications of our analysis with those of a more recent literature that has studied monetary policy constrained by the zero bound on nominal interest rates. Finally, our model economy is perturbed by two stochastic disturbances: a credit shock and a preference shock, to capture credit and interest-rate risk, respectively. These are the two most relevant risks in thinking about the consequences of recent asset purchases by central banks in advanced economies.

Consider a closed-economy model with a continuum of measure one of households all sharing the following utility:

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \xi_t \left[\frac{C_t^{1-\rho}}{1-\rho} - \int_0^1 \frac{(L_t(j))^{1+\eta}}{1+\eta} dj \right] \tag{7}$$

where E_t denotes the standard conditional expectation operator, β is the intertemporal discount factor with $0 < \beta < 1$, ξ is a stochastic disturbance, which affects the intertemporal preferences of households and is assumed to follow a Markov process, with transition density $\pi_\xi(\xi_{t+1}|\xi_t)$ and initial distribution f_ξ. We assume that (π_ξ, f_ξ) is such that $\xi \in [\xi_{\min}, \xi_{\max}]$.[15] C is a consumption bundle of the form

$$C \equiv \left[\int_0^1 C(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}};$$

---

[14]See Lucas (1984) for further discussion of the usefulness of this class of models for monetary theory and more recently Sargent (2014).

[15]We restrict our analysis to exogenous stochastic processes with finite state space.

$C(j)$ is the consumption of a generic good j produced in the economy and θ, with $\theta > 1$, is the intratemporal elasticity of substitution between goods while ρ, with $\rho > 0$, is the risk-aversion coefficient; $L(j)$ is hours worked of variety j which is only used by firm j to produce good j while η is the inverse of the Frisch elasticity of labor supply, with $\eta > 0$. Each household supplies all the varieties of labor used in the production.

The timing of markets' opening follows that of Lucas and Stokey (1987). In a generic period t, the asset market opens first, followed by the goods market. There is a financial friction since only money can be used to purchase goods and only in the goods market. In the asset market, households can adjust their portfolio according to

$$M_t + \frac{B_t + X_t}{1 + i_t} + Q_t Z_t \leq B_{t-1} + X_{t-1} + (1 - \varkappa_t)D_{t-1} +$$
$$\int_0^1 W_{t-1}(j)L_{t-1}(j)dj + T_t + \Phi_{t-1} + (M_{t-1} - P_{t-1}C_{t-1}). \quad (8)$$

Households invest their financial wealth in money, M_t – a non-interest bearing asset issued by the central bank which provides liquidity services – in treasury bills, B_t, or central bank's reserves, X_t, which both carry the same risk-free nominal return given by i_t. Finally households can lend or borrow using long-term securities, Z_t, at a price Q_t.[16] To shorten the writing of (8), we are including only those securities, among the ones traded, that households can exchange externally with the treasury and central bank. In addition, in each period, households can trade with each other in a set of state-contingent nominal securities spanning all states of nature which they face in the next period. It is assumed that the payoffs of these securities are enough to "complete" the financial markets.[17]

In the case of long-term lending or borrowing, the stochastic disturbance $\varkappa_t$ on the right-hand side of (8) captures the possibility that long-term assets or liabilities can be partially seized by exogenous default; in particular $\varkappa_t$ follows a Markov process with transition density $\pi_\varkappa(\varkappa_{t+1}|\varkappa_t)$ and initial distribution $f_\varkappa$. We assume that $(\pi_\varkappa, f_\varkappa)$ is such that $\varkappa \in [0,1)$. Moreover, the security available has decaying coupons: by lending Q_t units of currency at time t, geometrically decaying coupons are delivered equal to $1, \delta, \delta^2, \delta^3...$ in the following periods and in the case of no default.[18] Given this structure, the stock of long-term securiteis

---

[16]While $M_t, X_t, B_t \geq 0$, households can borrow or lend freely in the long-term fixed-rate security. To simplify the analysis the long-term security is issued either by the households or by the treasury.

[17]But not to undo the financial friction.

[18]See among others Woodford (2001).

held at time t is $D_t = Z_t + \delta(1 - \varkappa_t)D_{t-1}$. Hence the budget constraint (8) can be written as

$$M_t + \frac{B_t + X_t}{1 + i_t} + Q_t D_t \le B_{t-1} + X_{t-1} + (1 - \varkappa_t)(1 + \delta Q_t)D_{t-1} +$$

$$\int_0^1 W_{t-1}(j)L_{t-1}(j)dj + T_t + \Phi_{t-1} + (M_{t-1} - P_{t-1}C_{t-1}). \quad (9)$$

In the budget constraints (8) and (9), $W(j)$ denotes wage specific to labor of quality j. Wage income for each variety of labor j, $W_{t-1}(j)L_{t-1}(j)$, and firms' profits, Φ_{t-1}, of period $t-1$ are deposited in the financial account at the beginning of period t; T_t are lump-sum transfers net of taxes received from the treasury. Unspent money in the previous-period goods market is deposited in the financial account.

When asset market closes, goods market opens and households can use money to purchase goods according to

$$M_t \ge P_t C_t, \quad (10)$$

where P_t is the consumption-based price index associated with the consumption bundle C_t.

The households' problem is subject to initial conditions $B_{t_0-1}, X_{t_0-1}, D_{t_0-1}, M_{t_0-1}$ and a borrowing limit of the form[19]

$$\lim_{T \to \infty} E_t \left[R_{t,T} \mathcal{W}_T \right] \ge 0, \quad (11)$$

looking forward from each time t where $R_{t,T}$ is the nominal stochastic discount factor that is used to evaluate nominal wealth $\mathcal{W}_T$ in a generic contingency at time T with respect to nominal wealth at time t, with $T > t$. It is also required for the existence of a well-defined intertemporal budget constraint that

$$E_t \left\{ \sum_{T=t}^{\infty} R_{t,T+1} \left[P_T C_T + \frac{i_T}{1 + i_T} M_T \right] \right\} < \infty \quad (12)$$

looking forward from any date t, since there is no limit to the ability of households to borrow against future income.[20]

Households choose consumption, hours worked and asset allocations to maximize utility (7) under the constraints (9), (10), (11) and (12), given the initial conditions. The optimal choice with respect to consumption, assuming an interior solution, requires that

$$\xi_t C_t^{-\rho} = (\varphi_t + \beta E_t \lambda_{t+1}) P_t, \quad (13)$$

---

[19]There are also initial conditions on $\int_0^1 W_{t_0-1}(j)L_{t_0-1}(j)dj, \Phi_{t_0-1}$ and $P_{t_0-1}C_{t_0-1}$ but we assume that they sum to zero as in equilibrium.

[20]It is important to note that the expression in the curly bracket of (12) is never negative since consumption, money holdings, prices and interest rates are all non-negative.

where λ_t and φ_t are the respective non-negative Lagrange multipliers associated with the constraints (9) and (10). The first-order condition with respect to money holdings

$$\lambda_t - \varphi_t = \beta E_t \lambda_{t+1}, \tag{14}$$

implies in (13) that the marginal utility of nominal wealth is simply given by $\lambda_t = \xi_t C_t^{-\rho}/P_t$, which is positive.

The optimality conditions with respect to the holdings of short-term treasury bills or central bank's reserves price the nominal interest rate according to

$$\frac{1}{(1+i_t)} = E_t R_{t,t+1}, \tag{15}$$

where the equilibrium nominal stochastic discount factor is $R_{t,t+1} = \beta \lambda_{t+1}/\lambda_t$, as implied by the optimality conditions – not reported here – with respect to the state-contingent securities. Combining (13)–(15) it follows

$$\varphi_t = \frac{i_t}{1+i_t}\lambda_t \tag{16}$$

from which $i_t \geq 0$, since $\varphi_t \geq 0$ and $\lambda_t > 0$. The complementary slackness condition on the constraint (10) can be written as

$$\varphi_t(M_t - P_t C_t) = 0.$$

The first-order condition with respect to lending or borrowing using long-term fixed-rate securities requires

$$Q_t = E_t[R_{t,t+1}(1 - \varkappa_{t+1})(1 + \delta Q_{t+1})]. \tag{17}$$

Finally the first-order condition with respect to labor can be written as

$$\xi_t(L_t(j))^\eta = \beta W_t(j) E_t \lambda_{t+1},$$

for each variety j. Combining the latter with (14) and (16), we can write the marginal rate of substitution between labor and consumption, for each variety j, as

$$\frac{(L_t(j))^\eta}{C_t^{-\rho}} = \frac{1}{1+i_t}\frac{W_t(j)}{P_t}, \tag{18}$$

which is shifted by movements in the nominal interest rate, reflecting the financial friction. Wage income, indeed, can be used to purchase goods only with one-period delay.

To conclude the characterization of the household's problem, a transversality condition

applies and therefore (11) holds with equality, given the equilibrium nominal stochastic discount factor $R_{t,T} = \beta^{T-t}\lambda_T/\lambda_t$.

3.1 Firms

We assume that there is a continuum of firms of measure one, each producing one of the goods in the economy. The production function is linear in labor $Y(j) = L(j)$. Given preferences, each firm faces a demand of the form $Y(i) = (P(i)/P)^{-\theta}Y$ where in equilibrium aggregate output is equal to consumption

$$Y_t = C_t. \tag{19}$$

Firms are subject to price rigidities as in the Calvo model. A fraction of measure $(1 - \alpha)$ of firms with $0 < \alpha < 1$ is allowed to change its price. The remaining fraction α of firms indexes their previously-adjusted prices to the inflation target $\bar{\Pi}$. Adjusting firms choose prices to maximize the presented discounted value of profits under the circumstances that the prices chosen, appropriately indexed to the inflation target, will remain in place until period T with probability α^{T-t}:

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\lambda_T \left[\bar{\Pi}^{T-t}P_t(j)Y_T(j) - (1 - \varrho)W_T(j)Y_T(j) \right],$$

where ϱ is a constant subsidy on firms' labor costs. The optimality condition implies

$$\frac{P_t^*(j)}{P_t} = \mu \frac{E_t \left\{ \sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\lambda_T \left(\frac{P_T}{P_t}\frac{1}{\bar{\Pi}^{T-t}} \right)^{\theta} W_T(j)Y_T \right\}}{E_t \left\{ \sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\lambda_T P_t\bar{\Pi}^{T-t} \left(\frac{P_T}{P_t}\frac{1}{\bar{\Pi}^{T-t}} \right)^{\theta} Y_T \right\}} \tag{20}$$

in which we have used the demand function $Y(i) = (P(i)/P)^{-\theta}Y$ and have defined $\mu \equiv \theta(1 - \varrho)/(\theta - 1)$. We can also replace in the previous equation $\lambda_t = C_t^{-\rho}\xi_t/P_t$ and $W_t(j)/P_t$ from (18) together with the demand function, $Y(i) = (P(i)/P)^{-\theta}Y$, to obtain

$$\left(\frac{P_t^*}{P_t} \right)^{1+\theta\eta} = \mu \frac{E_t \left\{ \sum_{T=t}^{\infty}(\alpha\beta)^{T-t} \left(\frac{P_T}{P_t}\frac{1}{\bar{\Pi}^{T-t}} \right)^{\theta(1+\eta)} (1 + i_T)Y_T^{1+\eta}\xi_T \right\}}{E_t \left\{ \sum_{T=t}^{\infty}(\alpha\beta)^{T-t} \left(\frac{P_T}{P_t}\frac{1}{\bar{\Pi}^{T-t}} \right)^{\theta-1} Y_T^{1-\rho}\xi_T \right\}}$$

where P_t^* is the common price chosen by the firms that can adjust it at time t.

Calvo's model further implies the following law of motion of the general price index

$$P_t^{1-\theta} = (1-\alpha)P_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta}\bar{\Pi}^{1-\theta}, \tag{21}$$

through which we can write the aggregate supply equation as

$$\left(\frac{1-\alpha\Pi_t^{\theta-1}\bar{\Pi}^{1-\theta}}{1-\alpha}\right)^{\frac{1+\theta\eta}{1-\theta}} = \mu\frac{E_t\left\{\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left(\frac{P_T}{P_t}\frac{1}{\bar{\Pi}^{T-t}}\right)^{\theta(1+\eta)}(1+i_T)Y_T^{1+\eta}\xi_T\right\}}{E_t\left\{\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left(\frac{P_T}{P_t}\frac{1}{\bar{\Pi}^{T-t}}\right)^{\theta-1}Y_T^{1-\rho}\xi_T\right\}}. \tag{22}$$

3.2 Budget constraints of treasury and central bank

To complete the description of the model we specify the budget constraints of treasury and central bank.

The treasury can finance its deficit through short-term and long-term debt, respectively B_t^G and D_t^G at the prices $1/(1+i_t)$ and Q_t. It faces the following flow budget constraint

$$Q_t D_t^G + \frac{B_t^G}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t)D_{t-1}^G + B_{t-1}^G - A_t - T_t^C$$

given initial conditions $D_{t_0-1}^G$, $B_{t_0-1}^G$ where T_t^C represents the transfers between the treasury and the central bank.[21] The primary surplus A_t is defined as:

$$A_t \equiv -T_t - \varrho\int_0^1 W_t(j)L_t(j).$$

Central-bank net worth – the difference between its assets and liabilities – is given by

$$N_t \equiv Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t^C - \frac{X_t^C}{1+i_t}, \tag{23}$$

since the central bank can hold assets in the form of short-term and long-term fixed-rate securities, B_t^C and D_t^C respectively.

The portfolio of assets in (23) is financed by issuing non-interest bearing liabilities – the sum of money M_t^C and net worth itself N_t – and reserves, X_t^C. Net worth evolves over time depending on the profits that are not distributed to the treasury:

$$N_t = N_{t-1} + \Psi_t^C - T_t^C \tag{24}$$

---

[21]It is worth reminding that to simplify the analysis we have assumed that there is only one long-term security which is issued either by the private sector or by the treasury. In particular, without losing generality, we assume $D_t^G = 0$ if $D_t < 0$ and $D_t \geq 0$ if $D_t^G > 0$.

where Ψ_t^C are central bank's profits, which depend on the composition of its balance sheet:[22]

$$\Psi_t^C = \frac{i_{t-1}}{1+i_{t-1}}(B_{t-1}^C - X_{t-1}^C) + [(1-\varkappa_t)(1+\delta Q_t) - Q_{t-1}]\,D_{t-1}^C. \tag{25}$$

They can also be written as

$$\Psi_t^C = i_{t-1}(N_{t-1} + M_{t-1}) + (r_t - i_{t-1})Q_{t-1}D_{t-1}^C \tag{26}$$

having used the definition $(1+r_t) \equiv (1+\delta Q_t)(1-\varkappa_t)/Q_{t-1}$.[23] Central bank's profits depend on two components: the first captures the revenues obtained by issuing non-interest bearing liabilities;[24] the second component, instead, represents the excess gains or losses of holding long-term securities with respect to a riskless portfolio. Since the realized excess return on these securities can be negative, the latter component can be as well negative – the more so the larger are the holdings of long-term securities – producing income losses for the central bank. In our model, these losses can happen in two cases: $i)$ when a credit event realizes; $ii)$ when short-term interest rates move unexpectedly.

Combining (23) and (24), we can write the central bank's flow budget constraint:

$$Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t^C - \frac{X_t^C}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t)D_{t-1}^C + B_{t-1}^C - X_{t-1}^C - M_{t-1}^C - T_t^C,$$

given initial conditions $D_{t_0-1}^C, B_{t_0-1}^C, X_{t_0-1}^C, M_{t_0-1}^C$.

In equilibrium, short-term debt issued by the government is held either by the private sector or the central bank

$$B_t^G = B_t + B_t^C,$$

while long-term debt held by the central bank is issued either by the private sector or the treasury

$$D_t^C = D_t^G - D_t.$$

Equilibrium in the market of money and reserves implies

$$M_t = M_t^C$$

---

[22]We abstract from the dividends that the central bank gives to the member banks. In the US, this amounts to 6% of capital (see Carpenter et al., 2015).

[23]The definition is clearly only valid for positive asset prices.

[24]Later we discuss the critical role played by the non-negativeness of the overall non-interest bearing liabilities for central bank's solvency.

and

$$X_t = X_t^C$$

respectively.

Finally, state-contingent securities are traded in zero-net supply within the private sector.

3.3 Equilibrium

Here, we describe in a compact way the equations which characterize the equilibrium allocation.

The Euler equations for short-term and long-term securities, (15) and (17), imply that

$$\frac{1}{1+i_t} = E_t \left\{ \beta \frac{\xi_{t+1} Y_{t+1}^{-\rho}}{\xi_t Y_t^{-\rho}} \frac{1}{\Pi_{t+1}} \right\}, \tag{27}$$

and

$$Q_t = E_t \left\{ \beta \frac{\xi_{t+1} Y_{t+1}^{-\rho}}{\xi_t Y_t^{-\rho}} \frac{(1 - \varkappa_{t+1})(1 + \delta Q_{t+1})}{\Pi_{t+1}} \right\}, \tag{28}$$

respectively, in which we have used the equilibrium value $\lambda_t = \xi_t C_t^{-\rho}/P_t$, the resource constraint (19) and we have defined the inflation rate $\Pi_t \equiv P_t/P_{t-1}$.

The cash-in-advance constraint (10) together with the resource constraint (19) implies

$$M_t \geq P_t Y_t, \tag{29}$$

while the complementary slackness condition is given by

$$i_t(M_t - P_t Y_t) = 0. \tag{30}$$

Aggregate-supply is described by

$$\left(\frac{1 - \alpha \Pi_t^{\theta-1} \bar{\Pi}^{1-\theta}}{1 - \alpha} \right)^{\frac{1+\theta\eta}{1-\theta}} = \frac{F_t}{K_t} \tag{31}$$

where F_t and K_t are defined recursively as:

$$F_t = \mu(1 + i_t)\xi_t Y_t^{1+\eta} + \alpha\beta E_t \left\{ \Pi_t^{\theta(1+\eta)} \bar{\Pi}^{-\theta(1+\eta)} F_{t+1} \right\} \tag{32}$$

$$K_t = \xi_t Y_t^{1-\rho} + \alpha\beta E_t \left\{ \Pi_t^{\theta-1} \bar{\Pi}^{1-\theta} K_{t+1} \right\}. \tag{33}$$

To the end of characterizing optimal policy in this model it is also convenient to add the

variable Δ_t, an index of relative-price dispersion which follows the law of motion

$$\Delta_t = \Delta\left(\frac{\Pi_t}{\bar{\bar{\Pi}}}, \Delta_{t-1}\right) \equiv \alpha\left(\frac{\Pi_t}{\bar{\bar{\Pi}}}\right)^{\theta(1+\eta)}\Delta_{t-1} + (1-\alpha)\left(\frac{1 - \alpha\Pi_t^{\theta-1}\bar{\Pi}^{1-\theta}}{1-\alpha}\right)^{\frac{\theta(1+\eta)}{\theta-1}}. \tag{34}$$

The bound (12) can be written as

$$E_t\left\{\sum_{T=t}^{\infty} \beta^{T+1-t}\xi_{T+1}Y_{T+1}^{-\rho}\left[Y_T + \frac{i_T}{1+i_T}\frac{M_T}{P_T}\right]\right\} < \infty, \tag{35}$$

in which we have used also (29) while the transversality condition, with equality, completes the demand side of the model

$$\lim_{T \longrightarrow \infty} E_t\left[\beta^{T-t}\frac{\xi_T Y_T^{-\rho}}{P_T}\left(M_T + \frac{B_T + X_T}{1+i_T} + Q_T D_T\right)\right] = 0, \tag{36}$$

which is derived from (11) where we have used $R_{t,T} = \beta^{T-t}\lambda_T/\lambda_t$, $\lambda_t = \xi_t C_t^{-\rho}/P_t$ and the resource constraint (19) together with the fact that the state-contingent securities are traded in zero-net supply within the private sector: the transversality condition therefore constrains just the long-run behavior of the "outside" assets held by the households.

The treasury and central-bank budget constraints are given by

$$Q_t D_t^G + \frac{B_t^G}{1+i_t} = (1 - \varkappa_t)(1 + \delta Q_t)D_{t-1}^G + B_{t-1}^G - A_t - T_t^C \tag{37}$$

and

$$Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t - \frac{X_t}{1+i_t} = (1 - \varkappa_t)(1 + \delta Q_t)D_{t-1}^C + B_{t-1}^C - X_{t-1} - M_{t-1} - T_t^C \tag{38}$$

respectively, where equilibrium in the asset markets implies that

$$B_t^G = B_t + B_t^C \tag{39}$$

$$D_t^G - D_t = D_t^C. \tag{40}$$

It is important to note that the transversality condition (36) does not imply two separate transversality conditions for the treasury and the central bank, but only one aggregate transversality condition on the net consolidated liabilities of both institutions which together with the flow budget constraints (37) and (38) implies a consolidated intertemporal budget constraint. We do not explicitly write this constraint since it is already implied by the set of equations written above.

To complete the characterization of the rational expectations equilibrium we need to specify the monetary/fiscal policy regime. First we note that excluding the bounds (35) and (36) and the complementary-slackness condition (30) there are eleven equilibrium conditions for the seventeen unknown stochastic processes $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t, X_t, B_t, B_t^C, B_t^G, D_t,$ $D_t^C, D_t^G, A_t, T_t^C\}$ given the definition $\Pi_t \equiv P_t/P_{t-1}$, implying that the monetary/fiscal policy regime should specify six additional equations. In particular, the monetary/fiscal policy regime specifies six of the sequences $\{i_t, M_t, X_t, B_t^C, B_t^G, D_t^C, D_t^G, A_t, T_t^C\}$, possibly as functions of some other endogenous variables and/or of exogenous state variables.[25]

Definition 1 *A **model** rational expectations equilibrium is a collection of stochastic processes* $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t, X_t, B_t, B_t^C, B_t^G, D_t, D_t^C, D_t^G, A_t, T_t^C\}$, *satisfying each of the conditions in equations (27) to (40) at each time $t \geq t_0$ (and in each contingency at t) consistently with the specification of a monetary/fiscal policy regime and given the definition $\Pi_t \equiv P_t/P_{t-1}$, the non-negativity constraint on the nominal interest rate $i_t \geq 0$, the stochastic processes for the exogenous disturbances $\{\xi_t, \varkappa_t\}$ and initial conditions $\Delta_{t_0-1}, M_{t_0-1}, X_{t_0-1}, B_{t_0-1}^C,$ $B_{t_0-1}^G, D_{t_0-1}^C, D_{t_0-1}^G$.[26]*

The above definition of rational expectations equilibrium may appear unconventional given most of the analysis in the literature using similar class of models. Indeed, it is often the case that one can determine the equilibrium output, inflation and interest rate by focusing on a smaller set of equations and variables. Note, however, that this is not a general feature of this class of models, but it actually depends on the specification of the monetary/fiscal policy regime. To understand this observation, consider the equilibrium conditions (27) to (35). Since (30) is a complementary-slackness condition and (35) is a bound, there are seven equations in the eight unknown stochastic processes $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t\}$, leaving the possibility to specify one of the stochastic processes $\{i_t, M_t\}$. A special case is defined by what we call *conventional monetary policy*, in which one of the stochastic processes $\{i_t, M_t\}$ is specified as a function of the other endogenous variables Y, Π, Q, F, K, Δ and/or exogenous state variables. A classical Taylor rule is included in this specification. Complementary to this definition, we also specify a *transfer policy* in which both the stochastic processes $\{A_t, T_t^C\}$ are specified as functions of the other endogenous variables and/or of exogenous states. Therefore we are left with the specification of three of the sequences $\{X_t, B_t^C, B_t^G, D_t^C, D_t^G\}$ to complete the characterization of the monetary/fiscal policy regime. In this work we limit

---

[25] As it will be clear later, some sequences cannot be specified independently from each other.

[26] In the definition, the time-t component of the endogenous stochastic process is meant to be a function of the history of shocks, $s^t \equiv (s_t, s_{t-1}, s_{t-2}, ..., s_{t_0})$ and the initial conditions $\Delta_{t_0-1}, M_{t_0-1}, X_{t_0-1}B_{t_0-1}^C, B_{t_0-1}^G, D_{t_0-1}^C, D_{t_0-1}^G$. The state s_t is a vector including ξ_t and χ_t and other exogenous state variables, which can be specified by the monetary/fiscal policy regime. It can also include sunspot disturbances.

our attention to regimes in which three of the sequences $\left\{B_t^C, B_t^G, D_t^C, D_t^G\right\}$ are specified as functions of the other endogenous variables and/or of exogenous state variables. It should be noted that (37) implies that $\left\{B_t^G, D_t^G\right\}$ cannot be chosen independently given that a *transfer policy* is specified.

Definition 2 *A **conventional monetary policy** specifies one of the stochastic processes* $\left\{i_t, M_t\right\}$ *as function of the other endogenous variables* Y, Π, Q, F, K, Δ *and/or of exogenous state variables. A **transfer policy** specifies both the stochastic processes* $\left\{A_t, T_t^C\right\}$ *as functions of the other endogenous variables and/or of exogenous state variables. A **balance-sheet policy** specifies three of the stochastic processes* $\left\{B_t^C, B_t^G, D_t^C, D_t^G\right\}$ *as functions of the other endogenous variables and/or of exogenous state variables, but not* $\left\{B_t^G, D_t^G\right\}$ *independently.*

Given this premise we now introduce the notion of *benchmark* rational expectations equilibrium.

Definition 3 *A **benchmark** rational expectations equilibrium is a collection of stochastic processes* $\left\{Y_t, \Pi_t, \ i_t, Q_t, F_t, K_t, M_t, \Delta_t, X_t, B_t, B_t^C, B_t^G, D_t, D_t^C, D_t^G, A_t, T_t^C\right\}$ *together with a **conventional monetary policy**, a **transfer policy** and a **balance-sheet policy** such that i)* $\left\{Y_t, \Pi_t, \ i_t, Q_t, F_t, K_t, M_t, \Delta_t\right\}$ *satisfy each of the conditions in equations (27) to (35) at each time $t \geq t_0$ (and in each contingency at t) consistently with the specification of the **conventional monetary policy**, given the definition $\Pi_t \equiv P_t/P_{t-1}$, the non-negativity constraint on the nominal interest rate $i_t \geq 0$, the stochastic processes for the exogenous disturbances $\{\xi_t, \varkappa_t\}$, the initial condition Δ_{t_0-1} and ii)* $\left\{X_t, B_t, B_t^C, B_t^G, D_t, D_t^C, D_t^G, A_t, T_t^C\right\}$ *satisfy each of the conditions in equations (36) to (40) at each time $t \geq t_0$ (and in each contingency at t) consistently with the specification of the **transfer** and **balance-sheet policies**, given the same stochastic processes* $\left\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t\right\}$ *of part i) and initial conditions $M_{t_0-1}, X_{t_0-1}, B_{t_0-1}^C, B_{t_0-1}^G, D_{t_0-1}^C, D_{t_0-1}^G$.*

The definition of *benchmark* equilibrium clarifies that any *benchmark* equilibrium is a *model* equilibrium. However, the reverse is not true because the general class of monetary/fiscal policy regimes consistent with Definition 1 is broader than the regimes that can be spanned by the various possible specifications of *conventional monetary policy, transfer* and *balance-sheet policies* of Definition 3.[27] It is not the plan of this work to characterize all possible monetary/fiscal policy regimes. However, the definition of *benchmark* equilibrium

---

[27]It should be noted that our definition of *benchmark* equilibrium can also allow for multiple collections of stochastic processes $\left\{Y_t, \Pi_t, \ i_t, Q_t, F_t, K_t, M_t, \Delta_t\right\}$ to satisfy each of the conditions in equations (27) to (35) at each time $t \geq t_0$ (and in each contingency at t) consistently with the same specification of conventional monetary policy. Uniqueness of equilibrium is neither an issue for our results nor something that we address in this work.

is useful not only to understand how it is possible to determine output, inflation and other variables by just focusing on a small set of equations, but also to study a Proposition of Neutrality of alternative compositions of central-bank balance sheet. We now turn to this analysis.

4   Proposition of Neutrality

We are interested in studying the relevance of the composition and size of the central bank's balance sheet for equilibrium. To this end, we should be careful not to mistakenly conclude for a relationship between macroeconomic variables and unconventional policies when instead something else in the specification of the monetary/fiscal policy regime has contextually changed with the implementation of unconventional monetary policy, leading to the variation in the macroeconomic variables observed in equilibrium. In this respect, the definition of *benchmark* equilibrium is helpful to make the right comparison and build an appropriate Proposition of Neutrality to study.

Consider indeed a *benchmark* rational expectations equilibrium and the associated *conventional monetary policy, transfer policy* and *balance-sheet policy*. Maintain the same *conventional monetary policy* and *transfer policy*. Let instead the specification of three stochastic processes among $\left\{B_t^C, B_t^G, D_t^C, D_t^G\right\}$ vary. These alternative *balance-sheet* regimes are said to be "neutral" with respect to the equilibrium allocation $\left\{Y_t, \Pi_t,\ i_t, Q_t, F_t, K_t, M_t, \Delta_t\right\}$ if the latter collection of processes is still a *benchmark* rational expectations equilibrium. This means that consistently with the same *transfer policy* and the new *balance-sheet* regime there is some $\left\{\tilde{X}_t,\ \tilde{B}_t,\ \tilde{B}_t^C, \tilde{B}_t^G, \tilde{D}_t, \tilde{D}_t^C, \tilde{D}_t^G, \tilde{A}_t, \tilde{T}_t^C\right\}$ satisfying each of the conditions in equations (36) to (40) at each time $t \geq t_0$ (and in each contingency at t) given the same stochastic processes $\left\{Y_t, \Pi_t,\ i_t, Q_t, F_t, K_t, M_t, \Delta_t\right\}$, *conventional monetary policy* and initial conditions.[28] This Proposition of Neutrality should be valid for any benchmark equilibrium and any appropriately-bounded specification of three of the stochastic processes $\left\{B_t^C, B_t^G, D_t^C, D_t^G\right\}$ consistent with the definition of *balance-sheet policy*.

Proposition 1 (*Proposition of Neutrality of alternative balance-sheet policies*) *Given any* **benchmark** *rational expectations equilibrium, the collection of stochastic processes* $\left\{Y_t, \Pi_t,\right.$ $\left. i_t, Q_t, F_t, K_t, M_t, \Delta_t,\right\}$ *of each* **benchmark** *rational expectations equilibrium is consistent*

---

[28]It is worth noting that in the new equilibrium the stochastic processes $\left\{\tilde{A}_t, \tilde{T}_t^C\right\}$, which are functions of the history and the initial conditions, can vary with respect to the original *benchmark* equilibrium, although the specifications of the functional forms of $\left\{A_t, T_t^C\right\}$ characterized by the *transfer policy* are not changed. To clarify this point recall the examples provided by rules (5) and (6) in Section 2, which specify $\left\{A_t, T_t^C\right\}$ as functions of other endogenous variables, and in particular of variables related to the *balance-sheet policy*.

*with another **benchmark** rational expectations equilibrium for the same **conventional monetary policy** and **transfer policy** and for any appropriately bounded **balance-sheet policy**.*

A key feature of the Proposition of Neutrality is that the specifications of the functional forms of the *transfer policy* – referring to both the stochastic processes $\left\{A_t, T_t^C\right\}$ as functions of other endogenous or state variables – and of the *conventional monetary policy* – referring to one of the stochastic processes $\left\{i_t, M_t\right\}$ again as function of other endogenous or state variables – are kept constant across the comparison while what it is varied is the functional form of three of the processes $\left\{B_t^C, B_t^G, D_t^C, D_t^G\right\}$. Among the various alternatives, it is possible to consider combinations in which

$$Q_t \tilde{D}_t^G + \frac{\tilde{B}_t^G}{1 + i_t} = L_t^G$$

$$Q_t \tilde{D}_t^C + \frac{\tilde{B}_t^C}{1 + i_t} = A_t^C$$

capturing the first equation the irrelevance of varying the maturity composition of treasury's debt while the second the irrelevance of varying the maturity composition of the central-bank assets; L_t^G and A_t^C are respectively the equilibrium paths of the treasury's liabilities and central-bank assets consistent with the initial *benchmark* equilibrium. The above two restrictions allow, for example, to specify alternative functional forms for the processes $\left\{D_t^C, D_t^G\right\}$. Another possibility is to vary D_t^C and B_t^C in a way that not only the maturity composition of the assets of the central bank changes but also the overall size implying a path $\tilde{A}_t^C$ different from the initial one A_t^C.[29]

We show that the Proposition of Neutrality holds in our model only conditional on some specifications of the *transfer policy*. This is not surprising. Indeed Wallace (1981) proved his irrelevance result of open-market operations in an overlapping-generation monetary model by relying on a particular *transfer policy*, and Sargent and Smith (1987) and Eggertsson and Woodford (2003) restated it in models with a non-pecuniary return of money, like ours.[30]

The road map of our analysis is as follows. First we analyze a class of *transfer policies* for which the Proposition of Neutrality holds, then we move to discuss alternative specifications of $\left\{A_t, T_t^C\right\}$, implying violations of the neutrality result, which are more of practical interest since they correspond to current institutional arrangements, in particular between central bank and treasury. In our analysis, we pay particular attention to the welfare consequences

---

[29]Having varied the specification of $\left\{B_t^C, D_t^C\right\}$, one is left with the possibility of specifying an alternative functional form for only one of the two stochastic processes $\left\{B_t^G, D_t^G\right\}$.

[30]Unlike our model, all these works consider a consolidated balance sheet for treasury and central bank.

of *balance-sheet policies* by analyzing whether the equilibrium maximizing the welfare of the consumers (7) varies depending on alternative *balance-sheet* regimes and *transfer policies*. The welfare criterion can be also rewritten as

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \xi_t \left[\frac{Y_t^{1-\rho}}{1-\rho} - \frac{Y_t^{1+\eta}}{1+\eta} \Delta_t \right]. \tag{41}$$

The **model** optimal policy can be simply characterized by focusing first on a small set of equations and then adding appropriate *transfer* and *balance-sheet policies* to support it as an equilibrium.

Definition 4 *The **benchmark** optimal policy is: i) a collection of stochastic processes $\{Y_t^*, \Pi_t^*, i_t^*, Q_t^*, F_t^*, K_t^*, M_t^*, \Delta_t^*\}$ that maximizes (41) and satisfies each of the conditions in equations (27) to (35) at each time $t \geq t_0$ (and in each contingency at t), given the definition $\Pi_t \equiv P_t/P_{t-1}$, the non-negativity constraint on the nominal interest rate $i_t \geq 0$, the stochastic processes for the exogenous disturbances $\{\xi_t, \varkappa_t\}$, given an initial condition Δ_{t_0-1} and ii) some collections of stochastic processes $\{X_t^*, B_t^*, B_t^{*C}, B_t^{*G}, D_t^*, D_t^{*C}, D_t^{*G}, A_t^*, T_t^{*C}\}$ satisfying each of the conditions in equations (36) to (40) at each time $t \geq t_0$ (and in each contingency at t) given the stochastic processes $\{Y_t^*, \Pi_t^*, i_t^*, Q_t^*, F_t^*, K_t^*, M_t^*, \Delta_t^*\}$ and initial conditions $M_{t_0-1}, X_{t_0-1}, B_{t_0-1}^C, B_{t_0-1}^G, D_{t_0-1}^C, D_{t_0-1}^G.$*[31]

In the above characterization of optimal policy the equilibrium conditions (28) and (29) are not really constraints for the path of the other endogenous variables mentioned in part i) of Definition 4 but just determine, respectively, the price of long-term bonds and money supply – provided the latter satisfies the bound (35).[32] This implies that the state $\varkappa$ influences only the stochastic process $\{Q_t^*\}$ under the optimal *benchmark* policy. Unlike standard analysis in the literature, the Euler equation (27) can be a restriction on the optimal path of inflation and output because the financial friction matters directly into the aggregate-supply equation through the nominal interest rate. The definition of *benchmark* optimal policy parallels that of *benchmark* equilibrium and indeed its determination can be understood in a similar way. First one can look at a small set of constraints and choose the optimal path $\{Y_t^*, \Pi_t^*, i_t^*, Q_t^*, F_t^*, K_t^*, M_t^*, \Delta_t^*\}$, then a *monetary/fiscal policy* regime can be found to "support" it as a *model* equilibrium.

Proposition 2 *The **benchmark** optimal policy is the best policy among **model** equilibria.*

---

[31] We could also add ϱ, the subsidy on firms' labor cost, as instrument of policy but this will not change the qualitative results of our analysis. In the numerical examples we are going to set it consistently with the efficient steady state.

[32] When $i_t = 0$, condition (29) determines a lower bound on money supply.

Proof. It follows by its definition that the *benchmark* optimal policy is a model equilibrium. To prove that it is also the *model*'s optimal policy, just consider that it allows to choose optimally the welfare-relevant stochastic processes $\{Y_t, \Delta_t\}$ by restricting attention to a smaller set of constraints than those included in the definition of *model* equilibrium. Therefore, it cannot be improved by another *model* equilibrium. ■

We can formulate a similar Proposition of Neutrality by asking whether the *benchmark* optimal policy is feasible regardless of central bank's *balance-sheet policies*. In this respect, our analysis will conclude that either i) there are some transfer policies for which the optimal policy coincides with the *benchmark* optimal policy irrespective of the portfolio composition of the central bank or ii) there are some other transfer policies for which the optimal policy still coincides with the *benchmark* optimal policy only when the central bank holds short-term assets otherwise welfare decreases for other portfolio compositions, or iii) there are some transfer policies for which the optimal policy does not coincide with the *benchmark* optimal policy, and welfare may be increased by lengthening the maturity structure of the central bank's assets, although never beyond that of the *benchmark* optimal policy. The bottom line is that unconventional monetary policies are never welfare improving if the reference point is the *benchmark* optimal policy. On the contrary, in line with point iii) above, welfare can increase starting from sub-optimal allocations.

To further clarify some of the previous debate on the effects of unconventional monetary policy in this class of models, note that if the *benchmark* optimal policy requires in some contingencies that $i_t^* = 0$ there will be multiple paths of money supply (provided all satisfy $M_t^* > P_t^* Y_t^*$) that can support the same optimal path of the other variables mentioned in the first part of Definition 4. These different paths of money supply can be consistent at the same time with alternative portfolio compositions of the central bank's assets, thereby including also long-term securities, implying some other stochastic processes $\{X_t^*, B_t^*, B_t^{*C}, B_t^{*G}, D_t^*, D_t^{*C}, D_t^{*G}, A_t^*, T_t^{*C}\}$ consistent with part *ii*) of the same Definition 4. But all these alternative paths and compositions are clearly not welfare-improving as Definition 4 and Proposition 2 show, since they support the same stochastic processes for the welfare-relevant variables.[33]

---

[33]See also Eggertsson and Woodford (2003). Auerbach and Obstfeld (2005) show that policies raising the money supply at the zero-lower bound consistently with $M_t > P_t Y_t$ can have an effect on current price level and output since they affect the price level and output once the economy exits from the zero-lower bound. They can also influence the duration of the trap. However, these effects rely on a change in policy (what we called *conventional monetary policy*) that lasts after the trap ends. Instead, the neutrality result holds if after the end of the trap the *conventional monetary policy* is not changed, as in the case of the optimal policy described in the paragraph (see also Robatto, 2014). Buiter (2014) shows that an expansion in the stock of base money can have permanent wealth effect even in a permanent liquidity trap provided money is not seen as a liability by the central bank.

4.1 Case I: Passive fiscal policy and passive remittances' policy

We start with the case mostly studied in the literature, in which the Proposition of Neutrality holds. The literature usually proceeds to make assumptions about the "consolidated" behavior of government, including central bank and treasury.[34] We instead keep the two institutions as separate since this is key, in our analysis, to characterize departures from the Proposition of Neutrality. We start by defining a regime in which fiscal policy is passive, in line with the literature, but focusing on the behavior of the treasury instead of that of the consolidated government.

Definition 5 *Under a passive fiscal policy the stochastic path of real primary surplus* $\left\{\frac{A_t}{P_t}\right\}$
is specified to ensure that the following limiting condition

$$\lim_{T \longrightarrow \infty} E_t \left[\tilde{R}_{t,T} \left(Q_t \frac{D_t^G}{P_t} + \frac{1}{1+i_t} \frac{B_t^G}{P_t} \right) \right] = 0 \tag{42}$$

is satisfied looking forward from each date $t \geq t_0$ *(and in each contingency at* t*) together with the sequence of equilibrium conditions* (37) *for any appropriately bounded* D_{t-1}^G, B_{t-1}^G, *any appropriately bounded stochastic process* $\left\{T_t^C\right\}$ *and for any stochastic processes* $\{Y_t, \Pi_t, i_t, Q_t, F_t,$
$K_t, M_t, \Delta_t\}$ *satisfying the conditions of part i) of Definition* 3 *consistently each with a specified conventional monetary policy.*[35]

According to the definition above, the real primary surplus is set in a way that the expected present discounted real value of treasury liabilities converges to zero for any stochastic processes $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t\}$ satisfying the equilibrium conditions (27) to (35) given a (and for any) *conventional monetary policy*. In equation (42), we have defined the real stochastic discount factor as $\tilde{R}_{t,T} \equiv \beta^{T-t} \xi_T Y_T^{-\rho} / \xi_t Y_t^{-\rho}$. We show in the Appendix that a fiscal rule for the real primary surplus of the form

$$\frac{A_t}{P_t} = \bar{a} - \frac{T_t^C}{P_t} + \phi \left[\frac{(1+r_t)Q_{t-1}D_{t-1}^G + B_{t-1}^G}{P_t} \right] \tag{43}$$

for some $\bar{a}$ and ϕ, with $0 < \phi < 1$, is consistent with the definition of passive fiscal policy.[36] In the above equation, recall that we have defined the gross nominal return on long-term debt as $(1 + r_t) \equiv (1 + \delta Q_t)(1 - \varkappa_t)/Q_{t-1}$. According to the rule, an increase in the outstanding

---

[34]This is the case of Wallace (1981), Sargent and Smith (1987), Eggertsson and Woodford (2003), Robatto (2014).

[35]There is indeed a caveat to consider in Definition 5: that D_{t-1}^G, B_{t-1}^G and the stochastic processes $\left\{T_t^C\right\}$ are appropriately bounded. Otherwise, the required primary surplus can be too large implying a negative consumption. In the analysis that follows we always rule out this possibility.

[36]The parameter $\bar{a}$ should be appropriately bounded to avoid that consumption is negative. It can be zero.

real market value of treasury debt – of whatever maturity – signals an adjustment in the path of primary surpluses needed to repay it. Furthermore, the rule is such that the treasury does not have to rely on central bank's remittances to repay its debt. In line with this, a reduction in the remittances from the central bank implies an immediate and specular increase of the primary surplus. If the central bank is reducing the transfers to the treasury, the treasury should immediately increase lump-sum taxes on the private sector to "support" the same equilibrium allocation for output, inflation and interest rate. This may seem a strong requirement but it is not just a peculiarity of the above rule. Indeed, we can consider another way to state the above definition of passive fiscal policy by noting that (42) together with (37) and (27)-(28) implies that

$$\frac{B_{t-1}^G}{P_t} + (1+r_t)\frac{Q_{t-1}D_{t-1}^G}{P_t} = E_t \sum_{T=t}^{\infty} \tilde{R}_{t,T}\left[\frac{A_T}{P_T} + \frac{T_T^C}{P_T}\right].$$

Under passive fiscal policy, the real primary surplus adjusts in a way to ensure that the above equation holds under any stochastic processes $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t\}$ satisfying the equilibrium conditions (27) to (35) given a *conventional* monetary policy and for any appropriately bounded stochastic process $\{T_t^C\}$ and bounded D_{t-1}^G, B_{t-1}^G. A reduction in the present discounted value of remittances from the central bank should reflect under a passive fiscal policy a specular increase in the present discounted value of the primary surpluses and therefore in the present discounted value of the taxes levied on the private sector.

A passive fiscal policy has direct implications for the equilibrium path of central bank's net worth. Indeed, equation (42), together with the equilibrium condition (36), requires that the expected present discounted value of the central bank's real net worth converge to zero in equilibrium:

$$\lim_{T\longrightarrow\infty} E_t\left[\tilde{R}_{t,T}\frac{N_T}{P_T}\right] = 0. \tag{44}$$

The latter equilibrium condition together with the flow budget constraint of the central bank (38) and (27)-(28) now implies the following intertemporal budget constraint

$$\frac{X_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} - \frac{B_{t-1}^C}{P_t} - (1+r_t)\frac{Q_{t-1}D_{t-1}^C}{P_t} = E_t \sum_{T=t}^{\infty} \tilde{R}_{t,T}\left[\frac{i_T}{1+i_T}\frac{M_T}{P_T} - \frac{T_T^C}{P_T}\right]. \tag{45}$$

The real market value of the outstanding net liabilities of the central bank at a generic time t should be backed by the present discounted value of the revenues obtained by issuing non-interest bearing securities – the pure seigniorage – net of the transfers between the central bank and the treasury. Interestingly, there could be rational-expectations equilibria in which net worth is negative, provided that the incoming seigniorage net of transfers is enough to

back the net liabilities of the central bank. However, the key observation is that, in general, (45) can restrict the path of prices, interest rates and other endogenous variables in a way that some stochastic processes $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t\}$ satisfying the equilibrium conditions (27) to (35) consistently with some *conventional monetary policy* are not equilibria, unless additional assumptions are added. To this end we now turn to define a passive policy of central bank's remittances, in a similar way to the above definition of passive fiscal policy and irrespective of the latter specification.

Definition 6 *Under a passive policy of central-bank remittances the stochastic path of real remittances $\left\{ \frac{T_t^C}{P_t} \right\}$ is specified to ensure that (44) is satisfied looking forward from each date $t \geq t_0$ (and in each contingency at t) together with the sequence of equilibrium conditions (38) for any appropriately bounded $X_{t-1}, B_{t-1}^C, D_{t-1}^C$ and for any sequence of stochastic processes $\left\{ Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t \right\}$ satisfying the conditions of part i) of Definition 3 consistently each with a specified conventional monetary policy.*[37]

Under a passive remittances' policy, a worsening of the market value of the central-bank net liability position signals an increase in seigniorage revenues or in transfers from the treasury. The combined regime given by the two passive policies defined above supports the Proposition of Neutrality:

Proposition 3 *Under a combined regime of passive fiscal policy and passive policy of central-bank remittances i) the Proposition of Neutrality holds and ii) the **benchmark** optimal policy is the **equilibrium** optimal policy.*

Proof. In the Appendix. ∎

Under the conditions stated in Proposition 3 whether the central bank purchases or not long-term securities, eventually recording losses for these operations, is irrelevant for the optimal path of inflation, output and interest rates. According to Definition 6, we could design many remittances' policies that can satisfy the definition. One possibility is the following:

$$\frac{T_t^C}{P_t} = \bar{T}^C + \frac{\Psi_t^C}{P_t} + \psi_t \frac{N_{t-1}}{P_t}, \tag{46}$$

for some parameter $\bar{T}^C$ and where $1 - \Pi_t < \psi_t \leq 1$ at each t. This rule, together with the law of motion of net worth (24), implies that real net worth is stationary and therefore (44) holds given the bounded stochastic process $\{\xi_t\}$ and any stochastic process $\{Y_t\}$ belonging to a *benchmark* equilibrium. One interesting feature of the rule (46) is the feedback response

---

[37]The appropriate boundedness of D_{t-1}^C, B_{t-1}^C is in line with the requirements of Definition 5 on the boundedness of $\{T_t^C\}$.

to central-bank profits Ψ_t^C: remittances to the treasury should follow one-to-one movements in profits, and in particular should fall when the central bank faces income losses. The second interesting aspect of the proposed rule is that it requires a reaction with respect to the (current) real value of previous-period net worth which should be as strong as $1 - \Pi_t$. In particular the reaction should be at least greater than zero if $\Pi_t < 1$.

Rule (46) is certainly special in order for the Proposition of Neutrality to hold, but it is even more special if we look at common central-bank practices. One interesting example to investigate is the *full treasury's backing regime*, already introduced in Section 2, where transfers from central bank to treasury are always equal to profits, $T_t^C = \Psi_t^C$.

Proposition 4 *A full treasury's backing regime, $T_t^C = \Psi_t^C$, is not in the class of passive remittances' policies.*[38]

Proof. This might seem surprising.[39] Indeed, consider the allocation $\Pi_t = \beta$, $i_t = 0$, $Q_t = 1/(1-\delta), Y_t = Y, F_t = F, K_t = K, \Delta_t = \Delta\left(\beta/\bar{\Pi}, \Delta\right)$ and $M_t \geq \beta^t PY$. It is easy to verify that this allocation satisfies the equilibrium conditions (27) to (35) as required by Definition 6 when ξ_t is constant and $\varkappa_t = 0$, given some non-negative Y, F, K, Δ and P. Moreover, a full treasury's backing regime implies that net worth is constant at $N_t = \bar{N}$ as shown by the law of motion (24). However, (44) is not satisfied because $\lim_{T \to \infty} E_t\left[\tilde{R}_{t,T} N_T / P_T\right] = \bar{N}/P_t$, which is not necessarily zero, unless $\bar{N} = 0$.[40] Therefore a full treasury's backing policy is not in the class of passive remittances' policies. ■

This result, however, does not imply that the Proposition of Neutrality does not hold, but only that some stochastic processes $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t\}$ satisfying the equilibrium conditions (27) to (35) are ruled out as equilibrium allocations under a full treasury's backing regime.

Proposition 5 *Under a passive fiscal policy and full treasury's backing regime i) the Proposition of Neutrality holds and ii) the optimal policy is still given by $\{Y_t^*, \Pi_t^*, i_t^*, Q_t^*, F_t^*, K_t^*, M_t^*, \Delta_t^*\}$, for any balance-sheet policy and assuming that $\bar{\Pi} > \beta$.*[41]

Proof. In the Appendix. ■

Rules (43) and (46), or $T_t^C = \Psi_t^C$ in place of the latter, clearly illustrate the transfer mechanisms that support the Proposition of Neutrality. If the central bank purchases some

---

[38] We are in any case omitting instances in which Ψ_t^C becomes so negative that the treasury needs to raise a primary surplus that violates the non-negative constraint on consumption as discussed in footnote 35.

[39] Moreover a full treasury's backing regime is clearly not in the class of rules identified by (46) for equilibrium gross inflation rates such that $\Pi_t < 1$, since the rule in this case requires $\psi_t > 0$.

[40] Recall that $\tilde{R}_{t,T} \equiv \beta^{T-t} \xi_T Y_T^{-\rho} / \xi_t Y_t^{-\rho}$ which is equal to β^{T-t} in this case, while $P_T = P_t \beta^{T-t}$.

[41] As detailed in the Appendix the condition $\bar{\Pi} > \beta$ ensures that the *benchmark* optimal policy is feasible under a *full treasury's backing regime*.

risky securities which were before in the hands of the private sector, that risk does not remain in the hands of the central bank since rule (46) ensures that the treasury transfers resources to the central bank in the case the risk materializes in negative profits, while rule (43) ensures that the treasury gets these resources from the private sector through higher lump-sum taxes. At the end, the risk remains in the hands of the private sector. Alternative transfer policies that break these linkages can challenge the result of neutrality. We turn to this analysis in the next section.

4.2 Case II: Passive fiscal policy but non-negative transfers from the central bank to the treasury

In this section, we evaluate the equilibrium consequences of the absence of treasury's support, meaning that the treasury never transfers resources to the central bank, $T_t^C \geq 0$. Still we maintain the assumption of passive fiscal policy. To intuit the implications of this restriction, consider a candidate equilibrium $\{Y_t^*, \Pi_t^*, i_t^*, Q_t^*, F_t^*, K_t^*, M_t^*, \Delta_t^*\}$ implied by the *benchmark* optimal policy.[42] We evaluate whether this allocation can be supported by the combination of passive fiscal policy and non-negative transfer policy. Given that fiscal policy is still passive, equations (42) and (36) require that (44) hold in equilibrium.

The candidate equilibrium is a feasible allocation if there are stochastic processes $\{X_t, T_t^C\}$, with $T_t^C \geq 0$, such that

$$\frac{X_{t-1}}{P_t^*} - \frac{B_{t-1}^C}{P_t^*} + \frac{M_{t-1}^*}{P_t^*} - (1 + r_t^*)\frac{Q_{t-1}^* D_{t-1}^C}{P_t^*} = E_t \sum_{T=t}^{\infty} \tilde{R}_{t,T}^* \left[\frac{i_T^*}{1 + i_T^*} \frac{M_T^*}{P_T^*} - \frac{T_T^C}{P_T^*} \right] \quad (47)$$

holds at all times and in each contingency given some specification of the stochastic processes $\{B_t^C, D_t^C\}$. The Proposition of Neutrality is valid if the above condition is satisfied for any specification of the processes $\{B_t^C, D_t^C\}$, given initial conditions $X_{t_0-1}, M_{t_0-1}, B_{t_0-1}^C, D_{t_0-1}^C$.

The central bank should be able to engineer a path of reserves together with a non-negative remittances' policy that is consistent with the optimal *benchmark* paths of inflation, output and other variables through the above equilibrium conditions. In practice, this requires to check that the infinite, but countable, set of intertemporal budget constraints (47) is satisfied at the optimal *benchmark* policy through a non-negative transfer policy and some path of reserves consistent with the flow budget constraint of the central bank.

An alternative way to write (47), using the definition of central bank's net worth and profits, is:

---

[42]We could follow a similar reasoning taking as a reference any *benchmark* equilibrium.

$$\frac{N_t^*}{P_t^*} + E_t \sum_{T=t}^{\infty} \tilde{R}_{t,T}^* \left(\frac{i_T^*}{1+i_T^*} \frac{M_T^*}{P_T^*} \right) = E_t \sum_{T=t+1}^{\infty} \tilde{R}_{t,T}^* \left(\frac{T_T^{*C}}{P_T^*} \right). \tag{48}$$

The left hand side of the above equation identifies the real value of the central bank given by the sum of its real net worth and the value of current and future resources that can be obtained by the monopoly power of issuing money. In equilibrium, it should be equal to the expected present discounted value of real transfers to and from the treasury. This rewriting clarifies the meaning of treasury's backing or passive remittances' policy. No matter what is the equilibrium value of the central bank, the treasury is ready to back it. If the left hand side is negative because net worth has reached a too low level in equilibrium, it is indeed the treasury that transfers resources to the central bank. In absence of treasury's support, this is not possible and the right-hand side of (48) becomes non-negative imposing a lower bound on the level that net worth can reach in equilibrium

$$\frac{N_t^*}{P_t^*} \geq -E_t \sum_{T=t}^{\infty} \tilde{R}_{t,T}^* \left(\frac{i_T^*}{1+i_T^*} \frac{M_T^*}{P_T^*} \right). \tag{49}$$

Net worth can become negative but should be backed by the current and future seignorage revenues. If equilibrium nominal interest rates are low or the economy is quasi cashless, the above lower bound is more stringent.

Unconventional purchases of long-term securities can bring the central bank's balance sheet in risky territories. Indeed, in absence of treasury's support, income losses translate directly into declining net worth

$$N_t^* = N_{t-1}^* + \Psi_t^{*C} - T_t^{*C} < N_{t-1}^*$$

which in turn can be inconsistent with (48) or even with (49). Key is again the specification of the transfer policy. Consider $T_t^C = P_t \bar{T}_t^C$ for some exogenous non-negative stochastic process $\bar{T}_t^C$ which can be even constant or zero at all times and substitute it on the right hand side of equations (47) and (48). Observe that it would be a coincidence if the allocation $\{Y_t^*, \Pi_t^*, i_t^*, Q_t^*, F_t^*, K_t^*, M_t^*, \Delta_t^*\}$ satisfies (47) in some contingency given the portfolio composition of the central bank and the assumed exogenous remittances' policy. It would be even harder to get consistency in all contingencies and for any possible composition of the central-bank balance sheet. Similarly, given that the right hand side of equation (48) is exogenously given, the equilibrium level reached by net worth should be backed by appropriate future seignorage revenues which are direct function of the monetary policy stance implied by the equilibrium. In the case of an inconsistency, there should be some price or quantity adjustment.

We have a theory of price (and quantity) determination which acts also through the

solvency condition of the central bank in which the composition of the central bank's assets matters.

4.2.1 A deferred-asset policy of central-bank remittances

The class of exogenous remittances' policy is a special one, since there is no contingent relationship with profits even in the upturn. In the practice of central banking there are several other possible specifications. An important example is that of the U.S. Federal Reserve which, in the case of negative profits, can issue a "deferred asset" that can be paid back by future positive profits. Only once the deferred asset is paid in full, remittances are rebated back to the treasury in the case of positive profits.[43]

Definition 7 *A deferred-asset policy of central-bank remittances is defined as* $T_t^C = \max(\Psi_t^C, 0)$ *only when* N_t *is above or equal to an initial threshold* $N_{t_0-1} = \bar{N} > 0$ *otherwise* $T_t^C = 0$.[44]

Assume first that the central bank holds only short-term assets, i.e. $D_t^C = 0$ at each time t. Equation (26) shows that profits depend only on the sum of the non-interest bearing liabilities times the foregone interest rate

$$\Psi_t^C = i_{t-1}(N_{t-1} + M_{t-1}). \tag{50}$$

If $N_{t_0-1} = \bar{N} > 0$, profits are positive and therefore also remittances. Equation (23) implies a constant and positive net worth $N_t = \bar{N}$ in all contingencies. Under a "normal" composition of the central bank's balance sheet and a deferred asset policy, whether the treasury is backing or not the central bank does not really matter since the constraint $T_t^C \geq 0$ never binds and net worth is stationary.

Turning to the analysis of an unconventional composition of the central-bank balance sheet, we can still prove a Proposition of Neutrality but only under very special conditions.

Proposition 6 *Under a passive fiscal policy and a deferred-asset policy of central bank's remittances i) the Proposition of Neutrality holds if and only if* $N_t = \bar{N}$ *in equilibrium for each* $t > \tau$ *(and in each contingency at t) and for some* τ *even far in the future and ii) the optimal policy is still given by* $\{Y_t^*, \Pi_t^*, i_t^*, Q_t^*, F_t^*, K_t^*, M_t^*, \Delta_t^*\}$ *for any balance-sheet policy if*

---

[43]See Carpenter et al. (2015). In our analysis we are abstracting from operating costs and standard dividends to member banks subscribing the capital of the central bank.

[44]Writing explicitly a deferred asset in the problem like a negative liability, as it is done in practice to avoid that the accounting value of net worth declines, does not really matter for the analysis.

and only if $N_t = \bar{N}$ in equilibrium for each $t > \tau$ (and in each contingency at t) and for some τ even far in the future, assuming that $\bar{\bar{\Pi}} > \beta$.[45]

Proof. In the Appendix. ∎

The necessary and sufficient condition of Proposition 6 is quite restrictive since requires that in equilibrium net worth returns to normal conditions in a finite period of time – although this time can be very far in the future – and in all contingencies. This requirement limits the set of possible equilibria for which neutrality holds. In particular, it is easy to see that a positive level of long-run net worth implies equilibria that should naturally satisfy the lower bound (49) before then.

To sharpen the intuition of the condition required in Proposition 6, note that a *deferred-asset regime* is in between two extreme regimes: i) one in which remittances are always zero $T_t^C = 0$, assuming that profits are always negative, and ii) the other in which remittances are positive and equal to profits, assuming that profits are always positive. In the latter case, net worth is constant and the solvency condition (47) does not restrict equilibrium prices and quantity since reserves appropriately adjust offsetting the different portfolio compositions at equilibrium prices as implied by

$$Q_t D_t^C + \frac{B_t^C}{1 + i_t} - M_t - \frac{X_t}{1 + i_t} = \bar{N}. \tag{51}$$

In particular, the necessary and sufficient condition ($N_t = \bar{N}$ in equilibrium for each $t > \tau$ and in each contingency at t and for some τ even far in the future) requires and ensures that there is a long run in which the central bank returns to "normal business", earning positive profits which are rebated to the treasury like in the extreme case ii). In this long run there is no issue of stationarity of net worth, and reserves appropriately adjust to support long-run equilibrium output and prices for alternative central-bank portfolio compositions. Key is that output and prices are not affected even in the short run. How is it possible? The central bank can absorb short-run losses by temporarily decreasing net worth and varying reserves, for a given equilibrium path of prices and output. These losses can be backed by future retained profits which add up until net worth returns to the initial condition. The intertemporal budget constraint (47) does not bind even in the short run. The central bank is able to absorb losses through a reduction by the same amount of the present discounted value of remittances, while keeping them non-negative in each period. At the end, these lower remittances are paid by higher taxes levied on the private sector.

---

[45]The condition $N_t = \bar{N}$ in equilibrium for each $t > \tau$ (and in each contingency at t) and for some τ even far in the future is different from the condition $\lim_{t \to \infty} N_t = \bar{N}$ a.s. since the latter includes the possibility that net worth converges to $\bar{N}$ only asymptotically.

What supports the Proposition of Neutrality in this case is therefore that short-run losses can be backed by future retained profits. Two non-trivial conditions, however, should be satisfied: i) that there is a long run in which the central bank is always profitable and ii) that losses only happen in the short run. Consistently with this intuition, we now provide two sufficient conditions for $N_t = \bar{N}$ to be true in equilibrium (for each $t > \tau$ and for some τ even far in the future) which are illustrative of cases of practical interest – underlining also their limitations – under which unconventional monetary policy does not affect equilibrium output and prices even in absence of treasury's support.

Proposition 7 *Consider a collection of stochastic processes* $\{Y_t, \Pi_t, \ i_t, Q_t, F_t, K_t, M_t, \Delta_t\}$ *satisfying the equilibrium conditions (27) to (35) given a conventional monetary policy and consider a deferred-asset policy of central-bank remittances; $N_t = \bar{N}$ in equilibrium (for each $t > \tau$ and in each contingency at t and for some τ even far in the future) if there is a time τ_1 even far in the future such that $N_{\tau_2} + M_t > 0$ at some $\tau_2 \geq \tau_1$ for each $t \geq \tau_2$ and either i) ξ_t and $\varkappa_t$ have absorbing states after τ_1 or ii) $D_t^C = 0$ at each $t \geq \tau_1$ and (in each contingency at t).*

Proof. In the Appendix. ∎

The sufficient condition ii) of Proposition 7 is clearly not in line with the Proposition of Neutrality since it constrains the possible specifications of the process $\{D_t^C\}$, although this happens only after a generic period τ_1 which can be very far in the future. However, both conditions are illustrative of situations of practical interest for which the Proposition of Neutrality can hold: central-bank losses should be limited in time and size.

In time because otherwise the intertemporal budget constraint (47) always restricts the path of output and prices conditional on the *balance-sheet* regime, requiring an appropriate monetary policy stance in order to smooth the path of reserves and achieve a stationary path of net worth.

In size because otherwise the profitability of the central bank can be permanently impaired, implying a violation of the condition $N_t + M_t > 0$ at some point in time given the equilibrium path $\{M_t\}$ and, most importantly, in the long run. As shown in (50) the long-run profitability is essentially related to the income produced by the ability of the central bank to finance its asset purchases using non-interest bearing liabilities. Accordingly, the overall amount of such liabilities should remain positive, implicitly imposing a (possibly negative) lower bound on the level that net worth can reach.

Our analysis also suggests some costs of monetary systems where cash disappears. Seigniorage is an important source of revenues which, as shown above, allows in a critical way to smooth over time periods of negative profits in a way that they do not constrain the ordinary

action of monetary policy. In cashless economies, seigniorage is zero and central bank loses its main source of revenue to back its net worth as (48) would require. Absent any treasury's support, central bank should rely on a positive level of net worth to recover from past losses, as shown by (49) and (50). This stricter bound can further constraint the action of monetary policy in the direction of avoiding negative profits for a prolonged period of time and may preclude the possibility of engaging in unconventional monetary policies.

4.2.2 Avoiding income losses

Net worth can decline in the equilibria discussed in Propositions 6 and 7 but at the end should return to normal levels. However, the duration of declining net worth might turn out to be so long that the central bank can decide to shorten it for several reasons, including political pressures because of zero remittances to the treasury or worries about a possible loss of independence. Even the concern that non-interest bearing liabilities can turn negative in some contingencies and for a prolonged period can force the central bank to reconsider its policy stance. In these circumstances, policy will deviate from the original plan or from the optimal *benchmark* policy.

Consider a central bank that tries to completely avoid periods of negative profits or declining net worth, $T_t^C = \Psi_t^C \geq 0$ implying $N_t = N_{t_0-1} = \bar{N}$ at each t. Given the definition of profits (26) and a generic stochastic process $\{D_t^C\}$, the central bank has no other tool available than appropriately constraining the action of conventional monetary policy in order to take into account the zero-lower-bound constraint on profits, Ψ_t^C. The equilibrium of the model is now subject to two zero-lower-bound constraints. There are two main results in this case: the Proposition of Neutrality never holds and optimal policy deviates from the optimal *benchmark* policy if the central bank holds long-term assets.[46] We have the following Proposition which characterizes the results:

Proposition 8 *Under a passive fiscal policy and a policy of central-bank remittances of the form $T_t^C = \Psi_t^C \geq 0$ i) the Proposition of Neutrality does not hold; ii) for generic balance-sheet policies the optimal policy is a) a collection of stochastic processes $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t\}$ that maximizes (41) and satisfies each of the conditions in equations (27) to (35) at each time $t \geq t_0$ (and in each contingency at t), given the definition $\Pi_t \equiv P_t/P_{t-1}$, the non-negativity constraints both on the nominal interest rate and on central-bank profits, $i_t \geq 0$ and $\Psi_t^C \geq 0$ – where Ψ_t^C is defined by (26) – the stochastic processes for the exogenous disturbances $\{\xi_t, \varkappa_t\}$, given initial conditions $i_{t_0-1}, N_{t_0-1}, M_{t_0-1}, Q_{t_0-1}, D_{t_0-1}^C$ and b) some collection of stochastic*

---

[46]If $D_t^C = 0$ at all times the lower-bound constraint on profits is never binding and the optimal policy conditional on this *transfer policy* coincides with the *benchmark* optimal policy.

processes $\{X_t, B_t, B_t^C, B_t^G, D_t, D_t^C, D_t^G, A_t\}$ *satisfying each of the conditions in equations (36), (37), (39), (40), (51) at each time $t \geq t_0$ (and in each contingency at t) given the stochastic processes* $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t\}$ *and initial conditions* $B_{t_0-1}^G, D_{t_0-1}^G.$[47]

4.3 Case III: Active fiscal policy

In the previous section, we have assumed limited treasury's backing of central bank's losses and shown that there are cases in which the central bank is able to absorb its losses without the need of changing monetary policy stance. However, one key assumption was the maintained ability of the treasury to receive or transfer income from the private sector contingently on the evolution of its own liabilities and of the remittances received from the central bank. Were the central bank experiencing income losses and therefore cutting remittances to the treasury, the treasury would offset the missing resources through appropriate increases of lump-sum taxes levied on the private sector.

We now limit the ability of the treasury to tax or transfer resources as needed and assume an active fiscal policy with a primary surplus that follows

$$\frac{A_t}{P_t} = a_t, \tag{52}$$

where a_t is a Markov process, with transition density $\pi_a(a_{t+1}|a_t)$ and initial distribution f_a. We assume that (π_a, f_a) is such that $a \in [a_{\min}, a_{\max}]$.

To get some intuition, we first assume that there is *full treasury's backing*, $T_t^C = \Psi_t^C$ for each t. As shown previously, this implies a constant central-bank net worth $N_t = N_{t_0-1} = \bar{N} > 0$. The set of equilibrium conditions (36) to (40) imply a consolidated intertemporal budget constraint

$$\frac{X_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} + (1+r_t)\frac{Q_{t-1}D_{t-1}}{P_t} = E_t \sum_{T=t}^{\infty} \tilde{R}_{t,T}\left[\frac{i_T}{1+i_T}\frac{M_T}{P_T} + \frac{A_T}{P_T}\right]$$

showing that the overall liabilities of the whole government should be backed by the expected present discounted value of seigniorage revenues and primary surpluses, as stressed by the literature on the fiscal theory of the price level.

We can further write the above intertemporal budget constraint as

$$\frac{B_{t-1}^G}{P_t} + (1+r_t)\frac{Q_{t-1}D_{t-1}^G}{P_t} - \frac{N_{t-1} + \Psi_t^C}{P_t} = E_t \sum_{T=t}^{\infty} \tilde{R}_{t,T}\left[\frac{i_T}{1+i_T}\frac{M_T}{P_T} + a_T\right], \tag{53}$$

---

[47]We still maintain the assumption $\bar{\bar{\Pi}} > \beta$.

having used the fiscal rule (52) together with the definitions of central bank's net worth and profits. Recall again that under full treasury's backing $N_{t-1} = \bar{N}$. Equation (53) shows clearly the fiscal consequences of unconventional central-bank operations. Income losses ($\Psi_t^C < 0$) on central-bank balance sheets should require an adjustment somewhere else in prices, output or seigniorage revenues in order for (53) to hold in equilibrium.

A reallocation of risks in the economy has fiscal consequences, given that the treasury is not passing central bank's losses to the private sector, and therefore can influence the equilibrium allocation.

More formally consider stochastic processes $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t\}$ satisfying the equilibrium conditions (27) to (35). The same stochastic processes should be consistent with the specification of the *conventional monetary policy* and with (53) under an active fiscal policy.[48] Moreover, it is easy to see that a *model* equilibrium is not neutral to alternative balance sheet policies. Indeed if an allocation $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t\}$ is consistent with (53) for a path $\{D_t^C\}$ it might not be consistent for an alternative path $\{\tilde{D}_t^C\}$ because central bank's profits can vary requiring a change in the path of some endogenous variables in (53).

Turning to the optimal *benchmark* policy, similar considerations suggest that only by chance it can also satisfy (53). Previous results in the literature on optimal debt management (see Angeletos, 2003), shows that in theory it is possible to "support" the *benchmark* optimal policy through an optimal choice of the maturity structure of treasury's debt.[49] But even in this case, alternative maturity structures or balance-sheet policies would imply a different optimal allocation.[50]

Proposition 9 *Under an active fiscal rule of the form (52) and given some specification of central-bank remittances' policy, the* **model** *equilibria and the welfare-maximizing model equilibrium are not invariant to alternative balance-sheet policies.*[51]

---

[48]In the case in which there are multiple collections of stochastic processes $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t\}$ satisfying the equilibrium conditions (27) to (35) for the same *conventional monetary policy,* the additional equilibrium condition (53) can reduce the multiplicity. Instead, when there is only one collection of stochastic processes consistent with (27) to (35) for the same *conventional monetary policy,* this allocation can be inconsistent with (53) and therefore not part of a model equilibrium.

[49]A necessary condition is that the number of securities issued by the treasury is at least as large as the number of states of nature at each point in time. But even in this case note that the right-hand side of (53) is a function of $\xi_t, \varkappa_t, a_t$ and their past values while the left-hand side, under the optimal *benchmark* policy, is a function of only $\xi_t, \varkappa_t$ and past values so that it is impossible to find a maturity structure that equates the two sides of the equation in all states of nature. Moreover, it is also known that on practical grounds the optimal maturity structure usually involves unrealistic short and long positions in the various securities.

[50]It can also be possible that the equilibrium optimal policy can be improved by an alternative composition of the central bank's portfolio even in the direction of including more long-term securities. However, in any case, welfare will be lower than under the *benchmark* optimal policy.

[51]We have not specified in details the policy of central-bank remittances since it is not much relevant for the general implications of the proposition. One could assume either a full treasury's backing regime, a deferred-asset regime or non-negative profits and obtain similar qualitative conclusions.

We leave to the numerical examples of next section to evaluate whether under optimal policy it is desirable that the central bank internalizes the fiscal consequences of its income losses and acts to reduce the duration of negative profits in response to the restriction imposed by equation (53) on equilibrium prices and quantities.

5   Evaluation of optimal policy

This section presents numerical examples on how events related either to interest-rate risk or credit risk affect the equilibrium dynamics of optimal policy under the alternative monetary and fiscal regimes outlined in previous section.[52]

We approximate the optimal policy problem using linear-quadratic methods around a non-stochastic steady state. The appendix shows that a second-order approximation of (41) around the efficient steady state implies the following simple loss function

$$L_{t_0} = \frac{1}{2}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[\hat{Y}_t^2 + \lambda_\pi(\pi_t - \bar{\pi})^2\right] \tag{54}$$

where λ_π is the relative weight attached to inflation volatility (defined in the appendix), while $\hat{Y}_t$ denotes deviations of output with respect to the efficient steady state, in which we have defined $\pi_t \equiv \ln\Pi_t$ and $\bar{\pi} \equiv \ln\bar{\Pi}$.[53] The minimization of the loss function is subject to a modified New-Keynesian Aggregate-Supply equation which is the result of a first-order approximation of equations (31) to (33)

$$\pi_t - \bar{\pi} = \kappa\hat{Y}_t + \zeta\hat{\imath}_t + \beta E_t(\pi_{t+1} - \bar{\pi}) \tag{55}$$

where κ and ζ are defined in the appendix, and $\hat{\imath}_t \equiv \frac{i_t - \bar{\imath}}{1+\bar{\imath}}$. The financial friction – which is usually absent in standard versions of the New-Keynesian Aggregate-Supply equation and is here captured by fluctuations in the nominal interest rate – acts as a cost-push shock in the AS equation.

---

[52]Results on sub-optimal policies like strict inflation targeting are available upon request as well as details of the numerical procedure.

[53]An interesting result, derived in the Appendix, is that the efficient steady state of the model is reached by setting the employment subsidy to $\varrho \equiv 1 - (1-1/\theta)/(1+\bar{\imath})$ where $\bar{\imath}$ is the steady-state level of the nominal interest rate. One needs to use only one instrument of policy to offset both the monopolistic distortion and the financial friction, since both create an inefficient wedge between the marginal rate of substitution between leisure and consumption and the marginal product of labor. Moreover, given this result, the steady-state level of the nominal interest rate can be different from zero, while the inflation rate can be set at the target $\bar{\Pi}$. The steady-state version of equation (15) relates the nominal interest rate to the inflation rate $\beta(1+\bar{\imath}) = \bar{\Pi}$.

The first-order approximation of (15) implies a standard Euler equation

$$\rho E_t(\hat{Y}_{t+1} - \hat{Y}_t) = \hat{\imath}_t - r_t^n - E_t(\pi_{t+1} - \bar{\pi}) \tag{56}$$

where the natural rate of interest is defined as

$$r_t^n \equiv \ln \xi_t - E_t \ln \xi_{t+1}$$

which varies because of the exogenous changes in the preferences shock. The optimal path of output, inflation and nominal interest rate solves the minimization of (54) under constraints (55)–(56), for each time $t \geq t_0$, given the zero-lower bound on the nominal interest rate. This optimal allocation corresponds to the first-order approximation of the *benchmark* optimal policy described in Definition 4. As discussed in the previous section, appropriate *monetary/fiscal policy regimes* are needed to support it, like a *passive transfer policy* – consisting of the passive fiscal regime specified in Definition 5 and the passive remittances' policy of Definition 6. The latter policy can also be substituted by a *full treasury's backing regime*, consistently with Proposition 5, under the condition $\bar{\Pi} > \beta$.

We first focus on interest-rate risk and disregard credit risk for the moment. In this respect, we simulate an economy which a time t_{-1} is already in a liquidity trap, because of a negative preference shock that hit sometime in the past. There is uncertainty on when the shock returns to normal conditions. Once uncertainty is resolved, the path of current and future short-term rates changes inducing an unexpected fall in the price of long-term securities which leads to income losses for the central bank, in the case it holds long-term assets. It should be clear that this example is very stylized since there is only one point in time in which the return on long-term securities can be different from the short-term rate causing losses or gains for the holders of long-term securities.

In the numerical example of Figure 1, the natural rate of interest at time t_{-1} is at -2% (with respect to a steady-state level of 4%). It is then expected to return to 4% with a constant probability of 10% in each period. In the Figure we assume that the ex-post duration of the shock is one year, implying that after one year there is an unexpected change in the path of short-term interest rates, leading to a fall in the return on long-term securities. The model is calibrated (quarterly) as follows. We set $\beta = 0.99$ and $\bar{\pi} = 0$, to imply a steady-state annualized nominal interest rate on bonds of about 4%. We calibrate the composition of central-bank balance sheet considering as initial steady state the situation in 2009Q3, when the economy had already been in a liquidity trap for about three quarters. Accordingly we set the share of money to total liabilities equal to 53%, the share of net worth to total liabilities to 1%, and the share of long-term asset to total assets to 72%.

This calibration implies that the steady-state quarterly remittances to the treasury (equal to profits) are equal to about 0.6% of the central bank's assets and that the central bank's net position on short-term interest-bearing securities (central bank reserves net of short-term asset holdings) is positive and amounts to about 18% of the central bank's balance sheet. Finally, the duration of long-term assets is set to ten years (accordingly, $\delta = .9896$) the relative risk-aversion coefficient to $\rho = 2$, the inverse of the Frisch-elasticity of labor supply to $\eta = 5$, the elasticity of substitution across goods to $\theta = 7.66$ (implying a 15% markup), the parameter α capturing the degree of nominal rigidity in the model implies an average duration of consumer prices of four quarters ($\alpha = 0.75$). As a result, the slope of the Phillips Curve is $\kappa = .015$.

The top panels of Figure 1 show the familiar result, already discussed in Eggertsson and Woodford (2003), that committing to a higher inflation for the periods after the liftoff of the natural rate of interest allows to limit the deflationary impact of the negative shock, despite the nominal interest rate cannot be cut as much as needed because of the zero floor. This commitment translates into maintaining the policy rate at the zero bound for several periods longer than the ex-post duration of the shock (in the specific case of Figure 1 it lasts three quarters longer).[54]

The bottom panels show instead the evolution of three key variables related to the balance sheet of the central bank: the quarterly remittances to the treasury T_t^C, which in this scenario are always equal to profits Ψ_t^C – defined by equation (26) – the central bank's net worth N_t, evolving according to (24) and the central bank's reserves (net of short-term assets) $(X_t - B_t^C)/(1 + i_t)$, all expressed as a share of the steady-state balance sheet of the central bank. As shown by Proposition 5, under Case I, the central bank's net worth remains constant at its initial level, 1% in the numerical example. The dynamics of profits (and remittances) instead mainly follow the return on long-term assets. As the shock to the natural rate returns back to steady state, the expectation that the nominal interest rate will jump up a few periods later is enough to bring down long-term asset prices and their return, thereby implying negative profits for the central bank. Under *full treasury's backing*, negative profits trigger a transfer of resources from the treasury to the central bank (negative remittances), so that net worth does not move. Central bank's reserves instead fall, as shown by the bottom-right panel as a consequence of the lower valuation of the long-term assets.

The solid red line in Figure 1 shows instead the response of an economy in which the *transfer policy* is a combination of a passive fiscal regime and a deferred-asset regime, as specified in Definition 7. This case is described by Proposition 6: the responses of inflation,

---

[54]Note, however, that in Eggertsson and Woodford (2003) there is also an unexpected shock at time t_0 which brings the economy into the liquidity trap while in our case the economy is already in a liquidity trap.
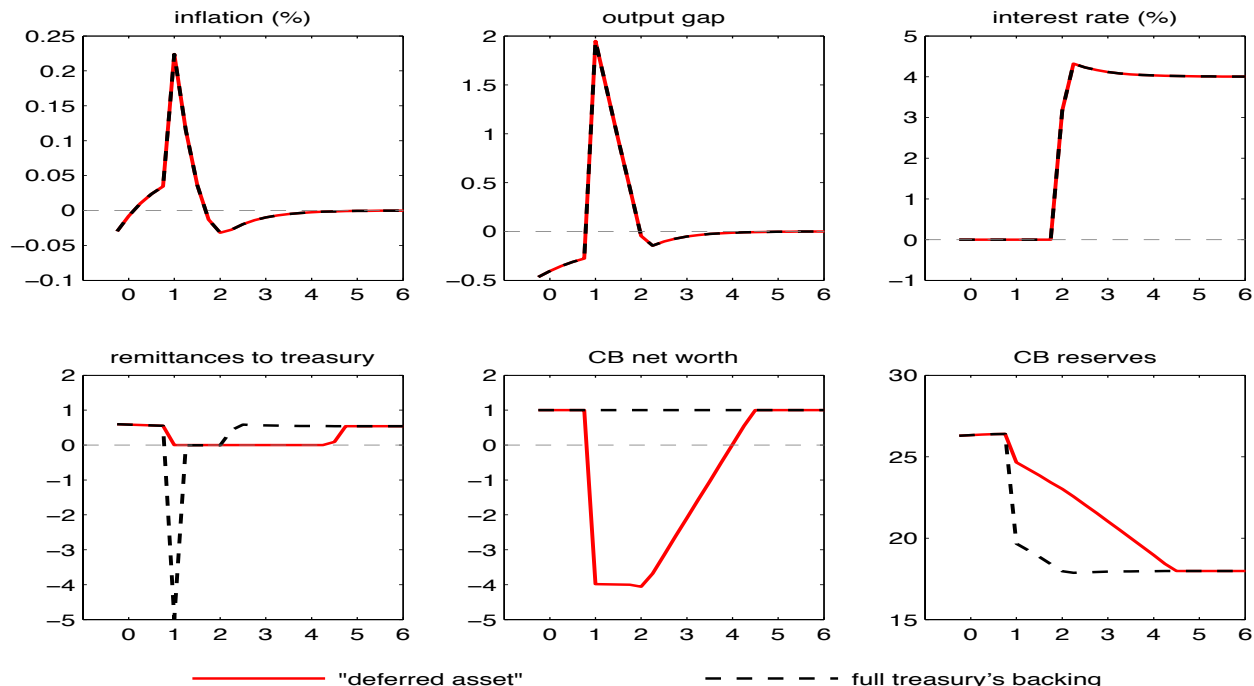
Figure 1: Equilibrium dynamics of selected variables under optimal monetary policy. The economy starts in a liquidity trap and the natural rate of interest is expected to go back from -2% to 4% with a 10% probability each period; ex-post reversal occurs after one year. Black dashed line: Passive fiscal policy and full treasury's backing of the central-bank liabilities (Case I). Red solid line: Passive fiscal policy and non-negative remittances to treasury ("deferred asset" specification of Case II). X-axis shows years.

output and the interest rate do not change. Indeed, the sufficient conditions of Proposition 7 are satisfied since losses are limited in size and time: in size since they are not so large to impair the long-run profitability of the central bank ($N_t + M_t > 0$ under the optimal benchmark policy), in time since uncertainty is resolved after one year. As central bank's profits turn negative,[55] remittances to the treasury fall to zero and stay at this level even when central bank's profits start to be positive as long as net worth is below the long-run level $\bar{N}$, thereby allowing the latter to converge back to $\bar{N}$ within a few quarters. After net worth is back at the initial value of 1%, central bank's profits are again rebated to the treasury. The implication is that central bank's reserves are temporarily higher than under full treasury's backing, and are paid back by next-period profits. Moreover, it is easy to see that the present discounted values of the remittances under the two regimes are the same.

The Proposition of Neutrality therefore holds in this example, and the central bank's balance-sheet composition does not matter for the optimal path of interest rate, inflation and output. Indeed, the dynamics of the latter three variables – displayed in the top panels of Figure 1 – are those of the *benchmark* optimal policy, and are therefore exactly identical

---

[55]The dynamics of central bank's profits under this scenario are the same as in the benchmark one, and are therefore captured by the dashed black line in the bottom-left panel of Figure 1.

to the ones arising under a *balance-sheet policy* in which $D_t^C = 0$ for all t. However, this simple example is certainly too stylized to conclude that interest-rate risk may not be an issue for the equilibrium outcome. Indeed, as mentioned, unexpected changes in the interest-rate path and non-zero excess returns on long-term assets occur only at one point in time, while in reality they can be more frequent. A proper evaluation should require a more careful projection analysis with a more realistic model economy as done in Carpenter et al. (2015).

To the end of further understanding the implications of violating the conditions of Proposition 7, we now shut down the preference shock ξ_t (assuming $\xi_t = 1$ at all times) and consider an economy hit at time t_0 by an unexpected credit shock implying default on a share $\varkappa$ of long-term debt. After period t_0 no other credit event or other shocks are expected or occur. As clear from equation (25), when a credit event happens the central bank might experience a substantial loss on its balance sheet. In absence of treasury's support, this loss can be strong enough to impair its long-run profitability under the *benchmark* optimal policy, even if uncertainty is fully resolved after t_0. The condition of part *i)* of Proposition 7 could be violated, meaning that it does not exist a $\tau_2 \geq t_0$ such that $N_{\tau_2} + M_t > 0$ for each $t \geq \tau_2$ conditional on the equilibrium paths implied by *benchmark* optimal policy, no matter how far in the future. Profits remain negative indefinitely. In this case, as argued in Section 4.2, the central bank finds it optimal to deviate from the *benchmark* optimal policy in order to restore long-run profitability and produce positive profits at some future point in time. Instead, if the credit event is not too strong, the Proposition of Neutrality still holds and the central bank is therefore able to return to the level $\bar{N}$ of net worth in a finite period of time.[56]

Figures 2 and 3 evaluate numerically this scenario under the same calibration as before. We consider two alternative sizes of the credit event: a default of 40% (i.e. $\varkappa = 0.40$, displayed by the solid line in Figures 2 and 3) and a default of 85% (i.e. $\varkappa = 0.85$, displayed by the dashed line in Figures 2 and 3). For our purposes, the main difference between these two shocks is that the first one is too weak to impair the central bank's long-run profitability under the *benchmark* optimal policy, while the second one is strong enough to compromise it irreversibly.

The solid line in the bottom right panel of Figure 2 shows that the stock of long-term assets in the balance sheet of the central bank is permanently reduced by 40% as a consequence of default: the market value of long-term assets in the hands of the central bank falls to about 43% of total assets. The other three panels however show that the *benchmark* optimal policy is still consistent with all the equilibrium conditions of the model, and in particular with long-run solvency of the central bank. Indeed, the solid line in Figure 3 implies that the fall

---

[56]In this example the *benchmark* optimal policy is simply given by $i_t = \bar{\imath}$, $\pi_t = \bar{\pi}$, and $\hat{Y}_t = 0$ and will be clearly achieved in presence of treasury's backing and under a passive fiscal policy.
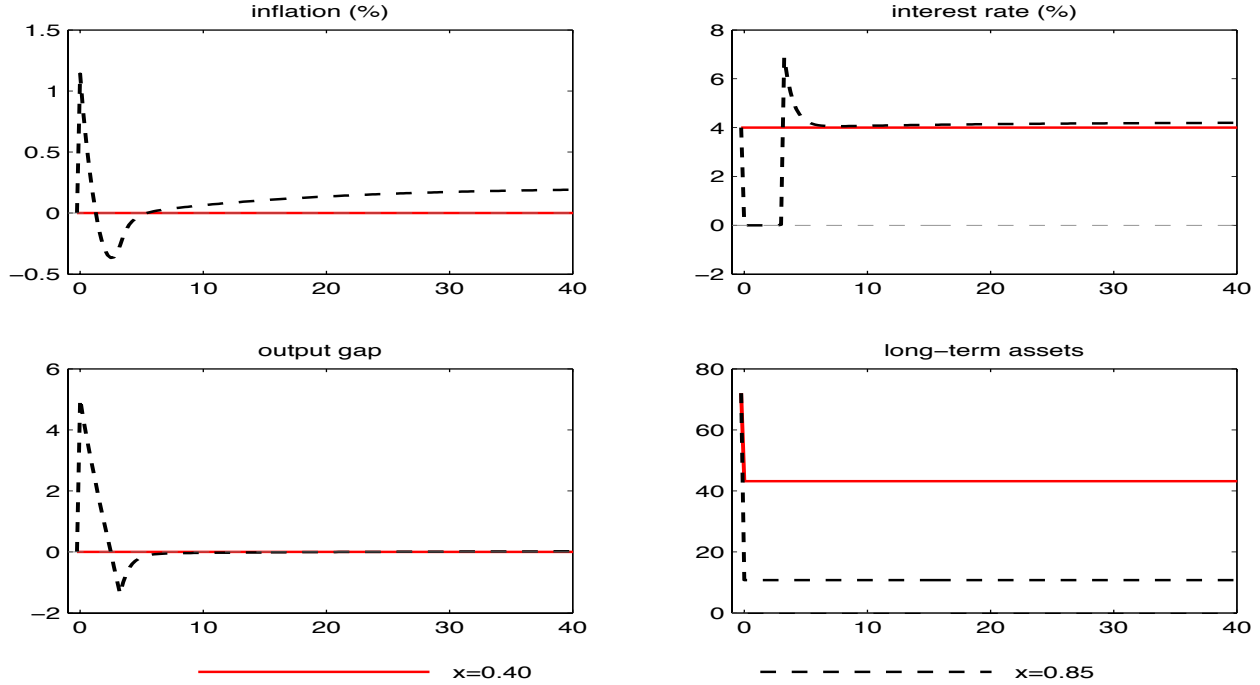
Figure 2: Response of selected variables, under optimal monetary policy, to a one-period credit event of alternative sizes. Red solid line: Credit event implies default on 40% of long-term assets. Black dashed line: Credit event implies default on 85% of long-term assets. X-axis shows years.

in nominal net worth that occurs when the shock hits, as a consequence of the income loss at t_0, is not enough to impair the ability of the central bank to produce positive gains from seigniorage in the future (i.e. $N_{\tau_2} + M_t > 0$ for some $\tau_2 \geq t_0$ and each $t \geq \tau_2$). Such positive profits, therefore, will be possible without the need for the path of nominal money supply to deviate from the *benchmark* optimal policy (second panel of Figure 3). Moreover, these gains will be used to repay the deferred asset over a period in which remittances are zero and net worth can be rebuilt (first and third panels of Figure 3, respectively).

Results substantially change if the credit event is sufficiently strong. In particular, a credit event due to a default of 85% of long-term assets implies, under the *benchmark* optimal policy, that the nominal stock of non-interest bearing liabilities, $N_t^* + M_t^*$, turns negative within the first two quarters, and follows a diverging path thereafter: the *benchmark* optimal policy, therefore, violates the long-run solvency condition of the central bank and is no longer an equilibrium – under a deferred-asset regime and in absence of treasury's backing. The dashed lines in Figures 2 and 3 show how to optimally deal with a shock of this size. The central bank should commit to progressively and permanently raise the stock of nominal money supply to compensate for the fall in nominal net worth. Such commitment will ensure that the stock of non-interest bearing liabilities eventually reverts to positive values and produces the profits needed to repay the deferred asset and rebuild net worth (although over an extremely
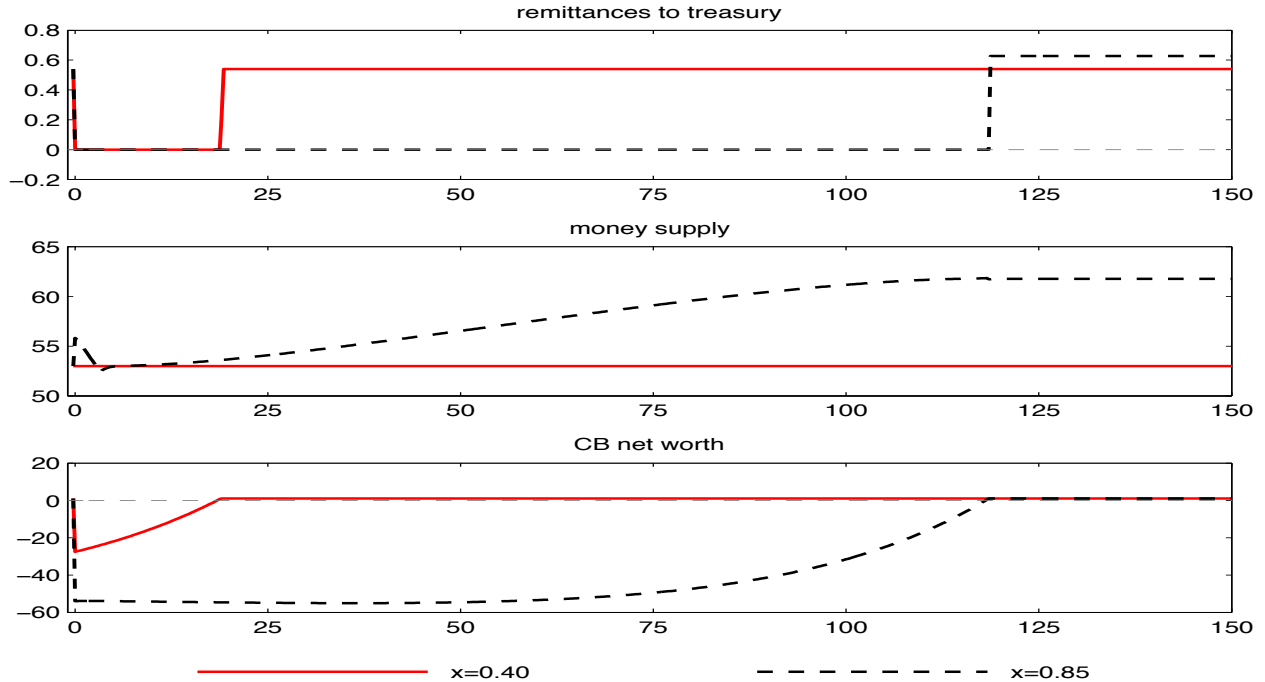
Figure 3: Response of selected variables, under optimal monetary policy, to a one-period credit event of alternative sizes. Red solid line: Credit event implies default on 40% of long-term assets. Black dashed line: Credit event implies default on 85% of long-term assets. X-axis shows years.

long time). To generate such a path of nominal money supply, the central bank should be accommodative enough to push up prices and inflation. In particular, as the dashed line in Figure 2 shows, inflation and output should go above their target on impact, which in turn requires the nominal interest rate to fall down to the zero-lower bound. Over time, moreover, the increase in inflation should be very persistent, and last until time τ_2 (not showed in the Figure), when the deferred asset is paid in full, net worth is back at the initial level $\bar{N}$ and nominal money supply can stabilize on a new, higher, level.

The above example is also very stylized, since uncertainty is immediately resolved, but it is illustrative of the concerns that could arise following unconventional purchases of long-term securities. Even if not necessary, the central bank can decide to shorten the duration of income losses. An extreme case, that we now investigate, is that of Proposition 8, in which the central bank seeks to avoid that profits turn negative at all. In this case, even mild credit events will have potentially sizable effects on the equilibrium dynamics of prices and quantities.

Figure 4 shows the effects of a one-period credit shock that implies a 10% default on long-term assets in a scenario in which the *transfer policy* consists of a passive fiscal regime and a remittance policy of the kind $T_t^C = \Psi_t^C \geq 0$ (red solid line) contrasted with the case of *full treasury's backing* (black dashed line). Consistently with Figures 2 and 3, a shock of this size is
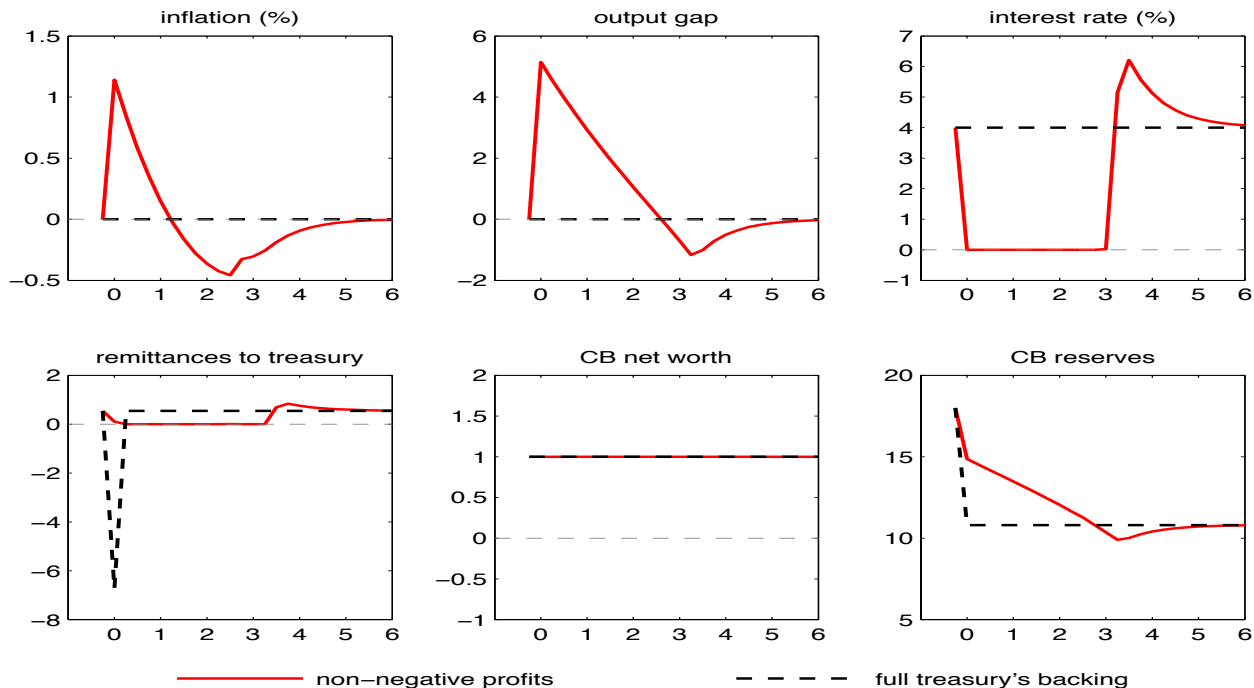
43

Figure 4: Response of selected variables, under optimal monetary policy, to a one-period credit event that implies default on 10% of long-term assets. Red solid line: Passive fiscal policy and non-negative central-bank profits. Black dashed line: Passive fiscal policy and full treasury's backing of the central-bank liabilities. X-axis shows years.

not strong enough to undermine the central bank's long-run profitability under the deferred-asset regime. However, if the central bank commits to avoid income losses altogether, the optimal equilibrium path of inflation, output and interest rate deviates substantially from the *benchmark* optimal policy that would instead arise under *full treasury's backing*. Interestingly, the short-run response of inflation, output and the risk-free rate is comparable, in size, to the one implied by a much stronger credit event under the deferred-asset regime, as shown in Figure 2: the interest rate falls to the zero-lower bound, where it stays for about 12 periods, inducing a persistent – though temporary – rise in inflation and output, supported by an analogous increase in nominal money supply (not showed). As a consequence, central bank's profits fall but only down to zero and stay there as long as the economy is in the liquidity trap. Central bank's reserves are naturally reduced to balance the permanent fall in the market value of long-term assets. In the long-run, however, the implications of a credit event of this size are very different from those shown by Figures 2 and 3: in particular, here the expansion in nominal money supply is only for few quarters, and therefore the inflationary consequences of the shock are short-lived.

We perform a similar analysis in the case of interest-rate risk. Figure 5 displays the equilibrium dynamics of selected variables under optimal policy in a scenario in which fiscal
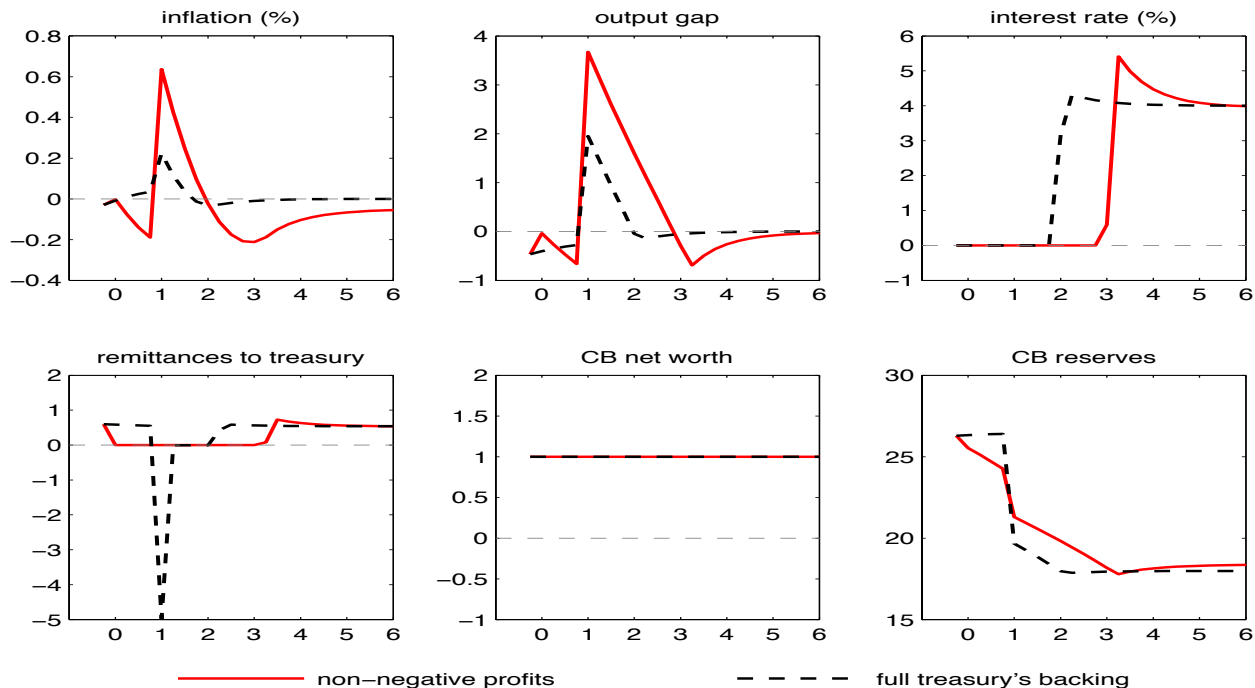
Figure 5: Equilibrium dynamics of selected variables under optimal monetary policy. The economy starts in a liquidity trap and the natural rate of interest is expected to go back from -2% to 4% with a 10% probability each period; ex-post reversal occurs after one year. Red solid line: Passive fiscal policy and non-negative central-bank profits. Black dashed line: Passive fiscal policy and full treasury's backing of the central-bank liabilities. X-axis shows years.

policy is passive and the central bank wants to avoid negative profits entirely (red solid line), and in one in which fiscal policy is passive and the central bank gets full treasury's backing from the treasury (black dashed line). The stochastic structure is the same as Figure 1 in which there is only the preference shock ξ. The non-negative constraint on central-bank profits requires to engineer a dynamic path for asset prices such that the long-term return does not display the sharp drop when the preference shock brings the natural rate of interest back to steady state. This requires committing to an interest rate path that remains at the zero-lower bound substantially longer than before, and implies (when r_t^n actually returns to steady state) an inflation rate three times higher and an output boom about twice as strong. The central bank's profits (and the corresponding remittances to the treasury), as a result, are zero and stay at zero several periods longer than the duration of the shock, following thereafter the path of the nominal interest rate with one period delay. Nominal money supply temporarily increases, when the natural interest rate rises back to 4%, to accommodate the output boom. To ensure that net worth stays constant, the central-bank reserves progressively decrease to their steady-state level, though along a substantially smoother path compared to the case of *full treasury's backing*. In a liquidity trap, therefore, a central bank committed to avoid income losses signals a change in its desired monetary policy stance towards temporarily
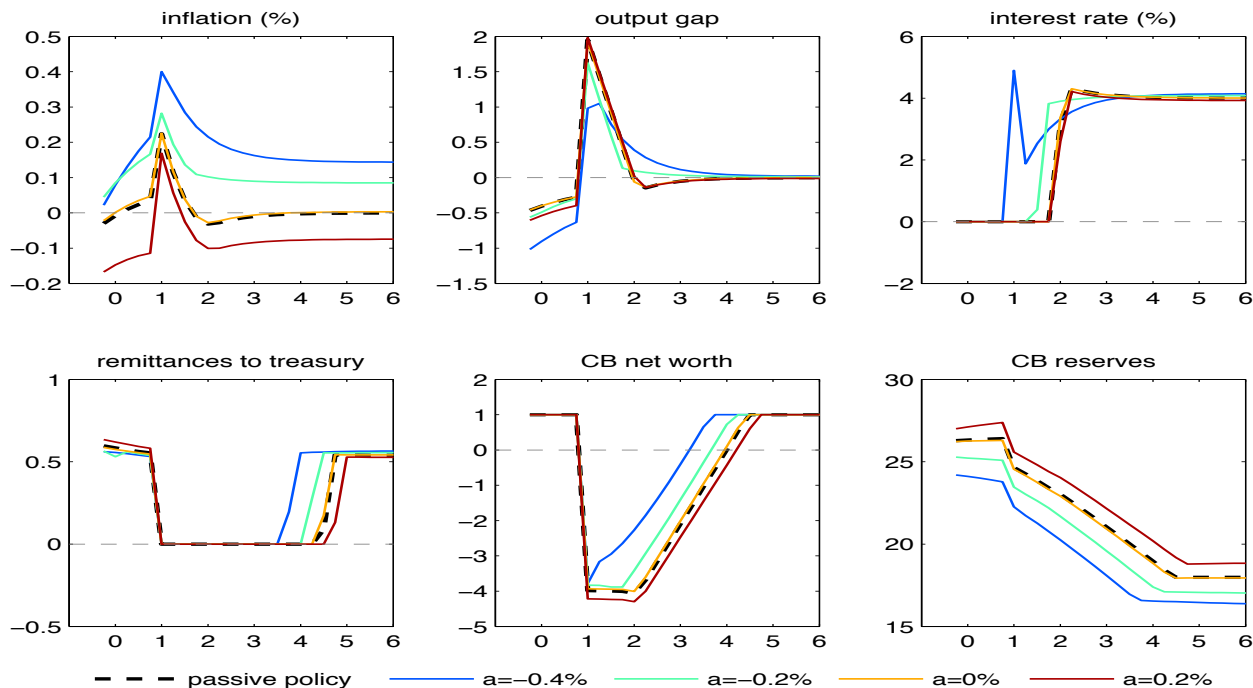
45

Figure 6: Equilibrium dynamics of selected variables under optimal monetary policy. The economy starts in a liquidity trap and the natural rate of interest is expected to go back from -2% to 4% with a 10% probability each period; ex-post reversal occurs after one year. Case III: Active fiscal policy and "deferred-asset" remittance policy, for alternative degrees of *fiscal strength*. Bold dashed line: Passive fiscal policy and "deferred-asset" remittance policy. X-axis shows years.

higher inflation and a delayed exit strategy.

Finally, as discussed in Section 4.3, the Proposition of Neutrality is clearly violated when fiscal policy follows an active rule of the form (52). In this case, indeed, seigniorage is an important source of revenues to back the liabilities of both the treasury and the central bank. The intertemporal budget constraints of the government become a relevant restriction for optimal policy.

Figure 6 evaluates the extent to which, under the deferred-asset regime, alternative degrees of fiscal strength might imply an endogenous response of monetary policy, in terms of duration of periods of lower net worth, relative to the case of passive fiscal policy. Consistently with the calibration discussed earlier, we set the ratio of long-term public debt to GDP in the initial steady state equal to $\bar{Q}\bar{D}^G/(4\bar{Y}\bar{P}) = 0.35$, in annual terms, as reported by the US Bureau of Public Debt for 2009Q3.[57]

In particular, the treasury sets the primary surplus as a constant share a of steady-state output: $A_t/P_t = \mathsf{a} \cdot \bar{Y}$. We capture alternative degrees of fiscal strength with different values of the share a. The interesting implication of Figure 6 is that, the smaller the resources that

---

[57]In particular, we consider the stock of publicly-held marketable government debt including securities with maturity above one year.

the treasury raises through primary surplus (i.e. the lower `a`), the more the central bank needs to optimally shorten the duration of the deferred asset, in order to be able to back the treasury's liabilities with positive remittances. To the end of implementing this policy the central bank needs to substantially deviate from the *benchmark* optimal policy. In particular, the central bank should raise permanently inflation – the more so the weaker is the fiscal policy stance – to increase its seigniorage revenues and absorb sooner the fall in net worth.

6 Conclusions

Starting from a proposition of neutrality of alternative compositions of central-bank balance sheet, we have discussed its generality investigating different fiscal/monetary policy regimes and stressing the consequences of these policies under cases of practical and theoretical interest which have been neglected so far by the literature. To preserve tractability, we kept the environment as simple as possible, at the cost of disregarding some important features that we now discuss more extensively.

Central banks around the world have very different accounting practices, capital requirements and transfer policies. A comprehensive analysis of all the various possibilities is out of the scope of this paper, though some alternative assumptions not made here could affect the results. One that deserves particular attention is the way purchases of long-term securities are accounted in the balance sheet and therefore in the profits-losses statement. We have evaluated them at the market value but some central banks do it at the historical value, like the Federal Reserve.[58] This can affect some of the numerical results of Section 5, in particular the duration of declining net worth and the size of the losses, but not the generality of the neutrality results.

We have restricted our focus on the equilibrium consequences of unconventional policies due to central bank's income losses. A broader view on the effects of these policies should address other features discussed by the recent literature. One class of models has included limits to arbitrage in private financial intermediation which translates into credit or term premia. These excess returns can be relaxed by central bank intervention through its ability to finance its purchases more easily than the private sector by expanding reserves. However these models assume some *ad hoc* costs for the central bank activity in financial intermediation, as also done in Curdia and Woodford (2011). One contribution of our analysis could be seen of providing some microfoundations for these costs. Another strand of literature emphasizes the

---

[58]In general the ECB uses a mark-to-market procedure with the possibility of inputting precautionary reserves in the case of gains. However, in the recent purchases of covered bonds and sovereign debt, it moved to an accounting system at historical costs.

benefits of unconventional monetary policies on the ground that the liabilities of the central bank can have an advantage in relaxing some liquidity (or collateral) constraints that bind the action of private agents.[59]

In our analysis, we have also dealt with credit risk but in an exogenous way. The challenge however is to model it endogenously – depending on the macroeconomic conditions and solvency of the issuers. A more suitable model is required, which goes beyond the scope of this work. However, one could still work in a simple model by postulating a reduced-form representation for credit losses as a function of aggregate macroeconomic variables and/or central bank's intervention. In the latter case, the model would gain another dimension to address the benefits of unconventional monetary policies.

A more relevant extension for emerging-market economies could be the modelling of reserves in foreign currency. In this case, capital losses can be consequence of exchange rate movements and can affect the conduct of monetary policy also for what concerns its effects on the exchange rate (see Jeanne and Svensson, 2007). In this respect, Adler et al. (2012) have shown that for emerging market economies deviations from standard interest-rate policies can be explained by concerns about the weakness of the central-bank balance sheet.

We have discussed our theoretical results in a cash-in-advance model *à la* Lucas and Stokey (1987) where the asset market opens before the goods market. The results of this paper are robust to other ways of modeling the liquidity friction like money in the utility function or through a cash-in-advance constraint in which the goods market opens before the asset markets.[60] The analysis can be extended also to cashless-limiting economies or to overlapping-generation monetary models.

In our model, the velocity of money is constant and unitary. Qualitative results can be robust to environments in which the velocity is endogenous. An interesting extension is to relate it to the balance-sheet position of the central bank. The possibility that a currency can be substituted with other means of payments, when the balance sheet deteriorates, can impair the long-run profitability of the central bank and leave the currency unbacked if there is no fiscal support.[61]

Finally, our analysis has emphasized the importance of the interaction between monetary and fiscal policy. In practice, it is hard to exactly tell when a regime of *full treasury's backing* is in place or even when fiscal policy is passive. It should be interesting to consider the

---

[59]See Kiyotaki and Moore (2012) and Benigno and Nisticò (2013).

[60]Details of the latter model are available upon request.

[61]Quinn and Roberds (2014) discuss the disappearance of the Florin as an international reserve currency in the late 1700s as a consequence of the central bank's income losses on non-performing loans. In Del Negro and Sims (2014), a specific transaction cost of holding money balances delivers a money demand function elastic with respect to the nominal interest rate. In their analysis real money balances can become zero for an interest rate above a certain finite threshold.

implications of uncertainty on the monetary and fiscal regimes, like in Leeper (2013), or the political dimension of the strategic interaction between treasury and central bank.

References

[1] Adler, Gustavo, Pedro Castro, and Camilo E.Tovar (2012). Does Central Bank Capital Matter for Monetary Policy? IMF Working Paper WP/12/60.

[2] Angeletos, George-Marios (2003). Fiscal Policy with Non-Contingent Debt and the Optimal Maturity Structure. Quarterly Journal of Economics 117:2.

[3] Archer, David and Paul Moser-Boehm (2013). Central Bank Finances. BIS Paper No. 71.

[4] Auerbach, Alan J. and Maurice Obstfeld (2005). The Case for Open-Market Purchases in a Liquidity Trap. American Economic Review, vol. 95(1): 110-137.

[5] Bank of England (2013). Annual Report 2012/13: Bank of England Asset Purchase Facility Fund Limited. Bank of England publication, 28 February 2013.

[6] Bassetto, Marco and Todd Messer (2013). Fiscal Consequence of Paying Interest on Reserves. Federal Reserve Bank of Chiacago, w.p. No. 2013-04.

[7] Benigno, Pierpaolo and Salvatore Nisticò (2013). Safe Assets, Liquidity and Monetary Policy. CEPR Discussion Paper No. 9767.

[8] Berriel, Tiago C. and Saroj Bhattarai (2009). Monetary Policy and Central Bank Balance Sheet Concerns. The B.E. Journal of Macroeconomics, Vol. 9(1), Contributions, Article 1.

[9] Buiter, William H. (2014). The Simple Analytics of Helicopter Money: Why It Works – Always. CEPR Discussion Paper No. 9998.

[10] Carpenter, Seth B., Jane E. Ihrig, Elizabeth C. Klee, Daniel W. Quinn, and Alexander H. Boote (2015). The Federal Reserve's Balance Sheet and Earnings: A Primer and Projections. International Journal of Central Banking, 11(2) 237-284.

[11] Christensen, Jens H.E., Jose A. Lopex and Glenn D. Rubebusch (2015). A Probability-Based Stress Test of Federal Reserve Assets and Income. Journal of Monetary Economics, forthcoming.

[12] Cúrdia, Vasco and Michael Woodford (2011). The Central-Bank Balance Sheet as An Instrument of Monetary Policy. Journal of Monetary Economics, 58(1) 54-79.

[13] Del Negro, Marco and Cristopher Sims (2015). When Does a Central Bank's Balance Sheet Require Fiscal Support? Journal of Monetary Economics, forthcoming.

[14] Eggertsson, Gauti and Michael Woodford (2003). The Zero Bound on Interest Rates and Optimal Monetary Policy. Brookings Papers on Economic Activity: 1, 212-219.

[15] Greenlaw, David, James D. Hamilton, Peter Hooper, and Frederic S. Mishkin (2013). Crunch Time: Fiscal Crises and the Role of Monetary Policy. Mimeo, University of California at San Diego.

[16] Hall, Robert. and Ricardo Reis (2015). Maintaining Central-Bank Solvency Under New-Style Central Banking. NBER Working Paper No. 21173.

[17] Jeanne, Olivier and Lars E.O. Svensson (2007). Credible Commitment to Optimal Escape from a Liquidity Trap: The Role of the Balance Sheet of an Independent Central Bank. American Economic Review 97, 474-490.

[18] Kiyotaki, Nobuhiro and John Moore (2012). Liquidity, Business Cycles, and Monetary Policy. NBER Working Paper No. 17934.

[19] Leeper, Eric (1991). Equilibria Under 'Active'and 'Passive' Monetary and Fiscal Policies. Journal of Monetary Economics 27 (1), 129-147.

[20] Leeper, Eric (2013). Fiscal Limits and Monetary Policy. NBER Working Paper No. 18877.

[21] Lucas, Robert E. Jr. (1984). Money in a Theory of Finance. Carnegie-Rochester Conference Series on Public Policy, 21, 9-46.

[22] Lucas, Robert E. Jr. and Nancy L. Stokey (1987). Money and Interest in a Cash-in-Advance Economy. Econometrica, 55(3): 491-513.

[23] Quinn, Stephen and William Roberds (2014). Death of a Reserve Currency. Atlanta Fed Working Paper 2014-17.

[24] Reis, Ricardo (2013). The Mystique Surrounding the Central Bank's Balance Sheet, Applied to the European Crisis. The American Economic Review, 103 (3): 135-140.

[25] Reis, Ricardo (2015). Different Types of Central Bank Insolvency and the Central Role of Seignorage. Journal of Monetary Economics, forthcoming.

[26] Robatto, Roberto (2014). Open Market Operations and Money Supply at Zero Nominal Interest Rates. Mimeo, Chicago University.

[27] Sargent, Thomas (1982). Beyond Demand and Supply Curves in Macroeconomics. American Economic Review, 72: 382-389.

[28] Sargent, Thomas (2011). Where to Draw Lines: Stability Versus Efficiency. Economica, 78: 197-214.

[29] Sargent, Thomas (2014). Robert E. Lucas, Jr.'s Collected Papers on Monetary Theory. Mimeo, New York University.

[30] Sargent, Thomas and Bruce D. Smith (1987). Irrelevance of Open Market Operations in Some Economies with Government Currency Being Dominated in Rate of Return. The American Economic Review, 77: 78-92.

[31] Sargent, Thomas and Neil Wallace (1981). Some Unpleasant Monetarist Arithmetic. Federal Reserce Bank of Minneapolis Quarterly Revies, 5(1): 1-17.

[32] Sims, Christopher (1994). A Simple Model for Study the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy. Economic Theory 4(3): 381-399.

[33] Sims, Christopher (2000). Fiscal Aspects of Central Bank Independence. Unpublished manuscript, Princeton University.

[34] Sims, Christopher (2005). Limits to Inflation Targeting. In Ben S. Bernanke and Michael Woodford (Eds.) *National Bureau of Economic Research Studies in Business Cycles,* vol. 32, ch. 7, Chicago: University of Chicago Press, pp 283-310.

[35] Sims, Christopher (2013). Paper Money. American Economic Review 103(2): 563-584.

[36] Smith, Adam (1806). "An Inquiry into the Nature and Causes of the Wealth of Nations, in three volumes," Vol.II. Edinnburgh: William Greech.

[37] Stella, Peter (1997). Do Central Banks Need Capital? IMF Working Paper WP/97/83.

[38] Stella, Peter (2005). Central Bank Financial Strength, Transparency, and Policy Credibility. IMF Staff Papers Vol. 52(2):

[39] Wallace, Neil (1981). A Modigliani-Miller Theorem for Open-Market Operations. The American Economic Review, 71(3): 267-274.

[40] Woodford, Michael (1994). Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy. Economic Theory 4: 345-380.

[41] Woodford, Michael (2001). Fiscal Requirements for Price Stability. Journal of Money, Credit and Banking 33(1): 669-728.

[42] Zhu, Feng (2004). Central Bank Balance Sheet Concerns, Monetary and Fiscal Rules, and Macroeconomic Stability. Unpublished Manuscript, Bank for International Settlements.

A   Appendix

We collect in this appendix some derivations and proofs.

A.1   Passive fiscal rule

We show that the fiscal rule

$$\frac{A_t}{P_t} = \bar{a} - \frac{T_t^C}{P_t} + \phi \left[\frac{(1+r_t)Q_{t-1}D_{t-1}^G + B_{t-1}^G}{P_t} \right] \tag{A.1}$$

satisfies the requirements of a passive fiscal policy of Definition 4.

First use the flow budget constraint (38) and write it in real terms:

$$Q_t \frac{D_t^G}{P_t} + \frac{1}{1+i_t} \frac{B_t^G}{P_t} = \frac{(1+r_t)Q_{t-1}D_{t-1}^G + B_{t-1}^G}{P_t} - \frac{A_t}{P_t} - \frac{T_t^C}{P_t}. \tag{A.2}$$

Using (A.1) into (A.2), we obtain

$$Q_t \frac{D_t^G}{P_t} + \frac{1}{1+i_t} \frac{B_t^G}{P_t} = (1-\phi) \left[\frac{(1+r_t)Q_{t-1}D_{t-1}^G + B_{t-1}^G}{P_t} \right] - \bar{a}. \tag{A.3}$$

Therefore we can write

$$E_{T-1} \left[\tilde{R}_{T-1,T} \left(Q_T \frac{D_T^G}{P_T} + \frac{1}{1+i_T} \frac{B_T^G}{P_T} \right) \right] =$$
$$(1-\phi) E_{T-1} \left[\tilde{R}_{T-1,T} \frac{(1+r_T)Q_{T-1}D_{T-1}^G + B_{T-1}^G}{P_T} \right] - E_{T-1} \left[\tilde{R}_{T-1,T} \right] \bar{a}. \tag{A.4}$$

Note first that the right-hand side of (A.4) can be written as

$$E_{T-1} \left[\tilde{R}_{T-1,T} \frac{(1+r_T)}{P_T} \right] = \frac{1}{P_{T-1}}$$

$$E_{T-1} \left[\tilde{R}_{T-1,T} \frac{1}{P_T} \right] = \frac{1}{(1+i_{T-1})P_{T-1}}$$

where we have used the equilibrium conditions (15) and (17), since $\tilde{R}_{T-1,T}/\Pi_T = R_{T-1,T}$. We can therefore write (A.4) as

$$E_{T-1} \left[\tilde{R}_{T-1,T} \left(Q_T \frac{D_T^G}{P_T} + \frac{1}{1+i_T} \frac{B_T^G}{P_T} \right) \right] =$$
$$(1-\phi) \left(Q_{T-1} \frac{D_{T-1}^G}{P_{T-1}} + \frac{1}{1+i_{T-1}} \frac{B_{T-1}^G}{P_{T-1}} \right) - E_{T-1} \left[\tilde{R}_{T-1,T} \right] \bar{a}. \tag{A.5}$$

Pre-multiplying equation (A.5) by $\tilde{R}_{T-2,T-1}$ and taking the expectation at time $T-2$, we can write it as

$$E_{T-2}\left[\tilde{R}_{T-2,T}\left(Q_T\frac{D_T^G}{P_T}+\frac{1}{1+i_T}\frac{B_T^G}{P_T}\right)\right]=$$
$$(1-\phi)E_{T-2}\left[\tilde{R}_{T-2,T-1}\left(Q_{T-1}\frac{D_{T-1}^G}{P_{T-1}}+\frac{1}{1+i_{T-1}}\frac{B_{T-1}^G}{P_{T-1}}\right)\right]-E_{T-2}\left[\tilde{R}_{T-2,T}\right]\bar{a}$$

in which we can substitute appropriately, on the right hand side, equation (A.5) lagged one period to get

$$E_{T-2}\left[\tilde{R}_{T-2,T}\left(Q_T\frac{D_T^G}{P_T}+\frac{1}{1+i_T}\frac{B_T^G}{P_T}\right)\right]=$$
$$(1-\phi)^2\left(Q_{T-2}\frac{D_{T-2}^G}{P_{T-2}}+\frac{1}{1+i_{T-2}}\frac{B_{T-2}^G}{P_{T-2}}\right)-(1-\phi)E_{T-2}\left[\tilde{R}_{T-2,T-1}\right]\bar{a}-E_{T-2}\left[\tilde{R}_{T-2,T}\right]\bar{a}.$$

After reiterating the substitution, we get

$$E_t\left[\tilde{R}_{t,T}\left(Q_T\frac{D_T^G}{P_T}+\frac{1}{1+i_T}\frac{B_T^G}{P_T}\right)\right]=$$
$$(1-\phi)^{T-t}\left(Q_t\frac{D_t^G}{P_t}+\frac{1}{1+i_t}\frac{B_t^G}{P_t}\right)-\bar{a}E_t\sum_{j=t+1}^{T}(1-\phi)^{T-j}\tilde{R}_{t,j}\quad\text{(A.6)}$$

in which, by taking the limit for $T\to\infty$, we obtain

$$\lim_{T\to\infty}E_t\left[\tilde{R}_{t,T}\left(Q_T\frac{D_T^G}{P_T}+\frac{1}{1+i_T}\frac{B_T^G}{P_T}\right)\right]=0,$$

provided $0<\phi<2$ and therefore also for $0<\phi<1$. In particular the term

$$\sum_{j=t+1}^{T}(1-\phi)^{T-j}E_t\tilde{R}_{t,j}=\xi_t^{-1}Y_t^\rho E_t\sum_{j=t+1}^{T}(1-\phi)^{T-j}\beta^j\xi_j Y_j^{-\rho}$$

on the right hand side of (A.6) converges to zero as $T\to\infty$ since the weights $(1-\phi)^{T-j}$ converge to zero, provided $0<\phi<2$, for small j while for j close to T, $\beta^j\xi_j Y_j^{-\rho}$ converges almost surely to zero for bounded stochastic processes ξ_j and under any *benchmark* equilibrium.

A.2   Proof of Propositions

A.2.1   Proof of Proposition 3

*Under a combined regime of passive fiscal policy and passive policy of central-bank remittances i) the Proposition of Neutrality holds and ii) the **benchmark** optimal policy is the **equilibrium** optimal policy.*

Proof. Consider the *passive transfer policy* in which the stochastic processes $\left\{A_t, T_t^C\right\}$ are set consistently with the respectively-defined passive policies. Consider the stochastic processes $\left\{Y_t, \Pi_t,\, i_t, Q_t, F_t, K_t, M_t, \Delta_t\right\}$ satisfying the equilibrium conditions (27) to (35) consistently with a given *conventional* monetary policy. Under the *passive transfer policy* the stochastic processes $\left\{A_t, T_t^C\right\}$ are specified to ensure that (37), (38), (42) and (44) hold for any stochastic processes $\left\{Y_t, \Pi_t,\, i_t, Q_t, F_t, K_t, M_t, \Delta_t\right\}$ satisfying the equilibrium conditions (27) to (35) and given any appropriately bounded stochastic processes $\left\{D_t^G, B_t^G, X_t, B_t^C, D_t^C\right\}$ and therefore for three independent specifications of the stochastic processes $\left\{B_t^C, B_t^G, D_t^C, D_t^G\right\}$ also consistent with (39) and (40).[62] Moreover, note that the sum of (42) and (44) given (39) and (40) implies (36). It follows that the stochastic processes $\left\{X_t, B_t, B_t^C, B_t^G, D_t, D_t^C, D_t^G, A_t, T_t^C\right\}$ satisfy each of the conditions in equations (36) to (40) at each time $t \geq t_0$ (and in each contingency at t) given initial conditions $M_{t_0-1}, X_{t_0-1}, B_{t_0-1}^C, B_{t_0-1}^G, D_{t_0-1}^C, D_{t_0-1}^G$ for the specified transfer policy $\left\{A_t, T_t^C\right\}$ and any appropriately-bounded specification of three of the stochastic processes $\left\{B_t^C, B_t^G, D_t^C, D_t^G\right\}$, given $\left\{Y_t, \Pi_t,\, i_t, Q_t, F_t, K_t, M_t, \Delta_t\right\}$. Therefore any $\left\{Y_t, \Pi_t,\, i_t, Q_t, F_t, K_t, M_t, \Delta_t\right\}$ satisfying the equilibrium conditions (27) to (35) consistently with a *conventional* monetary policy is part of a benchmark equilibrium for the same specified *passive transfer policy* $\left\{A_t, T_t^C\right\}$ and any appropriately-bounded specification of the *balance-sheet policy*: the Proposition of Neutrality holds. The above proof can be extended to the *benchmark* optimal policy implying that it is always an equilibrium under the *passive transfer policy*, regardless of the specification of *balance-sheet policy*. ∎

A.2.2   Proof of Proposition 4

Under a passive fiscal policy and full treasury's backing regime i) the Proposition of Neutrality holds and ii) the optimal policy is still given by $\left\{Y_t^*, \Pi_t^*, i_t^*, Q_t^*, F_t^*, K_t^*, M_t^*, \Delta_t^*\right\}$, for any balance-sheet policy and assuming that $\bar{\Pi} > \beta$.

Proof. Consider a collection of stochastic processes $\left\{Y_t, \Pi_t,\, i_t, Q_t, F_t, K_t, M_t, \Delta_t\right\}$ satisfying the equilibrium conditions (27) to (35) consistently with a *conventional* monetary policy and such that

$$\lim_{T \longrightarrow \infty} E_t \left[\beta^{T-t} \frac{\xi_T Y_T^{-\rho} P_t}{\xi_t Y_t^{-\rho} P_T}\right] = 0, \tag{A.7}$$

---

[62] We continue to assume that $\left\{B_t^G, D_t^G\right\}$ cannot be chosen independently.

looking forward from each $t \geq t_0$ and in each contingency at t. This is a non-empty set. Under a regime of passive fiscal policy the stochastic process $\{A_t\}$ is specified to ensure that (42) and (37) hold for any stochastic processes $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t\}$ satisfying the equilibrium conditions (27) to (35), consistently with a *conventional* monetary policy, and given any pair of appropriately bounded stochastic processes $\{D_t^G, B_t^G\}$. Note that (A.7) together with the implication of a full treasury's backing regime, i.e. that $N_t = \bar{N}$, and with (23), (39), (40), (42) implies (36) looking forward from each period $t \geq t_0$ and in each contingency at t. Moreover $N_t = \bar{N}$ implies that

$$Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t - \frac{X_t}{1+i_t} = \bar{N} \tag{A.8}$$

determining the stochastic process $\{X_t\}$ for all appropriately bounded stochastic processes $\{B_t^C, D_t^C\}$ and consistently with (38) and the same $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t\}$. Therefore $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t\}$ is part of a benchmark equilibrium supported by the specified *transfer policy* and this is true for any specification of three of the stochastic processes $\{B_t^C, B_t^G, D_t^C, D_t^G\}$ and therefore for any *balance-sheet policy*. Finally note that, if $\bar{\Pi} > \beta$, the *benchmark* optimal policy is also consistent with the above condition (A.7) given the boundedness of the process $\{\xi_t\}$. Indeed, if there are nominal rigidities, it is known that in the above model is costly to permanently deviate from $\bar{\Pi}$ and therefore the optimal path of inflation is not $\Pi_t = \beta$ at all times provided $\bar{\Pi} > \beta$.[63] This implies that the *benchmark* optimal policy is consistent with (A.7). Following a similar reasoning it can be supported by the specified *transfer policy*, regardless of the specification of *balance-sheet policy*. ∎

A.2.3 Proof of Proposition 6

Under a passive fiscal policy and a deferred-asset policy of central-bank remittances i) the Proposition of Neutrality holds if and only if $N_t = \bar{N}$ in equilibrium for each $t > \tau$ (and in each contingency at t) and for some τ even far in the future and ii) the optimal policy is still given by $\{Y_t^*, \Pi_t^*, i_t^*, Q_t^*, F_t^*, K_t^*, M_t^*, \Delta_t^*\}$ for any balance-sheet policy if and only if $N_t = \bar{N}$ in equilibrium for each $t > \tau$ (and in each contingency at t) and for some τ even far in the future, assuming that $\bar{\Pi} > \beta$.

Proof. Consider stochastic processes $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t\}$ satisfying the equilibrium conditions (27) to (35) given a *conventional* monetary policy such that (A.7) holds looking forward from each $t \geq t_0$ and in each contingency at t. This is a non-empty set. Under a regime of passive fiscal policy the stochastic process $\{A_t\}$ is specified to ensure that (37) and (42) hold for any stochastic processes $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t\}$ satisfying the equilibrium

---

[63]This is the first instance in the paper in which the assumption of nominal rigidity is used.

conditions (27) to (35) and given any pair of appropriately bounded stochastic processes $\left\{ D_t^G, B_t^G \right\}$. Now assume that $N_t = \bar{N}$ for each $t > \tau$ and for some τ even far in the future and in all contingencies, then equation (A.7), together with (23), (39), (40) and (42) implies (36) looking forward from each period $t \geq t_0$ and in each contingency at t. Moreover if $N_t = \bar{N}$ for each $t > \tau$ and for some τ even far in the future and in all contingencies implies that $\{X_t\}_{t \geq \tau}$ is going to be determined by

$$\frac{X_t - B_t^C}{1 + i_t} = \frac{(X_{t-1} - B_{t-1}^C)}{1 + i_{t-1}} + (Q_t D_t^C - Q_{t-1} D_{t-1}^C) - (M_t - M_{t-1}). \tag{A.9}$$

given the stochastic processes $\{i_t, Q_t, M_t\}_{t \geq \tau - 1}$, given any appropriately bounded specification of $\{D_t^C, B_t^C\}$ and any appropriately bounded initial condition $X_{\tau-1}$. For t such that $t_0 \leq t < \tau$, and given the stochastic processes $\{i_t, Q_t, M_t\}$, the stochastic process $\{X_t\}_{t_0}^{\tau-1}$ is determined either by

$$\frac{X_t - B_t^C}{(1 + i_t)} = (X_{t-1} - B_{t-1}^C) + Q_t D_t^C - (1 + r_t) Q_{t-1} D_{t-1}^C - (M_t - M_{t-1}), \tag{A.10}$$

or by (A.9), depending on whether $N_t < \bar{N}$ or $N_t = \bar{N}$, and is appropriately bounded for any appropriately bounded specification of $\{D_t^C, B_t^C\}$ and initial conditions $M_{t_0-1}, X_{t_0-1}, B_{t_0-1}^C, D_{t_0-1}^C$. Therefore if $N_t = \bar{N}$ for each $t > \tau$ and for some τ even far in the future and considering the specified *transfer policy*, it is possible to find some $\left\{ \tilde{X}_t, \ \tilde{B}_t, \ \tilde{B}_t^C, \tilde{B}_t^G, \tilde{D}_t, \tilde{D}_t^C, \tilde{D}_t^G, \tilde{A}_t, \tilde{T}_t^C \right\}$ satisfying each of the conditions in equations (36) to (40) at each time $t \geq t_0$ (and in each contingency at t) given the stochastic processes $\left\{ Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t \right\}$ and initial conditions, regardless of the *balance-sheet policy*. To show that $N_t = \bar{N}$ in equilibrium for each $t > \tau$ (and in each contingency at t) and for some τ even far in the future is also a necessary condition, consider instead that there is an history and a τ such that $N_t < \bar{N}$ for each $t > \tau$ given the stochastic processes $\left\{ Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t \right\}$ for which the Proposition of Neutrality holds. In equilibrium, given the assumed passive fiscal policy, (44) holds looking forward from each t and in each contingency. Therefore the intertemporal budget constraint

$$\frac{X_{t-1}}{P_t} - \frac{B_{t-1}^C}{P_t} + \frac{M_{t-1}}{P_t} - (1 + r_t) \frac{Q_{t-1} D_{t-1}^C}{P_t} = E_t \sum_{T=t}^{\infty} \tilde{R}_{t,T} \left[\frac{i_T}{1 + i_T} \frac{M_T}{P_T} - \frac{T_T^C}{P_T} \right] \tag{A.11}$$

should hold for each t, and in particular for any $t \geq \tau$ (and in each contingency at t) where τ can be very far in the future. However, for the history in which $N_t < \bar{N}$, it should be true that $T_t^C = 0$ for any $t \geq \tau$ under the deferred-asset regime. Given this implication, it is easy to see that (A.11) cannot hold for any appropriately bounded specification of $\{D_t^C, B_t^C\}$ and given $\left\{ Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t \right\}$. Therefore $N_t = \bar{N}$ in equilibrium for each $t > \tau$ (and

in each contingency at t) and for some τ even far in the future is a sufficient and necessary condition for the Proposition of Neutrality to hold. We can follow a similar reasoning for the *benchmark* optimal policy. First note that conditional on the passive fiscal policy and the "deferred-asset" policy, optimal policy is a collection of stochastic processes $\{Y_t^{**}, \Pi_t^{**}, i_t^{**}, Q_t^{**}, F_t^{**}, K_t^{**}, M_t^{**}, \Delta_t^{**}, X_t^{**}, B_t^{C**}, D_t^{C**}\}$ that maximizes (41) and satisfies each of the conditions in equations (27) to (35), (44) at each time $t \geq t_0$ (and in each contingency at t), the non-negativity constraint on the nominal interest rate $i_t \geq 0$, the stochastic process for the exogenous disturbance $\{\xi_t\}$, and the conditions in equations (A.9) or (A.10) depending on whether $N_t < \bar{N}$ or $N_t = \bar{N}$. The optimal policy problem is subject to initial conditions $\Delta_{t_0-1}, M_{t_0-1}, X_{t_0-1}, B_{t_0-1}^C, D_{t_0-1}^C$ and moreover there should be some collection of stochastic processes $\{B_t^{**}, B_t^{**G}, D_t^{**}, D_t^{**G}, A_t^{**}\}$ satisfying each of the conditions in equations (37), (39), (40), (42) at each time $t \geq t_0$ (and in each contingency at t) given the stochastic processes $\{Y_t^{**}, \Pi_t^{**}, i_t^{**}, Q_t^{**}, F_t^{**}, K_t^{**}, M_t^{**}, \Delta_t^{**}, X_t^{**}, B_t^{C**}, D_t^{C**}\}$ and initial conditions $B_{t_0-1}^G, D_{t_0-1}^G$. It is worth mentioning that we add $\{B_t^{C**}, D_t^{C**}\}$ as choice variables in the optimal policy problem because they are degrees of freedom that can be chosen to achieve the maximum welfare. It is indeed optimal to set $D_t^{C**} = 0$ since we already observed that in this case the constraint $T_t^C \geq 0$ is never binding, implying that the *benchmark* optimal policy is the equilibrium optimal policy for any specification of the stochastic process $\{B_t^C\}$ and of one of the stochastic processes $\{B_t^G, D_t^G\}$. The critical step is to show that it is also the optimal policy for any specification of the stochastic process $\{D_t^C\}$. A necessary and sufficient condition is $N_t = \bar{N}$ in equilibrium for each $t > \tau$ (and in each contingency at t) and for some τ even far in the future, provided $\bar{\Pi} > \beta$.

To evaluate the relevance of constraints (A.9) and (A.10) for the optimal policy, consider the Lagrangian problem of the optimal plan in which we evaluate the two additional constraints using the marginal utility of nominal income – λ_t – and attach to them Lagrangian multiplier $\beta^t \vartheta_t$ while considering as a choice variable $(X_t - B_t^C)/(1 + i_t)$. The first-order condition with respect to $(X_t - B_t^C)/(1 + i_t)$ is

$$\vartheta_t \lambda_t = \beta(1 + i_t) \sum_{s_{t+1}:\ N_{t+1} < \bar{N}} \pi(s_{t+1}|s^t) \vartheta(s_{t+1}|s^t) \lambda(s_{t+1}|s^t) +$$

$$\beta \sum_{s_{t+1}:\ N_{t+1} \geq \bar{N}} \pi(s_{t+1}|s^t) \vartheta(s_{t+1}|s^t) \lambda(s_{t+1}|s^t) \quad \text{(A.12)}$$

If $N_t = \bar{N}$ in equilibrium for each $t > \tau$ (and in each contingency at t) and for some τ even far in the future equation (A.12) collapses to

$$\vartheta_t = E_t R_{t,T} \vartheta_T$$

for each $t \geq \tau$ in which we used the equilibrium value of the nominal stochastic discount factor $R_{t,T} = \beta^{T-t}\lambda_T/\lambda_t$. The above equation has a stable solution $\vartheta_t = 0$ at all times $t \geq \tau$. This is clearly the optimal solution since it implies that the condition (A.9) is not a relevant constraint for the optimal policy problem for each $t \geq \tau$. Given that $\vartheta_\tau = 0$, it follows looking backward in (A.12) that $\vartheta_t = 0$ at all times and contingencies. Conditions (A.9) and (A.10) are not relevant constraints of the optimal policy problem. Furthermore, the restriction $\bar{\Pi} > \beta$ ensures that optimal policy satisfies (A.7) and then (44) having used the condition $N_t = \bar{N}$ in equilibrium for each $t > \tau$ (and in each contingency at t) and for some τ even far in the future. To see that the latter is also a necessary condition assume by contradiction that $N_t < \bar{N}$ for each $t > \tau$ and in some history given the candidate optimal stochastic processes $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t\}$ for which the Proposition of Neutrality holds. Similarly to previous steps, (A.11) should hold at each time and in each contingency. However, for the history in which $N_t < \bar{N}$ for each $t > \tau$ and for some τ even far in the future, it should follow that $T_t^C = 0$ for each $t \geq \tau$ given the deferred-asset regime. The equilibrium conditions (A.11) therefore cannot hold for any appropriately bounded specification of $\{D_t^C, B_t^C\}$ given $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t\}$. This completes the proof. ■

A.2.4 Proof of Proposition 7

Consider a collection of stochastic processes $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t\}$ satisfying the equilibrium conditions (27) to (35) given a conventional monetary policy and consider a deferred-asset policy of central-bank remittances; $N_t = \bar{N}$ in equilibrium (for each $t > \tau$ and in each contingency at t and for some τ even far in the future) if there is a time τ_1 even far in the future such that $N_{\tau_2} + M_t > 0$ at some $\tau_2 \geq \tau_1$ for each $t \geq \tau_2$ and either i) ξ_t and $\varkappa_t$ have absorbing states after τ_1 or ii) $D_t^C = 0$ at each $t \geq \tau_1$ and (in each contingency at t)

Proof. Consider the first condition, *i*). If ξ_t and $\varkappa_t$ have absorbing states starting from some point in time τ_1, then the nominal return on long-term securities is equal to the short-term risk-free nominal rate from that date onwards. Central bank's profits are given by (50). If conditional on the stochastic processes $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t\}$ there is a time $\tau_2 \geq \tau_1$ such that $N_{\tau_2} + M_{\tau_2} > 0$ then central-bank profits are positive at time τ_2, as shown in (50). If $N_{\tau_2} + M_t > 0$ for each $t \geq \tau_2$, profits are always positive starting from period τ_2. If it turns out that $N_{\tau_2} < \bar{N}$ the central bank can reach $N_t = \bar{N}$ in a finite period of time given that a *deferred-asset regime* allows zero remittances during this period. Once $N_t = \bar{N}$ remittances can be again rebated to the treasury keeping N_t constant at $\bar{N}$ in the long run. The proof of the second sufficient condition, *ii*), follows similar reasoning noting that if $D_t^C = 0$ at each $t \geq \tau_1$ central-bank profits are also given by (50). ■

A.3 Derivation of the loss function

In this appendix, we derive the second-order approximation to social welfare (41). The approximation is taken with respect to the efficient steady state. The latter maximizes (41) under the resource constraint (34). Since $\Delta_t \geq 1$, it is clear from (41) that at the efficient allocation $\Delta_t = 1$ and $\bar{Y}^{-\rho} = \bar{Y}^\eta$.

The decentralized equilibrium allocation instead implies, in the steady state, through equations (31) to (33), that $\mu\bar{Y}^{-\rho} = (1 - \bar{\varphi})\bar{Y}^\eta$. To implement the efficient allocation it is sufficient to have a subsidy such that $\mu/(1 - \bar{\varphi}) = 1$. This requires setting the employment subsidy $\varrho \equiv 1 - (1 - 1/\theta)(1 - \bar{\varphi})$. A second-order approximation of the utility flows in (41) around this steady state can be simply written as

$$U_t = \bar{U} + \bar{Y}^{-\rho}\left[(Y_t - \bar{Y}) - \frac{\rho}{2}\bar{Y}^{-1}(Y_t - \bar{Y})^2\right] - \bar{Y}^\eta\left[(Y_t - \bar{Y}) + \frac{\eta}{2}\bar{Y}^{-1}(Y_t - \bar{Y})^2\right] -$$
$$- \frac{1}{1+\eta}\bar{Y}^{1+\eta}(\Delta_t - 1) + \mathcal{O}(||\xi||^3)$$

since Δ_t is already a second-order term. The above expansion can be further simplified to

$$U_t = \bar{U} - \frac{1}{2}(\rho + \eta)\bar{Y}^{1-\rho}\hat{Y}_t^2 - \frac{1}{1+\eta}\bar{Y}^{1-\rho}(\Delta_t - 1) + \mathcal{O}(||\xi||^3) \qquad (A.13)$$

in which we have used

$$Y_t = \bar{Y}\left(1 + \hat{Y}_t + \frac{1}{2}\hat{Y}_t^2\right) + \mathcal{O}(||\xi||^3)$$

where $\hat{Y}_t \equiv \ln Y_t/\bar{Y}$. Note that the welfare function (41) can be written as

$$W_{t_0} = E_{t_0}\sum_{t=t_0}^\infty \beta^{t-t_0}\xi_t U_t$$

whose second-order approximation is simply

$$W_{t_0} = E_{t_0}\sum_{t=t_0}^\infty \beta^{t-t_0}[\bar{U} + (\xi_t - 1)\bar{U} + (U_t - \bar{U}) + (\xi_t - 1)(U_t - \bar{U})] + \mathcal{O}(||\xi||^3)$$

where we have normalized the steady-state of ξ_t to 1. From (A.13), note that $U_t - \bar{U}$ is a second-order term, it follows that the second-order approximation of the welfare can be simply written as

$$W_{t_0} = -E_{t_0}\sum_{t=t_0}^\infty \beta^{t-t_0}\bar{Y}^{1-\rho}\left[\frac{1}{2}(\rho + \eta)\hat{Y}_t^2 - \frac{1}{1+\eta}(\Delta_t - 1)\right] + \text{t.i.p.} + \mathcal{O}(||\xi||^3)$$

since $\bar{U}$ and $(\xi_t - 1)\bar{U}$ are terms independent of policy which enters in t.i.p.

Finally, by taking a second-order approximation of (34), and integrating appropriately across time, we obtain

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \hat{\Delta}_t = \frac{\alpha}{(1-\alpha)(1-\alpha\beta)} \theta(1+\eta)(1+\eta\theta) \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{(\pi_t - \bar{\pi})^2}{2} + \text{t.i.p.} + \mathcal{O}(||\xi||^3).$$

We can therefore write

$$W_{t_0} = -(\rho + \eta)\bar{Y}^{1-\rho} \cdot \frac{1}{2} E_t \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\hat{Y}_t^2 + \lambda_\pi (\pi_t - \bar{\pi})^2] \right\} + \text{t.i.p.} + \mathcal{O}(||\xi||^3)$$

from which the loss function (54) follows, and where we have defined

$$\lambda_\pi \equiv \frac{\theta}{\kappa},$$

$$\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \frac{(\rho + \eta)}{(1+\eta\theta)}.$$

In (55), the parameter ζ is defined as $\zeta \equiv \kappa(1+\bar{\imath})/(\rho + \eta)$ where $\bar{\imath}$ is the steady-state level of the nominal interest rate.