Optimal Inflation Target in an Economy with Menu Costs and Zero Lower Bound*

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Abstract

This paper studies the optimal inflation target in a medium-scale menu cost model with an occasionally binding zero lower bound on interest rates. I find that the optimal inflation target is 3%, larger than the rates currently targeted by the Fed and the ECB, and also larger than in existing models used for monetary policy analysis. When my model is consistent with the frequency and the distribution of price changes, resource misallocation does not increase greatly with inflation. The key element for this result is firms' idiosyncratic shocks, which is necessary to match the micro data. Inflation is therefore not as costly as in existing models, even though my model's business cycle implications are similar to the ones found in those models (e.g. Calvo) in normal times. For this reason, a higher inflation target, aimed at reducing the frequency and severity of recessions caused by the zero lower bound, is warranted.

JEL: E3, E5, E6.

Keywords: menu costs, (S,s) policies, monetary policy, inflation target.

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1 Introduction

Since the end of the 1980s, many countries around the world, including the U.S., have adopted a policy of inflation targeting, in which the central bank explicitly declares a medium-term inflation rate. An important question for monetary policy is: What is the inflation target a Central Bank should have?

Following the Great Recession, a number of economists have argued that a higher inflation target may be beneficial in the presence of a zero lower bound (ZLB) on nominal rates.[1] According to these economists, a higher inflation target would raise average nominal interest rates, thus giving central bankers more room to react in response to adverse macroeconomic shocks. A higher inflation target, however, would also have its costs. In existing sticky price models, the major cost of inflation is that it induces inefficient dispersion in relative prices and therefore productivity losses due to dispersion in firms' marginal product.

This paper quantifies these costs and benefits of a higher inflation target. Since the main cost of inflation is price dispersion, I discipline this cost with a menu cost model with idiosyncratic cost shocks that reproduces micro-price behavior at low and high levels of inflation.[2] The model also generates realistic business cycle dynamics, owing to a medium-scale macroeconomic model with a rich set of aggregate shocks and occasionally binding ZLB. Requiring that the model reproduces aggregate business cycles is critical because the main benefit of a higher inflation target is the reduction in business cycle volatility stemming from less frequent occurrence of ZLB episodes.

My main result is that the optimal inflation target in this environment is 3%, much greater than in existing pricing models—see Billi (2011) and Coibion and Gorodnichenko (2012). I show, for example, that the optimal inflation target in an otherwise identical Calvo pricing model would be 1%. The main reason why the optimal inflation target is higher than in previous studies is that the cost of inflation is much lower in my model. Despite this result, my model has similar business cycle implications in normal times. Inflation is less costly in my model because of the interaction between idiosyncratic shocks and menu costs.

To see why idiosyncratic shocks are critical to my argument, consider first an environment with menu costs but no idiosyncratic or aggregate shocks. In this model, all the prices are optimal at zero inflation target; hence price dispersion is zero.[3] For any positive level of inflation, the distribution of relative prices jumps from all the prices being optimal to a uniform distribution within the Ss bands; thus, there is positive price dispersion. This discontinuity at zero inflation, together with a first order increase in the width of the Ss bands with respect to inflation, implies that the model with only menu cost has a larger cost of inflation than the Calvo model at low levels of inflation—less than 3%—and a menu cost model with idiosyncratic shocks as I will explain below.

To see why menu costs are important for my argument, consider next my model with menu costs and idiosyncratic shocks. Since firms are exposed to real fluctuations of idiosyncratic marginal cost, misallocation and

[1]See Ball (2013), Blanchard, DellAriccia and Mauro (2010), and Williams (2009) for a revival of this old proposal by Summers (1991).

[2]see Blanco (2015) for a proof of a one-to-one mapping between micro-price statistics and inefficient dispersion of relative prices for any sticky price model.

[3]Strictly speaking, this result requires a very small probability of a free price change to generate a unique ergodic steady state.

productivity losses depend on the dispersion of markup. If a firm is hit by negative idiosyncratic cost shocks, then positive inflation decreases the firm's markups and offsets the idiosyncratic cost shock. In this case, higher inflation offsets idiosyncratic shocks, decreasing the dispersion of markups. In the other case, whenever a firm is hit by positive idiosyncratic cost shocks, higher inflation increases price dispersion. At zero inflation target, these two effects cancel each other, generating a zero first order effect of inflation to price dispersion—this property holds even in the Calvo model. Since with large idiosyncratic shocks, the width of the Ss bands and the symmetry of the distribution of relative prices are almost constant, this effect prevails for low levels of inflation in my menu cost model. The increase in the frequency of price changes coming from the firms hit by sufficiently large positive cost shocks is almost canceled out by the reduction of price changes coming from firms with large negative cost shocks. Thus, the low cost of inflation does not come from a significant increment in the frequency of price changes: raising inflation from zero to three percent inflation target increases the frequency of price changes from 10.05% to 10.3%.

I continue with an overview of the ideas in this paper and how they relate to other contributions in the literature.

Model for Optimal Inflation Target: The pricing model I use is a menu cost model in which producers have free random price change opportunities and a mixed normal distribution for firm shocks— Nakamura and Steinsson (2010) called this model the CalvoPlus model, which for simplicity I will refer to as the menu cost model. This model matches, among others things, small price changes, large size of average price changes and fat tails in the price change distribution in low inflation environments. Additionally, this model has the capacity to generate micro-price statistics like frequency of price change and the extensive margin component of inflation volatility in high inflation environments.

Since my goal is to study business cycle stabilization due to a lesser incidence of the ZLB, I depart from a standard menu cost model in several dimensions. First, monetary policy is endogenous and responds to aggregate shocks with a Taylor rule subject to an occasionally binding ZLB. Second, I introduce a rich set of aggregate shocks to productivity, government expenditure, risk premium, and nominal rates to generate realistic business cycle dynamics of output and inflation. Third, I assume Greenwood-Hercowitz-Huffman (GHH) preferences and intermediate inputs in production. These features allow the model to match the low sensitivity of inflation to output observed in the data. Together with the pricing model, these three features generate business cycle fluctuations of inflation that resemble the US economy.

Since the main trade-off comes from mean price dispersion versus consumption and labor volatility, I assume Epstein-Zin preferences that can capture the cost of business cycle without affecting the inter-temporal elasticity of substitution—see Alvarez and Jermann (2004).

Pricing Behavior Implications of the Model: My model is flexible enough that, with a suitable change of parameters, it nests the key pricing model used heretofore in the literature: the Calvo model, the Golosov and Lucas (2007) model, the Gertler and Leahy (2008) model, and intermediate combinations of these models, like the ones studied in Nakamura and Steinsson (2010) and Midrigan (2011).[4] I use the data to pin down its key parameters. In particular, I estimate the model to match the average price change distribution at 2% inflation target using monthly price quotes recollected for the UK's CPI. In order to do this, I clean the data with respect to sales, product substitution, out-of-season, outliers, and product-level heterogeneity. In addition to matching these low-inflation price statistics, the model can match pricing behavior in medium and high inflation environments, both across countries and across products. The first evidence my model matches is across countries: 1) the elasticity of frequency of price change with respect to inflation, and 2) the decreasing intensive component of inflation volatility with the level of inflation. The second evidence is across products. Since for the same product I have a large sampling across shops and regions—an unique property of the UK's CPI data—I can compute the frequency of price change, the standard deviation of price change, the skewness of price change, and the mean inflation for each product. I find a positive relation between frequency of price change and mean inflation—which the model quantitatively matches—but also an insignificant relation between skewness of price changes and inflation. Since inflation is costly if it generates a large asymmetry in the price change distribution, I don't find evidence of it exploiting product heterogeneity in inflation levels.

My paper contributes to the optimal inflation target level literature—see Schmitt-Grohé and Uribe (2010) for a description of the costs and benefits of a higher inflation target. The effect of the ZLB on the optimal inflation target has been studied in Walsh (2009), Billi (2011) and Coibion, Gorodnichenko and Wieland (2012). This paper extends their analysis by using a menu cost model with idiosyncratic shocks that reproduces micro price data.

Business Cycle Implications of the Model: In addition to matching the micro-price statistics, my model model also reproduces salient business cycle statistics in the data. In particular, I verify that the business cycle dynamics in my model are similar to the Calvo model without ZLB and a low inflation target. Hence, my model is consistent with the main model used by central banks for its ability to reproduce macroeconomic dynamics. To formalize this argument, I show that an econometrician with only aggregate time series could not distinguish which model generates the data in a finite sample though VARs or business cycle statistics.

The result of similar business cycle dynamics without ZLB depends on the pricing model and general equilibrium effects due to a Taylor rule. To explain the effect of the latter mechanism, notice that in my model monetary policy responds to inflation, so the effect of a steeper Phillips curve on equilibrium inflation volatility is partially offset by movements in the nominal interest rate. To see this argument, assume a structural shock that increases the output-gap. Since nominal interest rate depends on inflation, in the menu cost model nominal interest will increase by more than in Calvo; consequently, the equilibrium output-gap responds by less and therefore so does inflation.

I will offer a brief sketch of how my model works at the ZLB at low and high inflation targets. A liquidity trap is a situation where the real interest rate is too high. This leads to excessive saving, and since the nominal interest rate cannot decrease due to the ZLB, this exacerbates the depression of spending and output, which in turn creates

[4]Multi-product firms with fat tail idiosyncratic shocks are observational equivalent to the free random probability of adjustements.

more deflationary pressures. At a low inflation target, a depressed inflation implies deflation; thus firms' relative prices hit the upper Ss bands, and firms choose to reduce their price optimally. Hence, in the aggregate, a small measure of firms will make large price changes, with its respective impacts on inflation, real rate and output-gap; these changes then feed back to new price changes generating a large deflationary spiral. At a high inflation target, a depressed inflation implies low inflation, so firms' relative prices do not hit the upper Ss bands, and firms only change their price if they are hit by large idiosyncratic shocks or zero menu cost. Importantly, this mechanism relies mainly on the distribution of price changes and not on the total number of price changes.

The paragraph above makes it clear why the ZLB breaks the equivalence result of aggregate dynamics in the Calvo and my menu cost model. Without ZLB, a strong response from the monetary authority mitigates a steeper Phillips curve in equilibrium inflation dynamics. With a ZLB constraint for the monetary authority, this result is reversed: monetary authority's incapacity to respond to adverse macroeconomic shocks exacerbates a steeper Phillips curve in equilibrium inflation dynamics.

The equivalence between the Calvo model and my menu cost model is reminiscent of results in Gertler and Leahy (2008) and Midrigan (2011). My paper contributes to this literature by showing an equivalent set of results in a medium-scale general equilibrium model with a Taylor rule that can generate empirically realistic business cycles. There are several papers that study the macroeconomic consequence of the ZLB in a medium scale New Keynesian model—see Cuba-Borda (2014), Fernández-Villaverde, Gordon, Guerrón-Quintana and Rubio-Ramirez (2015) and Gust, López-Salido and Smith (2012)—with Calvo pricing. My contribution with respect to these papers is to extend their study of ZLB dynamics to a menu cost model.

Normative Implications of the Model: The optimal inflation target in my menu cost model is 3%; with the same preference and technology in a Calvo model, it is 1%. To understand the main reasons for this result, I begin from the observation that any sticky price model can be described with a set of wedges from the neoclassical growth model; thus the welfare depends on these wedges and the structural shocks that affect the feasible set of consumption and labor in the economy like productivity and government expenditure.

The first wedge, the labor wedge, comes directly from the stochastic process of output-gap. A higher inflation target decreases the business cycle volatility of the output-gap for two reasons. First, it decreases the probability of hitting the ZLB. Second, in the menu cost model, a higher inflation target reduces the extensive component of the deflationary spiral during liquidity traps, as I explain above. Additionally, at low levels of inflation targets, the average output-gap is lower. Thus, a higher inflation target for levels of inflation less than 4% increases the mean and decreases the volatility of the output-gap.

The second wedge, the productivity wedge, comes directly from the inefficient dispersion of the relative prices and its effect on the aggregate productivity. First, I show that strategic complementarities in the form of intermediate inputs generate a larger cost of inflation since they increase the elasticity from the inefficient dispersion of the relative prices to productivity. Second, I show that when I estimate the menu cost model to be consistent with pricing behavior, price dispersion increases 10 times less than in the Calvo between 0% and 6% inflation, even if 47% of price changes are due to random free price adjustments. Large idiosyncratic shocks, and not a large increase of the frequency of price changes, are critical for this result.

The cost of inflation in menu cost models has been analyzed by Burstein and Hellwig (2008), Alvarez, Gonzalez-Rozada, Neumeyer and Beraja (2011), Nakamura, Steinsson, Sun and Villar (2016) and Blanco (2015). Burstein and Hellwig (2008), Alvarez *et al.* (2011) and Nakamura *et al.* (2016) analyze the cost of inflation in an off-the-shelf menu cost model consistent with micro-data of high inflation environments. My paper contributes to this literature since I incorporate a menu cost model in a standard New Keynesian model to analyze policy trade-offs with a realistic cost of inflation.

Solution Method: There are two challenges to numerically solve a menu cost model in a medium scale DSGE New Keynesian model. First, the firm and aggregate equilibrium conditions have kinks, eliminating perturbation methods as a means to solve these economies. Thus, I rely on global projection methods. Due to the curse of dimensionality and since the Smoliak interpolation method does not work to approximate idiosyncratic equilibrium conditions, I developed a new interpolation method where I combine Smoliak interpolation with splines projection methods. I call this method Completed Smoliak Interpolation method.

Secondly, since the state of the economy is the distribution of relative prices, the solution method I use to compute this economy is Krusell-Smith. I find that the standard application of this algorithm fails in these economies, so I develop a modified version of this algorithm. In my model, Krusell-Smith consists of obtaining the inflation policy function in the simulation and using this function to solve aggregate and idiosyncratic equilibrium conditions. Thus, the inflation policy is obtained in the simulation *separately* from the solution of equilibrium equations. But replacing the Phillips curve with the inflation policy function, at the moment of solving aggregate equilibrium conditions, implies nominal and real interest rates that depend only on the state of the economy. Since nominal and real interest rates do not react to output-gap whenever solving idiosyncratic and aggregate equilibrium conditions, there is indeterminacy at the moment of solving aggregate conditions. To avoid this problem, I modify the Krusell-Smith algorithm to incorporate the intensive margin of the Phillips curve whenever solving the equilibrium conditions.

Section 2 describes the model. Section 3 presents the equilibrium conditions in a simplified version of the model and shows the modified Krusell-Smith algorithm together with the projection method I used. Section 4 calibrates the model. Section 5 analyzes the positive dimensions of the model with respect to micro-pricing behavior and business cycle dynamics and section 6 analyzes the optimal inflation target. Section 7 concludes.

2 Model

Time is discrete. There is a continuum measure one of intermediate firms indexed by $i \in [0,1]$, a final competitive firm, a representative household, a central bank and a government.

The final competitive firm produces output Y_t using intermediate firms' production y_{ti} subject to random idiosyncratic shocks A_{ti}

$$Y_t = \left(\int_0^1 \left(\frac{y_{ti}}{A_{ti}} \right)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}} \tag{1}$$

where the final output uses a Dixit-Stiglitz aggregator with elasticity γ.

Intermediate firms are monopolistically competitive. The intermediate good firm i produces output y_{ti} using labor l_{ti} and material n_{ti}, and the productivity of the firm is given by an idiosyncratic component A_{ti}, an aggregate component η_{tz} and a labor augmented productivity $\Gamma_t = (1+g)^t$ according to

$$y_{ti} = A_{ti}\eta_{tz}n_{ti}^{\alpha} \left(\Gamma_t l_{ti} \right)^{1-\alpha} \tag{2}$$

The firm's production function features intermediate inputs as in the data. This technological assumption adds real rigidities in the model: the firm's policy function depends on its own marginal cost and aggregate price level. There are two consequences of this assumption in the model: a flatter Phillips curve, which affects inflation dynamics and the magnitude of the deflationary spiral; and a larger elasticity of productivity losses with respect to inefficient price dispersion, which affects the cost of inflation.

Following the literature,[5] I refer to A_{ti} as a quality shock. On one hand, a decrease in A_{ti} increases the marginal product of the final producer, but at the same time, it reduces the marginal product of the intermediate producer. These two effects offset each other, in such a way that the marginal product of labor in firm i with respect to the final output is independent of A_{ti}. The main reason to add this quality shock in the Dixit-Stiglitz aggregator is to decrease the state space of the firm and the aggregate state of the economy (see section 3 for a detailed explanation).

The quality shock growth rate A_{ti} follows a mixed normal distribution given by

$$\Delta \log(A_{ti}) = \begin{cases} \eta^1_{t+1i} & \text{with prob. } p \\ \eta^2_{t+1i} & \text{with prob. } 1-p \end{cases} \quad ; \quad \eta^k_{ti} \sim_{i.i.d} N(0, \sigma_{ak}) \tag{3}$$

Idiosyncratic shocks allow the model to match the empirical facts with respect to firms' micro-price behavior. Moreover, as I show in section 6.3, the interaction between idiosyncratic shocks and menu cost is critical to have a low cost of inflation.

Firms face a stochastic physical cost of changing their price. Every time the firm changes her nominal price,

[5] This formulation was first used by Woodford (2009) to keep his model tractable. It was also used by Midrigan (2011), Midrigan and Kehoe (2011), Alvarez and Lippi (2012) and others.

she has to pay a *menu cost* given by θ_t units of labor. The menu cost is an *i.i.d.* random variable over time with the following process

$$\theta_t = \begin{cases} \theta & \text{with prob. } 1 - hz \\ 0 & \text{with prob. } hz \end{cases} \tag{4}$$

The menu cost is fixed to a constant number with a free price change opportunity with an i.i.d. probability. The random menu cost allows the model to generate small price changes—as in the data—that cannot be matched with a constant menu cost.

Households' preferences are given by

$$\begin{aligned} U_t &= u(C_t, L_t)(1-\beta) + \beta \mathbb{E}_t [U_{t+1}^{1-\sigma_{ez}}]^{\frac{1}{1-\sigma_{ez}}} \\ u(C_t, L_t) &= \bar{u}\,(1-\sigma_{np})^{-1} \left(C_t - \Gamma_t \kappa \,(1+\chi)^{-1} L_t^{1+\chi} \right)^{1-\sigma_{np}} \end{aligned} \tag{5}$$

where σ_{ez} encodes a measure of risk aversion. Period utility follows GHH preferences specification, where C_t is aggregate consumption and L_t is labor supply. The term Γ_t affects the disutility of labor and implies a balance growth path in the model. GHH preferences allow the model have low volatility of inflation, since it decreases the elasticity of the output with respect to the marginal cost for a given Frisch elasticity.

This paper computes the optimal inflation target; therefore, it is crucial to capture the benefit of business cycle stabilization. With this objective in mind, I depart from expected utility and I calibrate the Epstein-zin parameter parameter to match asset pricing facts with respect to the price of risk.

The consumer faces the following budget constraint given by

$$P_t C_t + B_t = W_t L_t + \int \Phi_{ti} di + \eta_{t-1q} R_{t-1} B_{t-1} + T_t \tag{6}$$

where W_t and P_t are the nominal prices of labor and consumption, Φ_{ti} are nominal profits for the intermediate producer and T_t are lump sum transfers from the government. B_{t-1} is the stock of one-period nominal bonds with a rate of return $R_{t-1}\eta_{t-1q}$, where the second element generates a wedge between the nominal interest rate controlled by the central bank and the return of the assets held by households. For this reason it is defined as a risk premium shock. This shock can be micro-funded as a net-worth shock in models with the financial accelerator and capital accumulation. Moreover, this shock allows my model to match the probability of hitting the ZLB.

The behavior of monetary policy is described by a Taylor rule given by

$$\begin{aligned} R_t^* &= \left(\frac{1+\bar{\pi}}{\beta(1+g)^{-\sigma_{np}}}\right)^{1-\phi_r} \left(R_{t-1}^*\right)^{\phi_r} \left[\left(\frac{P_t}{P_{t-1}(1+\bar{\pi})}\right)^{\phi_\pi} X_t^{\phi_y}\right]^{1-\phi_r} \left(\frac{X_t}{X_{t-1}}\right)^{\phi_{dy}} \eta_{rt} \\ R_t &= \max\left\{1, R_t^*\right\} \end{aligned} \tag{7}$$

where R is the nominal interest rate, $\bar{\pi}$ is the target inflation, η_r is a money shock, and X_t is the output-gap, i.e., the ratio between current output and the natural level of output defined in an economy with zero menu cost (an economy without price rigidities). This quantitative Taylor rule describes the behavior of monetary policy with binding and non-binding ZLB. I define R^* as the shadow interest rate. Importantly, the shadow interest rate can be below zero even after an economic recovery as optimal policy and empirical evidence in this framework suggests.

Aggregate output is equal to aggregate consumption plus government expenditure

$$Y_t - \int n_{ti} di = C_t + \eta_{tg} \tag{8}$$

where I used η_{tg} to denote government expenditure. The government follows a balanced budget each period.

Aggregate shocks follow an AR(1) given by

$$\log(\eta_{tj}) = (1-\rho_j)\eta_j + \rho_j \log(\eta_{t-1j}) + \sigma_j \epsilon_{tj} \quad \epsilon_{tj} \sim_{i.i.d.} N(0,1) \tag{9}$$

where $j \in \{r, q, z, g\}$. Next I will describe each agent's problem and the equilibrium definition.

Household: The representative consumer problem is given by

$$\max_{\{C_t, L_t, B_t, u_t, U_t\}_t} U_0 \tag{10}$$

subject to (5) and (6) for all periods. From the problem of the representative consumer we have the stochastic nominal discount factor

$$Q_{t+1} = \beta \left(\frac{U_{t+1}}{\mathbb{E}_t[U_{t+1}^{1-\sigma_{ez}}]^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}} \frac{U_c(C_{t+1}, L_{t+1})}{U_c(C_t, L_t)} \frac{P_t}{P_{t+1}} Q_t \tag{11}$$

with $Q_0 = 1$.

Final Firm Producer: The final producer problem is given by

$$\max_{\{Y_t, \{y_{ti}\}_i\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} Q_t \left(P_t Y_t - \int_0^1 p_{ti} y_{ti} di\right)\right] \tag{12}$$

subject to (1). Given constant return to scale and zero profits conditions, we have that the aggregate price level and the firm's demand are given by

$$P_t = \left(\int_0^1 (p_{ti} A_{ti})^{1-\gamma} \, di \right)^{\frac{1}{1-\gamma}} \qquad y_t(A_{ti}, p_{ti}) = A_{ti} \left(\frac{A_{ti} p_{ti}}{P_t} \right)^{-\gamma} Y_t \tag{13}$$

Intermediate Firm Producer: The intermediate firm's problem is given by

$$\begin{aligned} &\max_{p_{ti}} \mathbb{E} \left[\sum_{t=0}^{\infty} Q_t \Phi_{ti} \right] \qquad s.t. \\ &\Phi_{ti} = y_t \left(A_{ti}, p_{ti} \right) \left(p_{ti} - \iota (1 - \tau_{mc}) \frac{(W_t)^{1-\alpha} P_t^{\alpha}}{A_{ti} \eta_{tz}} \right) - I(p_{t-1i} \neq p_{ti}) W_t \theta_{ti} \end{aligned} \tag{14}$$

subject to (3), (4) and A_{-1}, p_{-1} given. Note that I've already included the optimal technique in the marginal cost of the firm with $\iota = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} + \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}$.

There is a subsidy to total cost denoted by τ_{mc} that allows my model to match two important objects for the optimal inflation target: the elasticity of labor misallocation across firms with respect to the relative price dispersion and the average level of markups. Without the cost subsidy, there is a trade-off between matching the mean markup or the demand elasticity.

Equilibrium definition *An equilibrium is a set of stochastic processes for (i) a policy for the representative consumer* $\{C, L, B, u, U\}_t$*; (ii) pricing policy functions for firms* $\{p_{ti}\}_t$ *and inputs demand* $\{n_{ti}, l_{ti}\}$ *for the monopolistic firms; (iii) final output and inputs demand* $\{Y_t, \{y_{ti}\}_i\}_t$ *for the final producer and (iv) nominal interest rate* $\{R\}_t$*:*

1. *Given prices,* $\{C, L, B\}_t$ *solve the consumer's problem in* (10).

2. *Given prices,* $\{Y_t, \{y_{ti}\}_i\}_t$ *solve the final good producer problem in* (12).

3. *Given the prices and demand schedule, the firm's policy* p_{ti} *solves* (14) *and inputs demand is optimal.*

4. *Nominal interest rate satisfies the Taylor rule* (7).

5. *Markets clear in each period:*

$$\begin{aligned} \int_0^1 (l_{ti} + I(p_{ti} \neq p_{t-1i}) \theta_{ti}) \, di &= L_t \\ Y_t - \int_0^1 n_{ti} di &= C_t + \eta_{tg} \end{aligned}$$

Notice that menu cost implies real resources in the economy and therefore it has welfare implication with respect to the optimal inflation target.

3 Equilibrium Conditions and Solution Method

This section describes the equilibrium conditions and the solution method to compute it in a simplified environment. Computing the equilibrium poses two challenges: 1) the combination of a Taylor rule for monetary policy and an infinite dimensional aggregate state requires a non-standard application and evaluation of Krusell-Smith; 2) both aggregate and idiosyncratic policy functions have kinks.

For exposition, and for exposition only, I simplify the model in several dimensions in this section: preferences are given by expected utility $\sigma_{ez} = 0$ with period utility $u(C, L) = \log\left(C - \frac{L^{1+\chi}}{1+\chi}\right)$; the only input of production is labor $y_i = A_i l_i$ and menu costs are constant; the risk premium shock is the only structural shock in the economy; and the Taylor rule is given by $R_t = \max\left\{\frac{1+\bar{\pi}}{\beta}\left(\frac{\Pi_t}{1+\bar{\pi}}\right)^{\phi_\pi}, 1\right\}$. I abstract from growth rate; thus $g = 0$. The equilibrium conditions of the main model are in the appendix C. The equilibrium conditions of the Calvo and the menu cost model together with the algorithm to compute the equilibrium in each model are in the online appendix sections F and G.

3.1 Equilibrium Conditions

Firm's Equilibrium Conditions: The relevant idiosyncratic state variable for firm i at time t is $\tilde{p}_{ti} = \frac{p_{ti}A_{ti}}{P_t}$, the relative price multiplied by productivity. For simplicity, I define this object as relative price. The assumption that productivity shocks also affect the demand of the intermediate input implies that the firm's static profits depend only on the relative price; thus, it is the only idiosyncratic state variable for the firm. Let $v(\tilde{p}_-, S)$ be the present discounted value of a firm with previous relative price $\tilde{p}_-$ and current aggregate state S. Then $v(\tilde{p}_-, S)$ satisfies

$$\begin{aligned} v(\tilde{p}_-, S) &= \mathbb{E}_{\Delta a}\left[\max_{\text{change,no change}}\left\{V^c(S), V^{nc}\left(\frac{\tilde{p}_- e^{\Delta a}}{\Pi(S)}, S\right)\right\}\right] \\ V^{nc}(\tilde{p}, S) &= mu(S)C(S)\tilde{p}^{-\gamma}(\tilde{p} - w(S)) + \beta\mathbb{E}_{S'}\left[v\left(\tilde{p}, S'\right)|S\right]\} \\ V^c(S) &= -\theta w(S)mu(S) + \max_{\tilde{p}}\left\{mu(S)C(S)\tilde{p}^{-\gamma}(\tilde{p} - w(S)) + \beta\mathbb{E}_{S'}\left[v\left(\tilde{p}, S'\right)|S\right]\right\} \end{aligned} \tag{15}$$

where $mu(S), C(S), w(S)$ and $\Pi(S)$ denote the marginal utility, aggregate consumption, real wage and inflation respectively. The timing of the firms' optimization problem is as follows: first, aggregate and idiosyncratic uncertainty are realized; then the firm has the option either to change the price or keep it the same. If it changes the price, it has to pay the menu cost θ.

The policy of the firm is characterized by two objects: (1) a reset price and (2) a continuation region. Let $P^*(S)$ be the reset price, i.e. the firm's relative price with respect to the aggregate price level. Then

$$P^*(S) = \max_{\tilde{p}}\left\{mu(S)C(S)\tilde{p}^{-\gamma}(\tilde{p} - w(S)) + \beta\mathbb{E}_{S'}\left[v\left(\tilde{p}, S'\right)|S\right]\right\} \tag{16}$$

The firm's relative price does not depend on the idiosyncratic shock; it only depends on the aggregate state of the economy and therefore it is the same across resetting firms. The continuation region is given by all relative prices such that the value of changing the price is less than the value of not changing the price. Let $\Psi(S)$ be the continuation region. Then

$$\Psi(S) = \{\tilde{p} : V^{nc}(\tilde{p}, S) \geq V^{c}(S)\} \tag{17}$$

Since the firm makes the pricing decision after aggregate and idiosyncratic shocks are realized, the firm's policy is given by changing the price and set a relative price equal to $P^*(S)$ if and only if $\frac{\tilde{p}_- e^{\Delta a}}{\Pi(S)} \notin \Psi(S)$.

As is typical in models with heterogeneity, the firm needs to forecast equilibrium prices and quantities and the aggregate state law of motion. If the firm knows these functions, then it has all the elements to take the optimal decision in (15).

Aggregate conditions: The aggregate conditions are given by the household optimality conditions, feasibility and the monetary policy rule

$$\begin{aligned}
mu(S) &= \beta R(S)\eta_q(S)\mathbb{E}_{S'}\left[\frac{mu(S')}{\Pi(S')}|S\right] && (18)\\
\kappa L(S)^{\chi} &= w(S) \quad ; \quad mu(S) = \bar{u}(C(S) - (1+\chi)^{-1} L^{1+\chi})^{-1} && (19)\\
R(S) &= \max\left\{\frac{1+\bar{\pi}}{\beta}\left(\frac{\Pi(S)}{1+\bar{\pi}}\right)^{\phi_\pi}, 1\right\} \quad ; \quad C(S) = \frac{L(S) - \Omega(S)\theta}{\Delta(S)} && (20)
\end{aligned}$$

where $\Delta(S)$ is labor productivity due to price dispersion given by

$$\Delta(S) = \int \tilde{p}^{\gamma} f(d\tilde{p}) \tag{21}$$

and $f(\tilde{p})$ is the distribution of relative prices after repricing and $\Omega(S)$ is the measure of firms changing the price.

Aggregate equilibrium conditions depend on two outcomes of the firm problem: inflation and price dispersion. Price dispersion only depends on the distribution of relative price and not the distribution of relative price and idiosyncratic productivity shocks. This is a direct consequence of the structure of the *quality* idiosyncratic shocks. To see this, notice that the technological assumptions over the idiosyncratic shocks imply that labor demand is given by

$$\int l_i(S)di = C(S)\left[\int \tilde{p}^{\gamma} f(d\tilde{p})\right] + \theta\Omega(S) \tag{22}$$

where $l_i(S)$ are firm's demand functions.

The key cross-equation restriction where the menu cost model deviates from the Calvo model is the cross-equation restriction with respect to inflation. The next proposition shows the equilibrium condition for inflation:

Proposition 1 *Define*

$$\mathcal{C}(S) = \left\{(\tilde{p}_-, \Delta a) : \frac{\tilde{p}_- e^{\Delta a}}{\Pi(S)} \in \Psi(S)\right\}$$

Inflation dynamic is given by

$$\Pi(S) = \left(\frac{1-\Omega(S)}{1-\Omega(S)P^*(S)^{1-\gamma}}\right)^{\frac{1}{1-\gamma}} \varphi(S)$$
$$\Omega(S) = \int_{(\tilde{p}_-,\Delta a)\notin \mathcal{C}(S)} f(d\tilde{p}_-)g(d\Delta a)$$
$$\varphi(S) = \left(\int_{(\tilde{p}_-,\Delta a)\in \mathcal{C}(S)} \frac{\left(\tilde{p}_- e^{\Delta a}\right)^{1-\gamma}}{1-\Omega(S)} f(d\tilde{p}_-)g(d\Delta a)\right)^{\frac{1}{1-\gamma}} \tag{23}$$

where $g(\Delta a)$ is the distribution of quality shocks innovations and $f(\tilde{p}_-)$ is the distribution of relative prices previous period.

I define $\varphi(S)$ as the menu cost inflation. The menu cost inflation characterizes the distribution of relative prices conditional on no price change. Since the mean of relative prices is fixed, it also reflects the relative position of the price changes; thus, the menu cost inflation reflects the magnitude of the "selection effect" after an aggregate shocks–see Golosov and Lucas (2007). Several properties are worth mentioning: 1) in the Calvo model menu cost inflation is identical to 1; 2) rising inflation increases the mean of the menu cost inflation since a larger set of firms change their price because they hit the lower Ss band.

Inflation is a function of three elements: the reset price, the firm's inaction set, and the distribution of relative prices from the previous period. Inflation depends on the forward-looking variables like the reset price and the continuation set—they are forward looking since they solve the firm's problem—and a backward-looking variable given by the distribution of relative prices, which is backward looking because it depends on the history of firms' previous choices.

Aggregate State: Given that inflation and price dispersion are the aggregation of the relative prices, the distribution of relative prices is the state in the economy. I denote with S the state of the economy with the law of motion $\Gamma(S'|S)$. Therefore, in this simplified economy, the state of the economy is $S = (f(\tilde{p}_-), \eta^Q)$ with the law of motion $\Gamma(S'|S)$.

3.2 Solution Method

To solve this model numerically, I modified the Krusell-Smith algorithm and I developed projection methods to approximate firm's policy function. Next, I describe each development.

Modification of Krusell-Smith Algorithm: Given that the distribution of relative prices is part of the state, I use the Krusell-Smith algorithm to solve this problem. However, the standard method to implement Krusell-Smith does not work for this problem. The main reason is the following: whenever solving equilibrium conditions, Krusell-Smith algorithm replaces a model's equilibrium conditions with an approximation of the equilibrium policies obtained in the simulation. In this model—as in many others—this could generate indeterminacy at the time of solving the aggregate equilibrium equations. Next, I explain briefly the steps in Krusell-Smith and where it fails.

Then, I briefly describe the modification of the Krusell-Smith algorithm and its evaluation in this model. See online appendix section M for an analytical example where I can apply Krusell-Smith at hand and show the problem in an specific example.

The Krusell-Smith algorithm consists of projecting price and quantities to a small set of moments of the distribution of relative prices—one of the states of this economy—and the exogenous state. Let us define S_{SK} the set of finite moments of the distribution in Krusell-Smith together with the exogenous shock η^Q, $\Gamma(S'_{SK}|S_{SK})$ the law of motion of the state, and $\Pi(S_{SK})$, $\Omega(S_{SK})$ and $\Delta(S_{SK})$ the projection of inflation and price dispersion with respect to the state. Formally, the algorithm is given by

1. Given $\Pi(S_{KS}), \Delta(S_{KS})$, $\Omega(S_{KS})$ and $\Gamma(S'_{KS}|S_{KS})$ solve aggregate conditions (18) to (20)
2. With the solution of (18) to (20), solve the firm's value function (15).
3. Simulate and update $\Pi(S_{KS})$, $\Delta(S_{KS})$, $\Omega(S_{SK})$ and $\Gamma(S'_{KS}|S_{KS})$. Check convergence. If $\Pi(S_{KS}), \Delta(S_{KS})$ and $\Gamma(S'_{KS}|S_{KS})$ don't converge, go to step 1.

To my knowledge, all Krusell-Smith formulations have this approach. The next proposition shows how the standard method generates multiplicity of solution of equilibrium equations at the step of solving aggregate conditions.

Proposition 2 *For any $\Pi(S_{KS}), \Delta(S_{KS}), \Gamma(S'_{KS}|S_{KS})$ and $\lambda > 0$, if*

$$\{mu(S_{KS}), C(S_{KS}), L(S_{KS}), R(S_{KS}), w(S_{KS})\} \tag{24}$$

is a solution for (18) to (20) , then $\{\lambda mu(S_{KS}), \tilde{C}(S_{KS}), \tilde{L}(S_{KS}), R(S), \tilde{w}(S)\}$ is a solution, where $\tilde{C}(S_{KS})$, $\tilde{L}(S_{KS})$ and $\tilde{w}(S)$ solves

$$\kappa \tilde{L}(S_{KS})^{\chi} = \tilde{w}(S_{KS}) \quad ; \quad \tilde{C}(S_{KS}) = \frac{\tilde{L}(S_{KS}) - \Omega(S_{KS})\theta}{\Delta(S_{KS})} \quad ; \quad \lambda mu(S_{KS}) = \bar{u}\left(\tilde{C}(S_{KS}) - \frac{\tilde{L}^{1+\chi}}{1+\chi}\right)^{-1} \tag{25}$$

Proof. It is easy to see that $\{\lambda mu(S_{KS}), \tilde{C}(S_{KS}), \tilde{L}(S_{KS}), R(S), \tilde{w}(S)\}$ satisfies all the equilibrium conditions. ■

From proposition 2 we can extract three observations. First, since I didn't make any assumption about S_{KS}, the main result in proposition 2 does not depend on the selected moments in the Krusell-Smith approximation. Second, the result does not depend on using global or projection methods. Third and more importantly, it is not saying that the economy has multiplicity of equilibriums, but rather it is saying that the method for computing the equilibrium generates multiplicity at the moment of solving aggregate equilibrium conditions.

To understand the implication of previous propositions, we need to understand how models with price rigidities work. Assume a deviation from the equilibrium with an increase in consumption. This increase in consumption raises the real marginal cost, and due to the Phillips curve, it also raises inflation. If the Taylor principle is satisfied, this change in inflation impacts the real rate, feeding back to consumption again, undoing the original increase. It is the general equilibrium effect between households, firms and the Central Bank that generate uniqueness of equilibrium. The Krusell-Smith algorithm breaks this general equilibrium effect at the moment of solving equilibrium equations since the Phillips curve is replaced by the inflation policy obtained in the simulation.

The main problem until now is that in the aggregate equilibrium conditions, there is no information on the relationship between inflation and real marginal cost, i.e. the Phillips curve. The solution I propose is to apply Krusell-Smith to the frequency of price change and menu cost inflation, and solve *jointly* the aggregate and idiosyncratic equilibrium condition. Even if this method generates some numerical challenges—which I have solved—it provides the central bank with the cross-equation restriction of the intensive margin of the Phillips curve, breaking the multiplicity mentioned above in the step 1. Through numerical computation, it seems a reliable method since it provides an unique solution whenever solving equilibrium conditions.[6]

Before describing the solution, I need to find the state. In models with nominal rigidities, the important object for the repricing of the firm is the markup, the ratio between the relative price and real marginal cost. Therefore I use real marginal cost in the previous period as the state in Krusell-Smith. This variable is significant to predict menu cost inflation. Moreover, I use price dispersion in the previous period. This variable approximates the second moment of the relative prices distribution and predicts menu cost inflation and itself. Next I describe the algorithm used to solve the model.

1. Guess $\Delta(w_-, \Delta_-, \eta^Q), \Omega(w_-, \Delta_-, \eta^Q), \varphi(w_-, \Delta_-, \eta^Q)$ as a function of the state.

2. Solve for the equilibrium conditions: the joint system of (15), (18) to (20), and (23). Get the law of motion for inflation and real wage $(mu, C, w, \Pi)(w_-, \Delta_-, \eta^Q)$, and the continuation set and reset price $(\Psi(w_-, \Delta_-, \eta^Q), P^*(w_-, \Delta_-, \eta^Q))$.

3. Simulate a measure of firms and compute $\{\Omega, \varphi, S\}_t$.

4. Project Ω, φ on the state. Check convergence. If not, update and go to step 2.

Note that price dispersion is the only law of motion of the state obtained in the simulation. Since the business cycle fluctuations of price dispersion are small, the law of motion for all the relevant endogenous states comes from solving the aggregate and idiosyncratic equilibrium conditions in step 2—not from the simulation. Secondly, even without the ZLB, I need to solve the model globally given the kinked property of idiosyncratic policies. Third, this method breaks the separability in the equilibrium solution. With the previous implementation of Krusell-Smith, it was possible to solve the *aggregate conditions separate from idiosyncratic conditions*, dividing the system into two sub-systems. Now, the aggregate and idiosyncratic equilibrium conditions must be solved together.

[6] Aggregate kinks and the large dimensionality of the aggregate state eliminate methods like Reiter (2009), and implemented in menu cost models by Costain and Nakov (2011).

Validity of the Modified Krusell-Smith Algorithm: I verify the validity of the Krusell-Smith algorithm checking the equilibrium conditions with the simulated inflation, price dispersion and frequency of price change.[7] For each variable, I construct the solution of the equilibrium equations with the Krusell-Smith projected inflation, price dispersion and frequency of price change and also with the simulated version of these variables. Then, for all the variables, I compute the difference between each way of computing them to verify the degree of Krusell-Smith approximation. Appendix C describes the construction of the errors for each variable and section N in the online appendix plots the time series. Table IV shows the errors of Krusell-Smith for the levels of inflation target from 0 t 6. For example, with simulated inflation, the standard deviation of the error of nominal interest rate over the standard deviation of nominal rates is 0.026% at zero inflation target with no ZLB and increases to 0.645% in the model with ZLB. At 6% inflation target, these errors increased to 0.082% and 0.072%. Thus, there is a good fit in the Krusell-Smith algorithm.

Projection Method: Due to the curse of dimensionality, I use the Smoliak sparse-grid method as in Judd, Maliar, Maliar and Valero (2014) with anisotropic construction. I found that the Smoliak interpolation method generates a good approximation for the aggregate conditions, but not for the firm's value function. Figure VIII in the online appendix shows firms' value function using the Smoliak interpolation method. As we can see in the figure, there is a bad fit with respect to the value function in different points of the state. This is because smooth polynomials cannot describe the kinks in the value function and because the sparse grid method does not generate enough grid points in the continuation region. The continuation region is the critical set that gives information about the value function. To solve this problem, I extended the method in Judd *et al.* (2014). The main idea is to use splines interpolation in the firm's idiosyncratic state variables and the Smoliak interpolation method in the firm's aggregate state variables. Figure VII in the online appendix plots firms' value function. See online appendix section L for more details of this projection method. I define this method as the Completed Smoliak Interpolation method.

4 Calibration

I calibrate the model to qualitatively evaluate the optimal inflation target. I solve the model for two different calibrations: (1) the menu cost model with finite menu cost and idiosyncratic quality shocks; and (2) the Calvo model with infinite menu cost and no idiosyncratic quality shocks. Solving the Calvo model provides me with a benchmark to compare my results against the menu cost model. The main reason I didn't use idiosyncratic shocks in the Calvo model is that price dispersion is infinite at levels of inflation target higher than 5%. Table I shows the model parameters for the menu model.

[7] I did not use the R^2 as most papers do, mainly because if a variable has a small effect on the ergodic set of the equilibrium equation, the fit of the projection do not inform us of the fit of the model. Hence, there is no significant law of motion of the aggregate state estimated in the simulation.

The strategy to calibrate the model is to divide the parameters into three separate sets: (1) preference and technology; (2) menu cost and idiosyncratic shocks; and (3) Taylor rule and aggregate shocks. I externally calibrate the parameters for preferences and technology with micro-evidence of their empirical counterpart. I estimate the random menu cost and the idiosyncratic shocks processes with the GMM method to match the price change distribution in the data at 2% inflation target. In this step, I use the steady state of the menu cost model and I verify ex-post that the model with business cycle reproduces similar facts with respect to the price change distribution. I calibrate the Taylor rule and the aggregate exogenous shocks to match their empirical counterpart during the period of the great moderation in the US economy estimated with Calvo pricing model. This methodology is correct since these two models generate similar business cycle dynamics without ZLB. I find it impossible to estimate all the parameters in the menu cost model since it takes an entire day to solve the model with business cycle for a given set of parameters.

Table I – Parameters Value and Targets

Parameter	Value	Target
Preferences and technology		
β : discount factor	$0.965^{1/12}(1+g)^{\sigma_{np}}$	Interest Rate 3.5%
σ_{ez} : encodes risk aversion	-100	Literature
σ_{np} : encodes inter-temporal elas.	1.5	Greenwood, Hercowitz and Huffman (1988)
χ : labor supply elasticity	0.5	Greenwood *et al.* (1988)
γ : demand elasticity	3	Micro-estimate (3-4)
α : share of intermediate inputs	0.5	intermediate inputs/total output of 0.7
g : growth rate	0.0017	2 % GDP growth rate
τ_{MC} : cost subsidy	0.26	Aggregate markup 10%
Random menu cost and quality shock processes		
(σ_a^1, σ_a^2) : idiosyncratic shock innovations	$(0.2075, 0.0145)$	GMM—price statistics at 2% inflation target
p : prob. of large idiosyncratic shock	0.0699	GMM—price statistics at 2% inflation target
hz : prob. of free price adjustment	0.0479	GMM—price statistics at 2% inflation target
θ : menu cost	0.305	GMM—price statistics at 2% inflation target
Taylor rule and aggregate shocks		
$(\phi_r, \phi_\pi, \phi_{mc}, \phi_{dmc})$: Taylor rule	$(0.86, 1.80, 0.64, 0.00)$	Del Negro, Schorfheide, Smets and Wouters (2007)
$(\rho_Z, \sigma_Z 100)$: productivity shocks	$(0.98, 0.055)$	Cociuba, Prescott and Ueberfeldt (2009)
$(\rho_G, \sigma_G 100)$: gov. expenditure shocks	$(0.95, 0.22)$	Del Negro *et al.* (2007)
$(\rho_R, \sigma_R 100)$: taylor rule shocks	$(0, 0.125)$	Del Negro *et al.* (2007)
$(\rho_Q, \sigma_Q 100)$: risk premium shocks	$(0.97, 0.055)$	International ZLB frequency and Coibion *et al.* (2012)

Preferences and Technology: A period in the model is a month. Thus, I choose $\beta = 0.965^{1/12}(1+g)^{\sigma_{np}}$ because it implies a risk-free annual interest rate of 3.5%. I calibrate $g = 0.0017$ to match the annual US growth rate of 2%. The GHH preferences parameters are set to $\sigma_{np} = 1.5$ and $\chi = 0.5$ as in Greenwood *et al.* (1988), which uses micro-evidence to calibrate these parameters. I set η_g to match the ratio of government expenditure over output equal to 0.25, the historic US average.

For the production function, I choose an elasticity between inputs γ equal to 3, a middle value in micro–estimates.[8] For the production technology, I set the elasticity with respect to materials equal to 0.5 to match

[8] See Barsky, Bergen, Dutta and Levy (2003), Nevo (2001) and Chevalier, Kashyap and Rossi (2000) for micro–estimates between 1

intermediate inputs over total output in the US economy of 50%. Finally, I calibrate τ_{mc} to match an aggregate markup of 10%.

I set the Epstein-Zin parameter equal to -100 to match previous calibrations of Epstein-Zin preferences in the literature.[9] Importantly, in my paper it is most critical not to overestimate the cost of business cycle. From Alvarez and Jermann (2004) we know that there is a one-to-one mapping between the cost of business cycle and the difference between the rates of return of two assets: 1) an asset that pays aggregate consumption with business cycle fluctuation; and 2) an asset that pays the trend consumption. This Epstein-Zin parameter setting ensures that I will not generate a counter-factual difference of rate of return in the previous two assets. Additionally, the equity premium and the bond premium are 1.29% and 0.4% in my model, also much lower than in the data. See online appendix section H for more details on these computations and on asset pricing implications of the model.

Whenever I compute the optimal inflation target in the Calvo model, I use indexation of prices with respect to business cycle fluctuation of inflation whenever inflation target is more than 3%. I set indexation between 0 and 50% for the Calvo model to satisfy stability in the global solution. Importantly, $\lambda = 0$ for the optimal inflation target.

Random Menu Cost and Quality Shock Stochastic Processes: I estimate the random menu cost and quality shock stochastic processes to match moments in the price change distribution together with the physical cost of menu cost.[10] There are 5 parameters to estimate: the menu cost (θ), the probability of zero menu cost (hz), and parameters for the stochastic process of the quality shocks given by ($\{\sigma_{ai}\}_{i=1,2}\,, p$). I choose these parameters to minimize the distance between a number of moments in the steady state of the menu cost model and in the data. Table II describes the moment in the data and in the model in the steady state with the estimated parameters. I choose the parameters so as to minimize the distance

$$\sum_{i=1}^{31} \text{weight}_i \left(\frac{\text{moment}_i^{data} - \text{moment}_i^{model}}{1 + abs(\text{moment}_i^{data})} \right)^2 \tag{26}$$

between a set of 31 moments in the model and in the data. For the moment in the data I choose the physical cost of the menu cost of 0.6% computed in Zbaracki, Ritson, Levy, Dutta and Bergen (2004) and Levy, Bergen, Dutta and Venable (1997). The physical cost of the price changes in the model is given by the model of $\theta\frac{\Omega_t - hz}{\eta_{tg} + C_t}$. The rest of the moments are with respect to the micro-price statistics computed with the UK's CPI price quotes . Next I describe this data and the main steps used to compute the moment in (26).

I use monthly price quotes recollected for the consumer price index (CPI) micro dataset of the United Kingdom's Office for National Statistics (ONS). This dataset offers several advantages. First, it is representative of the whole economy since it reflects all the prices in the consumer consumption basket. Second, it is publicly available from

and 5 and Burstein and Hellwig (2007) estimate an elasticity of substitution between 1.55 to 4.64.

[9] Rudebusch and Swanson (2012) use -73. Similar numbers were found in van Binsbergen, Fernandez-Villaverde, Koijen and Rubio-Ramirez (2008) and Caldara, Fernandez-Villaverde, Rubio-Ramirez and Yao (2012). Pariès and Loublier (2010) found a much larger value.

[10] The estimated parameters are robust if I exclude the physical cost of menu cost; see online appendix VII.

the time period 1996M1-2016M3. Third, it has similar micro-price facts as other low-inflation countries like the United States, Canada, and the European Union countries.[11]

The ONS office surveys prices for goods and services in the United Kingdom. In total, there are 31 million price quotes in the time period between 1996 and 2016 at monthly frequency. There is a large advantage to using the UK's CPI price quotes: since the ONS office constructs aggregate inflation by departing from product price index, they generate a large sampling across stores and regions for the same product. To give a concrete example, instead of simply constructing a single index for a large number of breakfast cereals, they sample prices of two particular brands of cereal across stores and regions in UK—cereal box 1 and 2. Thus, it is possible to download from ONS the product level CPIs to construct the aggregate CPI.

Since the price change distribution is critical to my calibration, and therefore to the cost of inflation, I apply several filters to render the data compatible with the model in the computation of price statistics. I apply standard filters used in previous studies like Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008): I complete price quotes for temporary missing observations (less than a year) and out-of-season with the last available price. I redefine an item as a new product whenever there are more than 12 consecutive missing observations. Moreover, I drop price changes with non-comparable product substitutions, outliers and months with changes in the VAT tax rate.

My model abstracts from sales and multi-product heterogeneity; hence, I computed the micro-price facts filtering sales and product heterogeneity from the data. See section B in the appendix for details in each filter and table VI in the online appendix where I report the data with each different filter.

- **Filter for sales:** The ONS data has a flag denoting sales for each price quote. I found that this flag does not characterize all temporal price decreases/increases where the price change and then comes back to the same level. Therefore, I drop all price changes with the sale flag and then I apply an additional filter that identifies sales: price changes where the price of the item before and after price changes is equal. Around 10% of the total price changes are identified with sales flags and another 10% of the total price changes are dropped with my filter.

- **Filter for product heterogeneity:** There is a large heterogeneity across different products. In order to control for heterogeneity, I generate a normalized price change distribution denoted by $g(\Delta\tilde{p})$ from the original price change distribution $f(\Delta p)$. If Δp_t^{ij} is the price change of product-location-shop i at time t of the product j, then I generate the standardize price change $\Delta\tilde{p}_t^{ij}$ as

$$\Delta\tilde{p}_t^{ij} = \frac{\Delta p_t^{ij} - \mathbb{E}[\Delta p_t^{ij}|i \in \text{product j}]}{\mathbb{S}td[\Delta p_t^{ij}|i \in \text{product j}]}\mathbb{S}td[\Delta p_t^{ij}] + \mathbb{E}[\Delta p_t^{ij}] \tag{27}$$

[11] See Alvarez, Dhyne, Hoeberichts, Kwapil, Bihan, Lünnemann, Martins, Sabbatini, Stahl, Vermeulen *et al.* (2006) and Kryvtsov and Vincent (2014).

where $\mathbb{E}[\Delta p_t^{ij}|i \in \text{product j}], \mathbb{S}td[\Delta p_t^{ij}|i \in \text{product j}]$ denotes the product-level mean and standard deviation of price changes and $\mathbb{S}td[\Delta p_t^{i}], \mathbb{E}[\Delta p_t^{i}]$ denotes the aggregate mean and standard deviation of price changes. In the online appendix section E, I show that this aggregation cleans heterogeneity in the Calvo, menu cost and Taylor models. Moreover, without this aggregation kurtosis is more the outcome of heterogeneity than of the pricing model. For example, the kurtosis of price changes without this transformation is 7.7, while with this transformation it is 3.9.

Each filter affects micro-price moments, and therefore the estimated parameters and the cost of inflation. Sales increases the frequency of price and the mean absolute price change, with the direct effect in the magnitude of idiosyncratic shocks, reducing the cost of inflation. As we can see in table VII in section D in the online appendix, idiosyncratic shocks without the filter of sales are twice as large as without this filter—columns 1 and 3. The heterogeneity filter affects the estimated ratio between the frequency of free price adjustment over total price changes: without this filter this ratio is 70% and with this filter this ratio is 47%. Intuitively, with this filter the model is more "menu cost" than "Calvo". Without the physical cost of menu cost in the GMM, a kurtosis of 7.7 implies 100% of free price adjustment.

Taylor rule and aggregate shocks: I use Del Negro *et al.* (2007) estimates at monthly frequency to calibrate the rest of the parameters since they restrict their estimate to the great moderation, the target in my model. Since Del Negro *et al.* (2007) ignore price dispersion from the output-gap definition in the Taylor rule, I use real marginal cost in the Taylor rule instead of output-gap to be consistent with the previous estimates—I re-scale the parameter of the Taylor rule with respect to the real marginal cost to be consistent with them. The only parameters I don't use from Del Negro *et al.* (2007) are those from the aggregate productivity process. I reproduce business cycle statistics of the linear trended output per hour computed in Cociuba *et al.* (2009) for the aggregate productivity.

It is difficult to find an empirical counter-part to the risk premium shocks; therefore, I partially follow the calibration strategy of Coibion *et al.* (2012). I calibrate the persistence of the risk premium shock to match the routinely high persistence of the risk premium in the financial series they use; thus, I target a quarterly persistence of the risk premium shock to 0.94—I use $\rho_q = 0.97$.

The innovations of the risk premium shocks are chosen to match the international evidence of the frequency of hitting the ZLB across countries. To calibrate the probability of hitting the ZLB, I construct an international database for CPI inflation and policy rates of the majority of countries around the world from the International Financial Statistics of the IMF and FRED. Appendix A describes the calculation in detail and figure VII describes mean inflation and mean probability of hitting the ZLB for each country and the average across countries.

The first criterion I use to compute the frequency of ZLB is to have a stable inflation target—as in my model. For example, a large body of economic research argues for a shift in trend inflation in the US during the period 1970-1980—see Coibion and Gorodnichenko (2011) and Cogley and Sbordone (2008). Thus, I discard time periods before 1988Q1, since before that date central banks did not incorporate implicit or explicit inflation targeting. The second criterion is to focus on moderate inflation, i.e. mean levels of inflation between 0 and 4%. Therefore, I drop all countries with average inflation more than 4% such as Mexico, Turky and Peru. In the sample, there are countries that hit the ZLB around 30% and 66% of the time, such as Switzerland and Japan, but also countries such as South Korea, New Zealand and Australia that hadn't hit the ZLB. I compute the average inflation across countries and the probability of hitting the ZLB. The mean inflation is 2.3% and the probability of hitting the ZLB is 14%. I calibrate the innovations of the risk premium shock to match this probability.

5 The Model At Work: Positive Implications of the Model

This section provides compelling evidence of the positive dimensions of the model before analyzing the optimal inflation target. On the micro-side, the model is able to generate the price change distribution observed in low inflation countries, as well as the relationship between the micro-price statistics and different levels of mean inflation. On the macro-side, the model has similar business cycle dynamics to Calvo—the main model used by central banks—at low inflation levels with no ZLB constraint. Moreover, the model without ZLB fits the US business cycle moments during the great moderation, under-predicting the volatility of inflation. Finally, at low inflation targets, the menu cost model generates larger deflationary spirals since there is a large selection effect at the moment the economy hits the ZLB.

5.1 Micro-Pricing Behavior

The price change distribution in low inflation economies has several characteristics that are robust across countries.[12] First, prices respond more to idiosyncratic reasons than aggregate. For example, almost one-half of the price changes are price decreases, and conditional on a price change, the average price change is around 10%. Since aggregate inflation is positive, low and with low volatility in low inflation economies, only idiosyncratic shocks can explain these facts. Second, there are a large amount of small and large price changes in the price change distribution that cannot be accounted for by an off-the-shelf menu cost model. For example, the 10th and 90th percentile of the absolute value of the price changes are around 1-3% and 20-25% across different studies.

These common facts across countries hold in the UK economy as table II describes. The mean price change in absolute value is 12% and the median is 1%. Therefore, prices respond more to idiosyncratic reasons than aggregate in UK, as they do in the rest the rest of the countries—for example, in US Midrigan (2011) describes exactly the same numbers. Additionally, the kurtosis is 3.9; and the 10th and 90th percentiles are 1% and 23%.

My menu cost model with idiosyncratic shocks can account for the price change distribution in UK—and therefore in low inflation economies. Several features of the my model allow to match the data. The random menu cost generates small price changes—almost 50% of price changes are given with zero menu cost. The mixed normal

[12]See Klenow and Malin (2010) for a review of the studies.

Table II – Price Change Distribution Data-Model at 2 Percent Inflation Target

moments	Data		Model	
	Raw	With filters	Steady State	Business Cycle
Absolute Value of Price Change				
Mean	0.105	0.122	0.129	0.131
Standar deviation	0.125	0.111	0.120	0.123
Price Change				
Mean	0.027	0.011	0.016	0.016
Standar deviation	0.161	0.165	0.176	0.179
Skewness	0.228	-0.007	-0.103	-0.121
Kurtosis	7.577	3.941	3.830	4.034
5th percentile	-0.228	-0.270	-0.294	-0.300
10th percentile	-0.136	-0.190	-0.212	-0.216
25th percentile	-0.025	-0.081	-0.075	-0.072
50th percentile	0.024	0.014	0.014	0.015
75th percentile	0.077	0.098	0.112	0.114
90th percentile	0.184	0.211	0.234	0.233
95th percentile	0.288	0.295	0.316	0.316
Frequency of price change	0.119	0.102	0.100	0.101
Cost of price adjustment*100	0.600	0.600	0.604	0.612
Ratio free/costly price adjustment	-100.000	-100.000	0.478	0.475

Selected moments of micro-price statistics in the GMM. Column 1 describes the price statistics with standard filters as Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008). Column 2 describes the price statistics computed with the filters explained in section 4. Columns 3 and 4 describe the price statistics in the model with and without business cycles.

distribution generates large price changes with relatively small Ss bands; without it, the model cannot generate large price changes. Finally, as I remark in section 4, since the price change distribution depends mainly on the idiosyncratic shocks, the model with business cycle dynamics generates the same price change distribution as the steady state economy—see columns 3 and 4 of table II.

Result 1 (Micro-price statistics at 2% Inflation Target I) *My menu cost model generates the price change distribution as in the UK economy.*

The Calvo model—or any pricing model—without idiosyncratic shocks cannot replicate the price change distribution. This property is the direct consequence of the fact that aggregate shocks are relatively small in comparison to idiosyncratic shocks, and therefore this model cannot reproduce the observed price distribution in the data.

Low inflation price statistics have the capacity to identify the ratio between the frequency of free adjustment and total adjustment alone. In table VII in the online appendix, I repeat the estimation with zero weight in the physical cost of menu cost. The estimated parameters are similar to the estimation with physical cost of price adjustment; the biggest difference is the frequency of zero menu cost that increases from 0.048 to 0.052. Since kurtosis of price changes in the data is 3.9 and the Calvo model has kurtosis of at least 6, the ratio between free price adjustment and total price adjustment should be less than one.

A question that arises is the following: is this calibration consistent with micro-pricing behavior at higher inflation environment? Next, I show that this is the case using international evidence and product level evidence.

International Evidence of Pricing Behavior at High Inflation: There are two empirical facts of pricing behavior at different levels of inflation: 1) the frequency of price changes increases with the level of inflation; 2) the intensive component of inflation volatility decreases with the level of inflation. To show the first fact, I construct a panel-data with frequency of price change and inflation across different countries—see table X in the online appendix. Figure XX in the online appendix plots a linear fit between the frequency and the inflation in the data and in the model without business cycle—I find that for levels of inflation more than 2% the frequency of price change with and without business cycle are almost identical. The model matches quantitatively the average sensitivity of the frequency of price changes with respect to inflation; even more, it underestimates this relation. Secondly, the model can match the decreasing intensive component in the business cycle fluctuations of inflation as Klenow and Kryvtsov (2008), Gagnon (2009) and Wulfsberg (2010) have shown in the data across different countries. See appendix P in the online appendix for more details.

Product-Level Evidence of Pricing Behavior at High Inflation: In the previous experiment I compare business cycle statistics with steady state statistics in the model. To have a better comparison data-model, I use product level price statistics. I compute average inflation and frequency with different products in UK's CPI. As I mention above, I can do this comparison due to the large sampling of the same product across regions and shops in the UK. As we can see in figure XXII-panel A, products with higher inflation have large frequency of price change and the model does a good job of capturing quantitatively this relation between mean inflation and mean frequency.

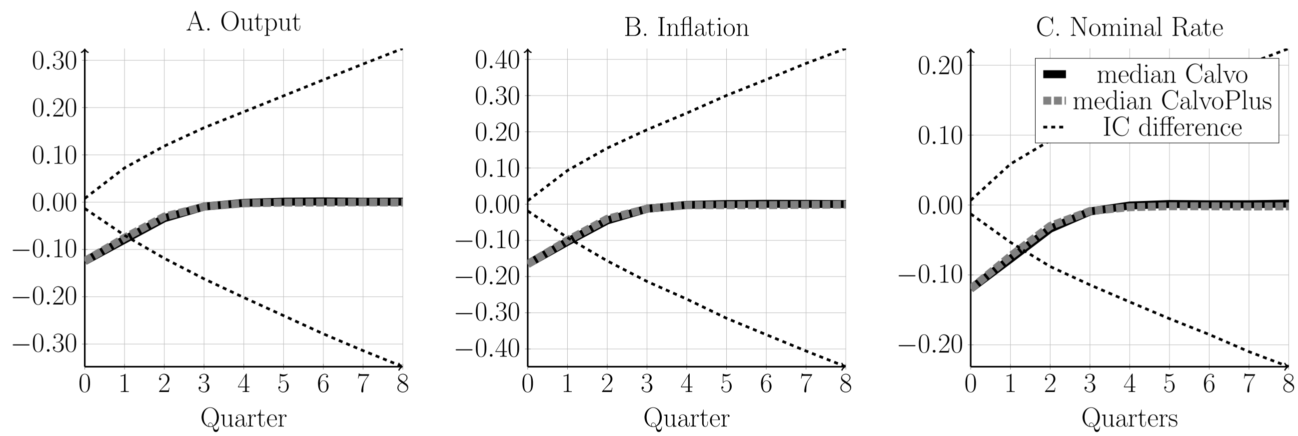
On top of a positive relation between the frequency of price changes and inflation, there is a positive relation between inflation and standard deviation of price changes and an almost zero relation between the skewness of price change and inflation across-product in UK. This is important, since if inflation generates a large price dispersion, then we would see a large asymmetry in the price change distribution captured in the skewness of price changes—the standard deviation of prices changes can capture this force if idiosyncratic shocks are constant across products, a strong assumption. For example, the Calvo model generates a large positive relation between mean inflation and skewness of price changes. Intuitively, the tails in the stopping time distribution translate to the tail of positive price changes. Instead, in the menu cost model skewness of price change could be decreasing or increasing with respect to inflation depending on the stochastic process of the idiosyncratic shocks, but there will definitely be a small sensitivity of skewness to inflation due to the Ss policy function. See online appendix P for more details.

Result 2 (Micro-price statistics high inflation target) *My menu cost model in the steady state generates an increasing frequency of price with similar magnitude as in the micro-data and international evidence. There is no evidence of a strong positive relation between skewness of price changes and mean inflation at product level.*

5.2 Equivalence Aggregate Dynamics at 2% Inflation Target Without Zero Lower Bound Constraint Between Calvo and Menu Cost Model

This section shows that the Calvo and my menu cost models have similar aggregate dynamics at 2% inflation without ZLB constraint on nominal interest rate. To formalize this argument, I show that an econometrician with only aggregate data could not distinguish which model generates aggregate time series in a finite sample with the same length of the great moderation. Moreover, both models have a good fit with the US macroeconomic time series during the great moderation—1984Q1-2006Q4—for a medium scale DSGE model.

Figure I – Impulse-Response to a Monetary Shock



Panel A to C describe the impulse response functions of the output, inflation and nominal interest rate to a monetary shock at 2% inflation target in the models without zero lower bound constraint to the nominal interest rate. The black solid and the gray dotted lines describe the median impulse-response functions of the Calvo and the CalvoPlus model respectively. The black dashed lines describe the 5 and 95 percentiles of the difference in the impulse-response functions between the two models over 5000 simulations of 22 years.

In order to show the claim of similar aggregate dynamics, I compute two sets of statistics: impulse-response functions with respect to the structural shocks, and business cycle statistics like standard deviation, persistence and correlations. Next I describe the impulse-response and the business cycle statistics.

Impulse-Response Function to Structural Shocks: I computed each impulse-response as an econometrician would; and I ask whenever these two models are different in a finite sampling of 22 years—the length of the great moderation. From random initial conditions, I simulate each model for the time period of 22 years and estimate a VAR with its respective impulse-responses; I repeat this procedure 5000 times. In both models, the impulse-responses are random variables with their respective confidence intervals that allow me to make statistical claims as an econometrician. See the online appendix section I for more details on the computation of the impulse-response.

Figure I plots the median impulse-response function with respect to the Taylor rule shock for the Calvo and the menu cost model, together with the [5, 95] interval confidence of the difference of the impulse-responses in the Calvo and the menu cost model. As this figures shows, the difference of the impulse-responses $[5, 95]$ interval confidence always covers zero for all the variables. The only difference is with respect to inflation: inflation has a larger impact effect in the menu cost model than in the Calvo model, but this effect is significant for only one quarter. This significant effect on the impact comes directly from the business cycle fluctuations of the menu cost inflation and not the reset price—see online appendix. The same behavior holds for productivity, government expenditure and risk premium shocks—see figures II, III and IV in the online appendix.

The fact that equilibrium dynamics are similar in both models does not imply that the Phillips curve in both models is similar, since a steeper Phillips curve is partially offset by movement in the nominal interest rate. To explain this argument, assume a structural shock that increases the output-gap. Since nominal interest rate depends on inflation, in the menu cost model nominal interest will increase by more with respect to the structural shocks; consequently, the equilibrium output-gap responds less and therefore so does inflation. The impulse response with respect to the risk premium shows this mechanism—see figure IV in online appendix. This result breaks with the ZLB on nominal interest rate.

Result 3 (Macro-dynamics in Menu Cost and Calvo model) *The difference of impulse-response functions of the Calvo and the menu cost models covers zero with a* $[5, 95]$ *interval confident of the menu cost model for US productivity, government expenditure, monetary shocks and risk premium shocks for all the variables minus inflation. Inflation is significant different only in the quarter after a structural shocks due to the menu cost inflation.*

Business Cycle Statistics: Table IX in the online appendix shows the business cycle statistics in the Calvo and the menu cost models.[13] Additionally, I include US business cycle statistics from the great moderation as an empirical guide.[14]

The menu cost model generates business cycle statistics close—or even equal—to the Calvo model. As we can see in the last four rows, the [5,95] percentiles of the difference of the standard deviation, persistence and correlation in both model always includes the zero. The main difference is that the menu cost model generates larger insignificant volatility of inflation and insignificant less persistence in inflation. Additionally, real marginal cost is less volatile in the menu cost model due to the different Phillips curve, but again this difference is insignificant.

Result 4 (Macro-dynamics in Menu Cost and Calvo model) *The difference of standard deviation, persistence, and correlations in the Calvo and menu the cost models covers zero with a* $[5, 95]$ *interval confident.*

How close are business cycle statistics in the model to those in the data? To compare the model with the data, I computed business cycle statistics in the US economy during the time period between the first quarter of 1984 and the last quarter of 2006 for the reasons described in section 4. As we can see, the model is able to reproduce all business cycle statistics in the data except for the volatility of labor and wages. Notice that with the assumptions

[13] See section I in the online appendix for a description of the computation of this table.

[14] It is difficult to have an empirical counterpart for this exercise, since in the data there is a ZLB on nominal interest rates. Nevertheless, I include the data to have a guidance of the magnitude of US business cycle.

over preferences, technology and the pricing model, the model does not over-predict the volatility of inflation; even more, it generates a lower volatility of inflation. This is important, since it will generate a quantitative significant deflationary spiral during ZLB periods.

The equivalence result does not hold with either of these two features: a ZLB constraint over nominal rate or a large positive inflation target. For a detailed explanation of how the ZLB breaks this equivalence between aggregate dynamics between models, see the next section. Additionally, higher inflation levels change the Phillips curve in both models, and therefore the aggregate dynamics.

5.3 Business Cycle Analysis with Zero Lower Bound

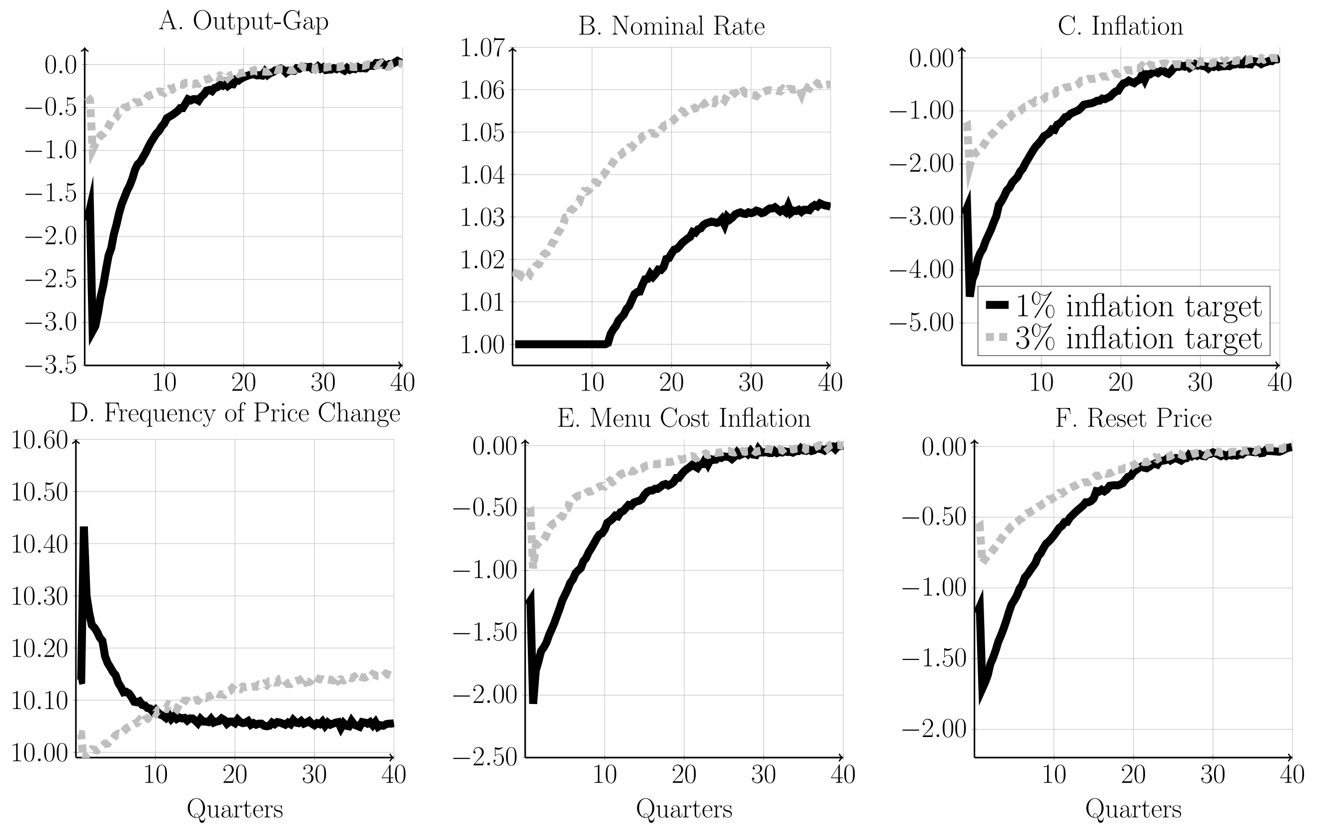
This section focuses on understanding the macroeconomic effect of the ZLB over aggregate dynamics; more specifically, it studies the interaction between state dependent pricing and a liquidity traps. To do the task at hand, I will first compute conditional impulse-response of my menu cost model at different levels of inflation target, and then I will analyze business cycle statistics in the Calvo and menu cost model. See online appendix section O for details on the computation on the impulse-response and business cycle statistics and more information on the business cycle dynamics.

What are the macroeconomic dynamics of my model during a period of a binding ZLB? When the ZLB is binding, the real interest rate is too high. This leads to excessive saving, and since nominal interest rate cannot decrease due to the ZLB, this exacerbates the depression of spending and output, which in turn creates more deflationary pressures. At low inflation targets, a depressed inflation implies deflation, thus firms' relative prices hit the upper Ss bands, and firms choose to reduced their price optimally. Hence, in the aggregate, a small measure of firms will make large changes to price, with respective impacts on inflation, real rate and output-gap; these changes then feedback to new price changes. This "domino" effect is so large that I introduce a value add-tax to mimic monetary policy at inflation targets less than 1% when monetary policy is binding[15], reducing the cost of the ZLB—see online appendix section G. At high inflation targets, a depressed inflation implies low inflation, thus firms' relative prices do not hit the upper Ss bands, and firms only change their price if they are hit by large idiosyncratic shocks or zero menu cost. Next I show this result with impulse-response functions and business cycle statistics.

Analysis of a Liquidity Trap With a Conditional Impulse-Response: Positive inflation targets and a ZLB constraint generate non-linearities in the aggregate dynamics. In non-linear models, the impulse-response function depends on the initial state—in the case of the menu cost model, the distribution of relative prices. Since in my model an economy hits the ZLB after a history of shocks that depress the output-gap and inflation, I compute the impulse-response of a risk premium shock conditional on low interest rates—interest rates lower than the 25th

[15]See Fernández-Villaverde *et al.* (2015) for another paper to introduce this tax. I found that without this tax the deflationary spiral does not converge.

Figure II – Conditional Impulse Response for Risk Premium Shock at 1% and 3% Percent Inflation Target



Panels A to F describe the non-linear median impulse response in the menu cost model at 1% and 3% inflation target. Panel A, C, D, E and F are in percentage deviation from the terminal value. Interest rate and inflation are in an annual base. The size of the innovation of the risk premium shock is $2\sigma_Q$. I use steady state elasticity between marginal cost and output gap to compute the output-gap from the marginal cost.

percentile of the ergodic distribution. The intuition of this criteria is that a low interest rate reflects a history of low inflation and output-gap.

Figure II shows the impulse-response after a risk premium shock to interest rate, output-gap, and the inflation components. All variables are expressed as log-deviation of the ergodic mean, except for the interest rate. The first property to notice is that for low interest rates, output-gap and inflation are depressed initially for both levels of inflation target. Additionally, the drop in inflation and output-gap are higher at low inflation targets after the exogenous shock with almost twice as much elasticity as at three percent inflation. Next, I will explain why.

At one percent inflation there is a *persistent* increase in the fraction of repricing firms. The key feature in menu cost models is that the increase in the fraction of repricing firms is not random across firms. This new mass of repricing firms hits the upper Ss band with a large downward price adjustment—around 10%. The new repricing firms decrease inflation and since the ZLB is binding, this initial drop in inflation increases real interest rates, even further depressing the output-gap and inflation.

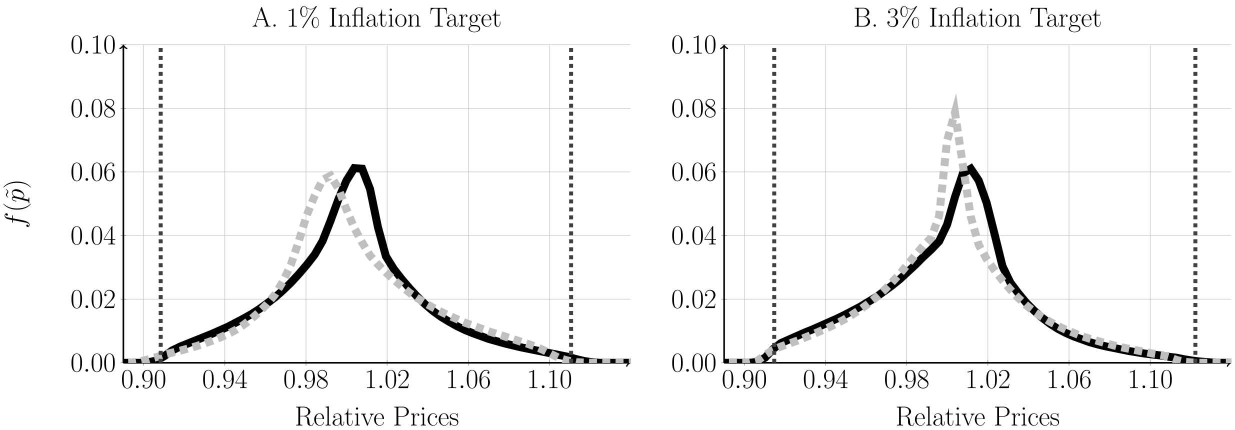
The variable that reflects the distribution of price changes is the menu cost inflation, the distribution of relative price conditional of no price changes. As we can see, menu cost inflation drops by 1% and total inflation drops by 1.5% percent. Thus, *more than half of the inflation drop is explained not by the reset price or the number of price changes, but by the distribution of price changes*. This is selection effect—see Golosov and Lucas (2007)—where a small increase in the frequency of price change (in this case less than 1%) can generate a large change in inflation, since the average price change is large. In my model, it is almost half of the deflation.

Why is the set of repricing firms so sensitive to the aggregate shock? To answer this question, we need to understand that a necessary condition for a low interest rate is a past history of low inflation, and in the case of one percent inflation target, deflation. Figure III describes the distribution of relative prices conditional of zero or positives rates. We can see that at 1 inflation target with positive rates, the distribution of relative price is symmetric around 1. This characteristic changes at the ZLB; the mode of the distribution is lower than 1, since the reset price responds to the negative macro-economic conditions, but even with a lower reset price, the distribution of relative price is tilted toward high relative price. Thus, there is a relatively high set of firms near the upper Ss bands, generating a large elasticity of the menu cost inflation to negative aggregate shocks.

Result 5 (ZLB Dynamics at 1 Percent Inflation) *When the economy hits the ZLB in the menu cost model at 1% inflation, there is a persistent increase in the frequency of price change for those prices selected to be large and negative. This increases the deflationary spiral, worsening the macroeconomic effect of shocks that depress output-gap. Moreover, inflation is more sensitive to aggregate shocks that depress output-gap, since the distribution of relative price is more tilted to the upper Ss bands when the economy is at the ZLB.*

At higher inflation target the menu cost model yields a much weaker deflationary spiral. First, as we can see in figure II, there is a decrease in the frequency of price change. The intuition for this result is clear: with positive

Figure III – Distribution of Relative Prices at 1% and 3% Inflation Target



Panel A to B describe the mean distribution of relative prices conditional of being at the ZLB or not, together with the average Ss bands at 1% and 3% inflation targets. The solid black line describes the mean distribution of rel. prices conditional on no binding ZLB and the dashed grey line describes the distribution of rel. prices conditional on a binding ZLB. Panel A describes the distributions at 1% inflation target and panel B describes the distribution at 3% inflation target. All the conditional ergodic means are given by $\frac{\sum_t f_t(\tilde{p}) I(X_t)}{\sum_t I(X_t)}$ where $X_t = \{R_t > 1, R_t = 1\}$.

inflation on average, when the economy hits the ZLB, there is a low inflation, *but not deflation*. Hence, the price changes are driven mostly by zero menu cost or by a sufficiently large sequence of idiosyncratic shocks.

In relative terms, at 1% inflation when the economy is at the ZLB, the distribution of relative prices is tilted toward the upper Ss bands. This effect is reversed at 3% inflation target; the distribution of price changes is symmetric around a relative price equal to one. We can see this property in the level of the menu costs inflation: since the mean annualized level is 1.12%, a drop of -1% implies a positive menu cost inflation; thus, there are firms still hitting the lower Ss band at the ZLB.

Result 6 (ZLB Dynamics at Three Percent Inflation) *When the economy hits the ZLB at 3 percent inflation in the menu cost model, there is no persistent change in the frequency of price change. Only firms with a zero menu cost or large idiosyncratic shocks change their prices. The distribution of relative prices is not tilted towards the upper Ss bands when the economy is at the ZLB.*

Since, at 3% inflation target, inflation dynamics at the ZLB are driven mainly by the zero menu cost or by large idiosyncratic shocks, this characteristic implies that ZLB dynamics should be similar to the aggregate dynamics in the Calvo model. In the online appendix, I compare the impulse-response in the menu cost and the Calvo model and I verify this conclusion.

In conclusion, a higher inflation target decreases the business cycle volatility of the output-gap in the menu cost for two reasons. First, and trivially, it decreases the probability of hitting the ZLB since the shocks that trigger the ZLB bound are larger. Second, in the menu cost model the inflation target determines the extent to which the aggregate price level reacts to aggregate shocks in periods when the ZLB is binding. At low inflation targets, the deflationary spiral is large since there is an increase in the selection effect at the ZLB. This effect decreases with higher inflation targets, since the distribution of price changes is driven mainly by the zero menu cost or large idiosyncratic shocks at the ZLB.

Table III – Business Cycle Moments With ZLB in Calvo and Menu cost Models

moment	Menu Cost 1 IT	Menu Cost 3 IT	Calvo 1 IT	Calvo 3 IT
Frequency and duration of ZLB				
Frequency	0.39	0.06	0.24	0.07
Mean duration, quarterly	8.10	2.27	5.13	3.02
Median duration, quarterly	4	2	3	2
95 percentile duration, quarterly	28	6	19	10
Output-Gap and Inflation Components				
Output-gap				
Mean deviation conditional on binding ZLB	-1.18	-0.81	-1.45	-1.18
Mean deviation conditional on no binding ZLB	0.77	0.05	0.46	0.08
Std conditional on binding ZLB	1.47	0.97	1.78	1.19
Std conditional on no binding ZLB	0.941	0.908	1.069	1.079
Inflation				
Mean deviation conditional on binding ZLB	-0.69	-0.51	-0.37	-0.43
Mean deviation conditional on no binding ZLB	0.45	0.03	0.12	0.03
Std conditional on binding ZLB	0.75	0.23	0.26	0.19
Std conditional on no binding ZLB	0.40	0.38	0.24	0.25
Reset price				
Mean deviation conditional on binding ZLB	-1.06	-0.83	-1.04	-1.26
Mean deviation conditional on no binding ZLB	0.68	0.05	0.33	0.09
Std conditional on binding ZLB	0.98	0.36	0.73	0.55
Std conditional on no binding ZLB	0.66	0.59	0.68	0.76
Menu cost inflation				
Mean deviation conditional on binding ZLB	-0.30	-0.22		
Mean deviation conditional on no binding ZLB	0.20	0.01		
Std conditional on binding ZLB	0.33	0.11		
Std conditional on no binding ZLB	0.57	0.17		
Frequency of price change				
Mean conditional on binding ZLB	10.43	10.04		
Mean conditional on no binding ZLB	10.06	10.22		

Mean deviations describe the log-deviation of the variable with respect to the mean condional of zero or positive rates. Conditional std describes the standard deviation of the variables in logs conditional of zero or positive rates.

Business Cycle Statistics with ZLB in the Menu Cost Model: Another way to analyze the positive implication of a higher inflation target is to analyze business cycle statistics. Table III describes business cycle statistics in the model with ZLB in the menu cost model at 1% and 3% inflation target. Additionally, to have a benchmark of comparison of the quantitative properties in the menu cost model, I also include the same statistics in the Calvo model.

The first row describes the frequency of the ZLB. In both models the frequency of hitting the ZLB decreases significantly from 1 to 3% inflation. As we can see in table III, the model has fat tails in the length of the ZLB at 1% inflation target, especially the menu cost model. For example, the median length is less than one year—with a mean of one and a half years—and the 95th percentile is 6 years. Since the structural shocks are normal, the fat tails are endogenous in the model. At higher levels of inflation target the mean and the tails of the duration of the ZLB decreases—by more in the menu cost model. As I explain with the impulse-response function, the aggregate price flexibility in periods at the ZLB increases significantly at low inflation targets.

The model cannot generate infrequent but long periods of ZLB as in the data. Since the aggregate shocks in my model follow an AR(1), the economy hits the ZLB whenever there is a large history of negative shocks. Dordal-i Carreras, Coibion, Gorodnichenko and Wieland (2016) explores the addition of fat tails in the aggregate shocks to match this feature in the data, and they find that this feature increases the optimal inflation target even more.

Rows 4 to 5 describe the moments of output-gap. The first property that we can see is that output-gap is not only lower at the ZLB but also more volatile. Thus the ZLB not only changes the conditional mean but also changes the conditional volatility of the output-gap. Importantly, in the menu cost model, increasing the inflation target decreases the conditional volatility of output-gap with binding and not binding ZLB.

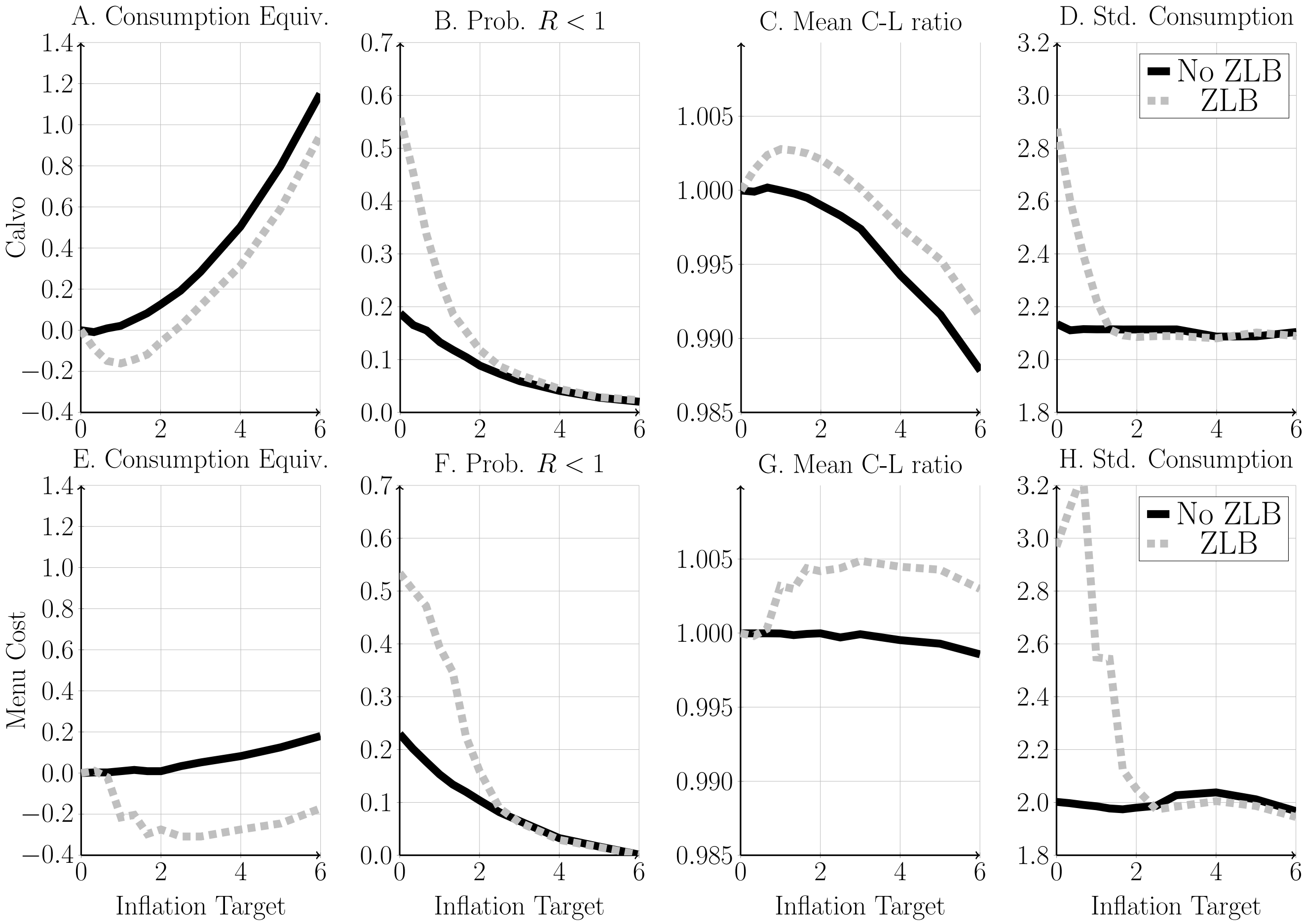
With higher inflation, the drop in output-gap is lower at the ZLB. The intuition of this result is as follows: with higher inflation only larger exogenous shocks can trigger the ZLB; thus if there weren't endogenous states, then the drop in the conditional mean would be higher with a rise in the inflation target. But since at low inflation targets the endogenous deflationary spiral is larger, there is an endogenous force that goes in the inverse direction. In this calibration, the second mechanism is stronger than the first one for the output-gap.

The last rows describe the business cycle moments of inflation and its components. As we can see, at 1% inflation target, inflation drops twice as much in the menu cost model as in the Calvo model and it is also twice as volatile. Since these quantitative properties do not hold for the reset price, this gap is covered by the menu cost inflation. Additionally, two properties hold: 1) inflation and menu cost inflation are more volatile at the ZLB at 1% inflation target and this property reverses at 3% inflation target; 2) mean frequency of price changes is higher at the ZLB at 1% inflation target and this property reverses at 3% inflation target. Finally, note that at 1% inflation target the average menu cost inflation at the ZLB is less than one; thus there are large price decreases on average when the ZLB is binding; but since the average menu cost inflation is 1.3 at 3% inflation target, the average menu cost inflation at the ZLB is more than one, with prices hitting the lower Ss band when the ZLB is binding.

6 The Optimal Inflation Target: Normative Implications of the Model

In this section, I quantitatively study the optimal inflation target in my menu cost model and compare these results with the Calvo model. The first section analyses welfare though basic moments in consumption, labor supply, and nominal interest. To better understand the welfare analyses, I continue the analyses through wedges of the New Keynesian model with respect to the Neoclassical Growth model. As I show below, different pricing models, different levels of inflation or even the ZLB affect welfare though these wedges. Finally, I show that the key to generate low costs of inflation is the interaction between menu cost and idiosyncratic shock.

Figure IV – Main Moments For Welfare in the Calvo and Menu Cost Model



Panels A to D describe the consumption equivalent, the probability of zero interest rate, the normalized consumption-labor supply ratio and the standard deviation of consumption in the Calvo model. Panels E to H describe the the same variables in the menu cost model. The black solid lines describe the moments without ZLB and the light-gray dotted lines describe the moments with ZLB.

6.1 The Optimal Inflation Target

The optimal level of inflation with ZLB constraint in my model is 3%; in the Calvo model is 1%. For this result, it is necessary to have the ZLB constraint on nominal interest rates since without it the optimal level of inflation is close to 0% in both models. Importantly, the target inflation is not equal to the mean inflation due to the ZLB constraint. In the case of the menu cost model, the average inflation in the optimal inflation is 3% and in the case of the Calvo model it is 0.33%.

Figure IV-panels A and E describe welfare using the consumption equivalence measure with respect to zero inflation. Each point in the y-axis reads the percentage increase in the consumption to equalize the welfare at zero inflation. If we define the following recursion

$$U^{\bar{\pi}}(S,\xi) = u\left(\left(1+\frac{\xi}{100}\right)C(S), L(S)\right) + \beta g^{1-\sigma_{np}} \mathbb{E}_{S'}\left[U^{\bar{\pi}}(S,\xi)^{1-\sigma_{ez}} \,\middle|\, S\right]^{\frac{1}{1-\sigma_{ez}}} \tag{28}$$

for welfare in state S with an increase in consumption given by ξ at the inflation target $\bar{\pi}$, then the consumption equivalence with respect to zero inflation is given by $\mathbb{E}\left[U^{\bar{\pi}}(S, \xi(\bar{\pi}))\right] = \mathbb{E}\left[U^{0}(S, 0)\right]$.

Without ZLB, increasing the inflation target from 0 to 6% in the Calvo model is equivalent to decreasing consumption by 1.1%, while this number falls to 0.2% in the menu cost. In these cases the consumption equivalent is increasing for all levels of inflations. With ZLB, the consumption equivalent is decreasing at low levels of inflation and then increasing. In the menu cost model the optimal inflation target is 3%, with a consumption equivalent with respect to zero inflation of more than 0.2%. In the Calvo model, the optimal inflation target decreases to 1% with a consumption equivalent of 0.2%.

Figure IV-panels B and F describe the probability of hitting the ZLB, with and without ZLB. Without ZLB, both models predict a similar probability of negative interest rates—slightly higher in the menu cost model. With ZLB, the probability of hitting the ZLB increases compared to a model without this restriction. The main intuition of this result is that the deflationary spiral endogenously increases the length of periods in the ZLB. Since the deflationary spirals are larger in the menu cost model, the probability of hitting the zero lower in the menu cost model is one-fourth larger in the menu cost model than in the Calvo model for levels of inflation more between 1% and 2% inflation targets—remember that I use a value-add tax in the menu cost model to mimic monetary policy for levels of inflation less than 1% to generate convergence of the deflationary spiral.

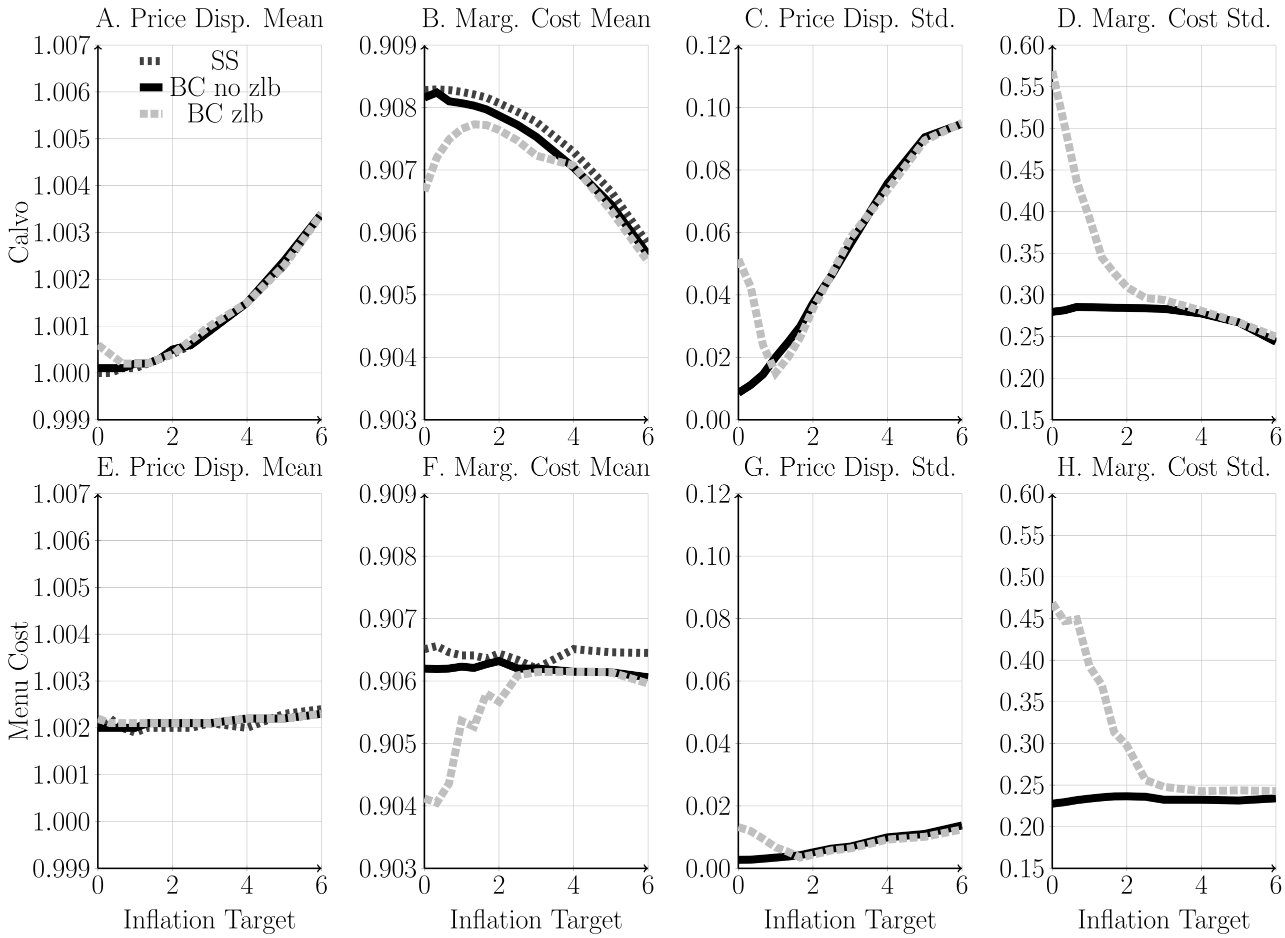
The main cost of a higher inflation target is a decrease in the consumption-labor ratio, as panel C and D in figure IV show. Without ZLB, the consumption-labor ratio explains a large proportion of the consumption equivalent: the consumption-labor ratio in the Calvo model falls by almost 1.3%, while the consumption-labor ratio in the menu cost falls by almost 0.1%. The ZLB generates an increasing consumption/labor ratio for low levels of inflation and then decreasing.

The main benefit of higher inflation is the decrease in the volatility of consumption and labor. There are two reasons for this effect. First, the economy hits the ZLB less often and therefore the nominal interest rate can accommodate aggregate shocks, decreasing the volatility of consumption. Second, a higher inflation target affects the Phillips curve in the two models and through the Taylor rule and the Euler equation affects the dynamic of consumption. This small quantitative effect is independent of the ZLB and depends mainly on the direct effect of different inflation targets on inflation dynamics in both models.

6.2 Welfare Analysis Though Wedges

Nominal rigidities generate three wedges with respect to the neoclassical growth model. The first wedge comes from firms' markups and their volatility. In the efficient allocation, firms' nominal marginal costs should be equal to the price, and therefore the real aggregate marginal cost should equal the efficient one. Monopolistic competition implies a real marginal cost less than one, since firms charge a positive markup (the markup is the inverse of the real marginal cost). Sticky prices together with monopolistic competition implies fluctuation in the

Figure V – First Two Moments of the Price Dispersion and Marginal Cost



Panels A to D describe the mean and standard deviation of the price dispersion and marginal cost of the Calvo Model. Panels E to H describe the mean and standard deviation of the price dispersion and marginal cost of the CalvoPlus Model. The dark-gray dashed line plots the steady state moments, the black solid line the moments without zero lower bound and the lightgray dotted lines plots the moment in the models with zero lower bound. The standard deviations are log-deviation with respect the mean and multiply by 100.

aggregate markup/real marginal cost, and therefore inefficient fluctuation in the marginal cost and consumption. The distortion from the first best given by markups is the first component of output-gap.

The second wedge comes from inefficient distortions in the relative prices across firms. In the first best it is optimal to have the same labor input across firms. Sticky price models create a misallocation in the input of production across firms, decreasing aggregate productivity.

The third wedge is a direct consequence of resources allocated for pricing decision. For each firm, there is a physical cost with respect to labor whenever the firm changes its price. In the aggregate, the total demand of labor comes from demand of labor for production and demand of labor for repricing.

The welfare in the Calvo and the menu cost model—or any pricing model—can be summarized in the stochastic process of real marginal cost, price dispersion and frequency of price change. Inflation targets, restrictions to the monetary policy or pricing models affect welfare through the stochastic process of these three wedges. The next proposition formalizes this claim in the economy with intermediate inputs, government expenditure and TFP shocks.

Proposition 3 (Welfare and Wedges) *Let $X = (\Delta, mc, FC, \eta_z, \eta_g)$ be the sufficiency statistics for consumption and labor and S the aggregate state of the economy. Then the welfare satisfies*

$$\begin{aligned}
KK(X) &= \frac{\alpha}{1-\alpha}\left(\frac{mc\ \eta_z}{\iota(1-\tau_{mc})}\right)^{1/(1-\alpha)} \qquad L(X) = \left(\frac{(1-\alpha)}{\kappa\alpha}KK(X)\right)^{1/\chi} \\
C(X) &= \frac{\eta_z}{\Delta}(L-FC)KK(X)^{\alpha}\left(1 - KK(X)^{1-\alpha}\frac{\Delta}{\eta_z}\right) - \eta_g(S) \\
U(S) &= \frac{\left(C(X(S)) - \kappa\,(1+\chi)^{-1}\,L(X(S))^{1+\chi}\right)^{1-\sigma_{np}}}{1-\sigma_{np}} + \beta g^{1-\sigma_{np}}\mathbb{E}_{S'}\left[U(S)^{1-\sigma_{ez}}\middle| S\right]^{\frac{1}{1-\sigma_{ez}}} \qquad (29)
\end{aligned}$$

where $FC(S)$ in the menu cost model is $\theta(\Omega(S) - hz)$ and in the Calvo model $FC(S) = 0$.

We can see in proposition 3 that productivity losses due to inflation are the outcome of the dispersion of relative prices and the degree of strategic complementarities in the form of intermediate inputs. Different pricing models affect the impact of inflation to the dispersion of relative prices, which is measured in Δ_t. Different degrees of strategic complementarities influence the direct effect of the dispersion of relative prices to TFP losses—in my model, the degree of complementarities is measured by α, the share of intermediate inputs. One of the main reasons I match the volatility of the inflation is because I calibrate my model to match the ratio of the value of total inputs over gross output; thus, I have a strong level of complementaries with the direct effect with respect to the elasticity TFP-price dispersion. Therefore, ignoring strategic complementarities as a foundation to generate low volatility of inflation gives an incorrect measure of cost of inflation.[16]

There are two quantitative properties in the model in proposition 3. First, fluctuation in the marginal cost affects labor supply with an elasticity of $\frac{1}{\chi(1-\alpha)} \approx 4$; hence, small fluctuations of marginal cost generate relatively large movements of labor and consumption. Second, fluctuations of consumption and labor affect total utility due to curvature of the period utility and the inter-temporal utility—the Epstein-Zin parameter.

Figure V describes the wedges in the Calvo and the menu cost model in the steady state and with business cycle with and without the ZLB. Next I describe the economic forces for the optimal inflation target through the first and second moments of these two wedges—I will ignore the physical cost of repricing since it is quantitatively insignificant.

Mean of the Price dispersion and Marginal Cost: Higher inflation target increases price dispersion in both models, but in the menu cost model this increase is small compared to the Calvo model. The key to generate a small cost of inflation is the interaction of menu cost and firm-level shocks—I will develop this argument in section 6.3. We can see in figure V that the increase of mean price dispersion in the Calvo model is third order while in

[16]In this paper, I have strategic complementarities with the technological assumption of intermediate input, but the result that strategic complementaries affect the elasticity of TFP with respect to the dispersion of relative prices holds with fixed inputs and Kimball aggregator.

the menu cost model it is fourth order even if 47% of price changes in the menu cost model at two percent inflation target are due to zero menu cost.

The mean marginal cost is decreasing with inflation in the model without ZLB and increasing-decreasing in the model with ZLB. To understand why, first let me define price gap as the difference between the current price and the static optimal price. Since the static profit function is asymmetric—it penalizes negative price gaps more than positive price gaps—the firm always prefers a positive rather than a negative price gap with the same magnitude. If inflation mean or volatility are higher, then the firm's relative price between price changes is more volatile and the firm increases the reset price as a precautionary motive to avoid negative price gaps, increasing the equilibrium level of markups. As with price dispersion, this force is smaller in the menu cost model because firms' profits function is almost symmetrical in the continuation set.

The increase in inflation volatility due to the ZLB is a key component of welfare since it affects the mean of price dispersion and marginal cost. At low inflation targets, since the frequency of the ZLB is high, the volatility of inflation is also large with its direct effect in the price dispersion. This is the main force for the optimal inflation in the Calvo model with low cost of business cycle, a trade-off between mean and volatility of inflation due to their impact on price dispersion. In my model with menu cost, this is not an important trade-off since price dispersion is almost insensitive to mean and volatility of inflation. Additionally, notice that the ZLB decreases the mean level of marginal cost due to the mechanism described in the paragraph above; thus the economy is more depressed on average. Finally, notice that both the mean price dispersion can be described at a high order of accuracy with the steady state price dispersion. I will use this result in the next section.

Standard Deviation of the Price Dispersion and Marginal Cost: The main benefit of higher inflation target is given by a lower volatility of real marginal cost. As we can see in figure V, the volatility of marginal cost largely decreases at higher inflation target since the nominal interest rate can react more strongly with respect to the structural shocks.

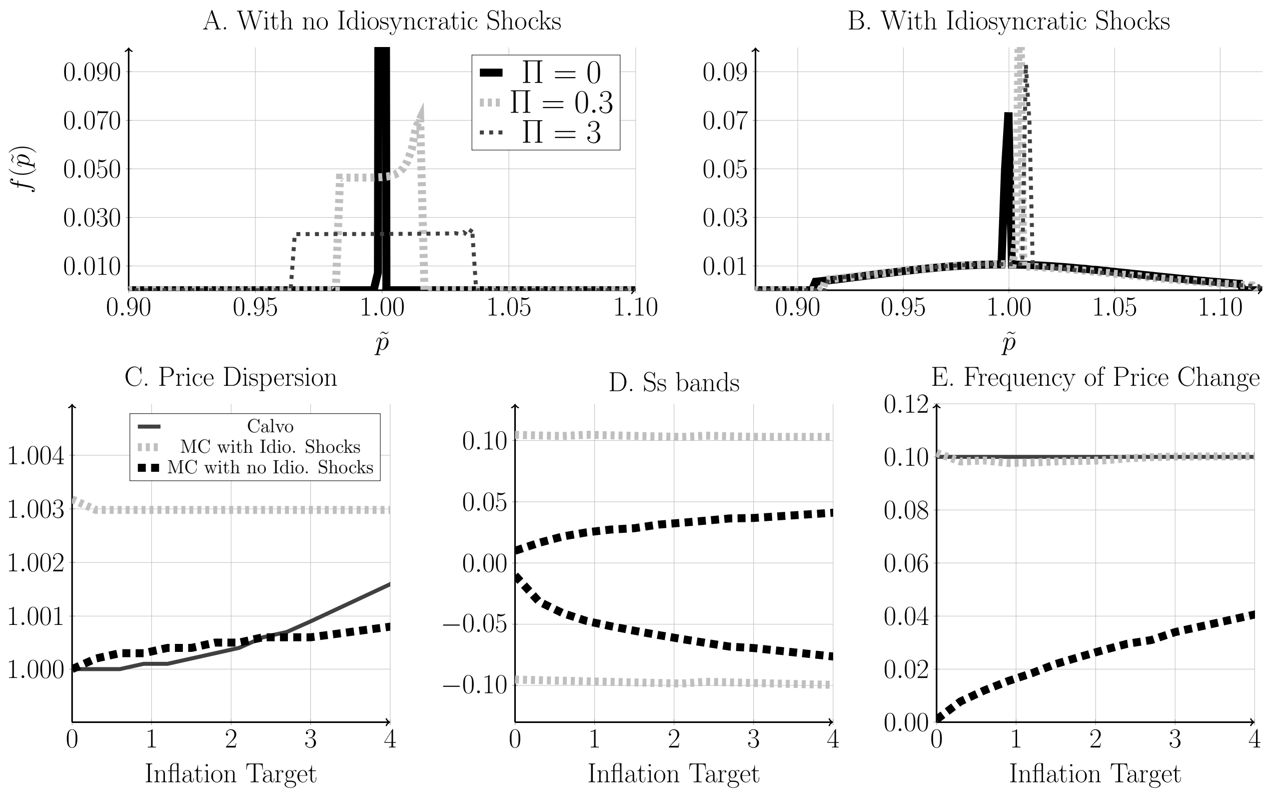
The volatility of price dispersion increases with respect to inflation target by an insignificant amount (at a fourth order) with almost no effect on welfare in the menu cost model. This mechanism does not hold in the Calvo model, where business cycle fluctuations of price dispersion are second order and therefore they are important to describe the optimal levels of inflation. This is an important property for welfare analysis. In the menu cost model, price dispersion is less sensitive with respect to the mean of inflation and volatility of inflation than the Calvo model.

6.3 Understanding the Interaction Between Idiosyncratic Shocks and Menu Cost

This section shows that a menu cost model without idiosyncratic shocks has more cost of inflation than the Calvo model for the relevant domain of optimal inflation target. Thus, a menu cost alone cannot generate a small inflation. Additionally, with empirical relevant firm-level shocks, the frequency of price change is almost insensitive to inflation.

To analyze the cost of inflation—the elasticity of price dispersion with respect to inflation—I focus on the steady state price dispersion, since, as I show in section 6.2, the steady state is a good approximation of the mean of the business cycle. Moreover, since real wage is a function of the inflation target, I isolate the effect of inflation with respect to price dispersion with a contingent tax on the marginal cost that keeps the real marginal cost constant. Thus, real wage and marginal cost are constant for all levels of inflation. Additionally, I use the calibration of the menu cost model with $\sigma_h = hz = 0$ to make the results as simple as possible.[17] All the statements hold with $\sigma_h, hz > 0$—See online appendix section J.

Figure VI – Steady State Distribution of Relative Prices



Panels A and B describe the distribution of relative price at 0%, 0.3% and 4% inflation inflation target. Panel C describes the price dispersion in the Calvo, and in the menu cost model with and without idiosyncratic shocks. Panel D describes the log-distance between the upper and lower Ss bands with respect to the reset price in the menu cost models and panel E describes the frequency of price in the three models.

First, I will show that a menu cost without idiosyncratic shocks generates more cost of inflation than Calvo. Figure VI-Panel C shows the price dispersion of the Calvo model and the menu cost with no idiosyncratic shocks. As we can see—see the online appendix for a formal proof—the price dispersion in the Calvo model is differentiable at zero and therefore it has a zero order effect with respect to inflation. This means that the effect of inflation to

[17] I choose the menu cost and the size of the idiosyncratic shock to match the frequency and the size of price change. Additionally, I set $hz = \epsilon$ to generate an unique ergodic distribution of relative prices.

price dispersion is second order. Instead, in the menu cost model with no idiosyncratic shocks, the price dispersion is discontinuous at zero and it has a positive first order for positive level of inflations. Next I show why.

Price dispersion has a positive jump at zero inflation in the menu cost model with no idiosyncratic shocks. The main reason is that since the length of the Ss bands is positive at all levels of inflation, the distribution of relative prices jumps from a probability atom at the optimal price at zero inflation target to an uniform distribution at any positive level of inflation. Figure VI shows this property of the distribution of relative prices at 0%, 0.3% and 3% inflation target. Additionally, the width of the Ss bands is strictly increasing with respect to inflation and since the mass of firms in the Ss bands are positive, there is a first order effect of increasing inflation.

Result 7 *[Cost of Inflation in Menu Cost with No Idiosyncratic Shocks] Price dispersion in the menu cost without idiosyncratic shocks and the Calvo model are zero at zero inflation target. There exists a positive $\bar{\Pi}$, s.t.*

$$\frac{\Delta^{MC,ns}(\Pi) - \Delta^{MC,ns}(1)}{\Delta^{C}(\Pi) - \Delta^{C}(1)} > 1 \tag{30}$$

for all levels of inflation between $(0, \bar{\Pi}]$ where $\Delta^{MC,ns}$ is price dispersion in the menu cost without idiosyncratic shocks and Δ^{C} is the price dispersion in Calvo.

As we see in figure VI, menu cost with idiosyncratic shocks has a relatively large price dispersion at zero inflation, due to idiosyncratic shocks, and a low elasticity of price dispersion with respect to inflation. There are two reasons for this result: 1) Ss are almost insensitive with respect to inflation at low inflation levels, thus the support of the distribution doesn't change much with inflation; and 2) the distribution of relative prices is almost symmetric at low inflation levels, thus the asymmetric impact of inflation is also small. To see these two effects, notice that using a first order Taylor approximation

$$\log(\Delta(\Pi)) - \log(\Delta(\Pi^*)) \propto (\mathcal{S}(\Pi^*) + \mathcal{D}(\Pi^*))(\Pi - \Pi^*) + o((\Pi - \Pi^*)^2) \tag{31}$$

with

$$\mathcal{S}(\Pi^*) = \int_{s(\Pi^*)}^{S(\Pi^*)} x^2 g_{\Pi}(x, \Pi^*) dx \quad ; \quad \mathcal{D}(\Pi^*) = (S(\Pi^*)^2 g(S(\Pi^*), \Pi^*) S'(\Pi^*) - s(\Pi^*)^2 g(s(\Pi^*), \Pi^*) s'(\Pi^*)) \tag{32}$$

where $g(x, \Pi)$ is the distribution of log-relative prices with domain $[S(\Pi), s(\Pi)]$.

Since the width of the Ss bands is constant—due to large idiosyncratic shocks—the $\mathcal{D}(\Pi^*)$ term is small. The second effect comes directly from the symmetry of the distribution of relative prices at zero inflation target. To see this, notice that firms are hit by a sequence of productivity shocks with positive or negative accumulated sum in a symmetric way. At zero inflation, an increase in the inflation increases the price dispersion of those firms with positive shocks, but also decreases price dispersion of those firms with negative shocks. At zero inflation, these two effects cancel each other with the direct implication of a low cost of inflation—in equation 32, it implies $g_\Pi(x,1) = -g_\Pi(-x,1)$, so $\mathcal{S}(1) = 0$. At positive inflation, large idiosyncratic shocks makes this effect quantitatively similar.

Result 8 *[Cost of Inflation in Menu Cost with Idiosyncratic Shocks] In the menu cost with idiosyncratic shocks, price dispersion is positive, continuous, and with zero first order effect at zero inflation.*

With the last result it is easy to see that the menu cost with idiosyncratic shocks is the model that generates the lowest cost of inflation, followed by Calvo and then the menu cost without idiosyncratic shocks, at low levels of inflation. Since the Calvo model with idiosyncratic shocks generates higher cost of inflation than the menu cost, it is the interaction between menu cost and idiosyncratic shocks that generates a low cost of inflation.

Finally, as figure VI shows, the menu cost model with idiosyncratic shocks does not generate low cost of inflation due to an increase of frequency of price change. We can see that at low inflation levels the frequency is almost constant as price dispersion. When calibrated to match micro-price statistics, idiosyncratic shocks are so large that inflation has almost no effect on either price dispersion or frequency at low levels of inflation.

7 Conclusion

Since the main cost of inflation in sticky price models is inefficient price dispersion, this paper uses a menu cost model with idiosyncratic shocks, founded on micro data, to find an optimal inflation target of 3%. In order to do this, I extend a random menu cost model with idiosyncratic shocks to a standard New Keynesian framework with a Taylor rule subject to a ZLB constraint and rich aggregate dynamics given by aggregate, government expenditure, money, and risk premium shocks. The main reason for this result is that price dispersion—the main cost in sticky price models—has low sensitivity with respect to inflation target. Moreover, the likelihood of hitting the ZLB constraint and the magnitude of the selection effect when the ZLB is binding are decreasing with the inflation target.

Going forward, it would be important to further explore the optimal inflation in different environments. For example, many countries with inflation targets are small open economies; does this feature affect the optimal inflation target? Are the structural shocks hitting these economies differently? Second, this paper focuses on price rigidities assuming a flexible wages. How does the optimal inflation change with sticky price and sticky wages? Does downward wage rigidity affect the optimal inflation target? Answering these questions in a framework similar to this paper and different from Calvo seems the correct environment to think about these questions.

In this paper, I didn't focus on the transition dynamics from the current inflation target to the optimal one and I use a Taylor rule to described monetary policy. These two assumptions could change the optimal inflation target. With respect to optimal monetary policy, this path has two important problems. First, there are no tools to solve optimal policy in a heterogeneous agents model with non-convexity at aggregate and idiosyncratic levels. Second, it is well known that optimal monetary policy under commitment generates unrealistic time series of nominal interest rate—see Clarida, Gali and Gertler (1999). Therefore, a set of realistic frictions that a central bank faces is important to have to set the interest rate.

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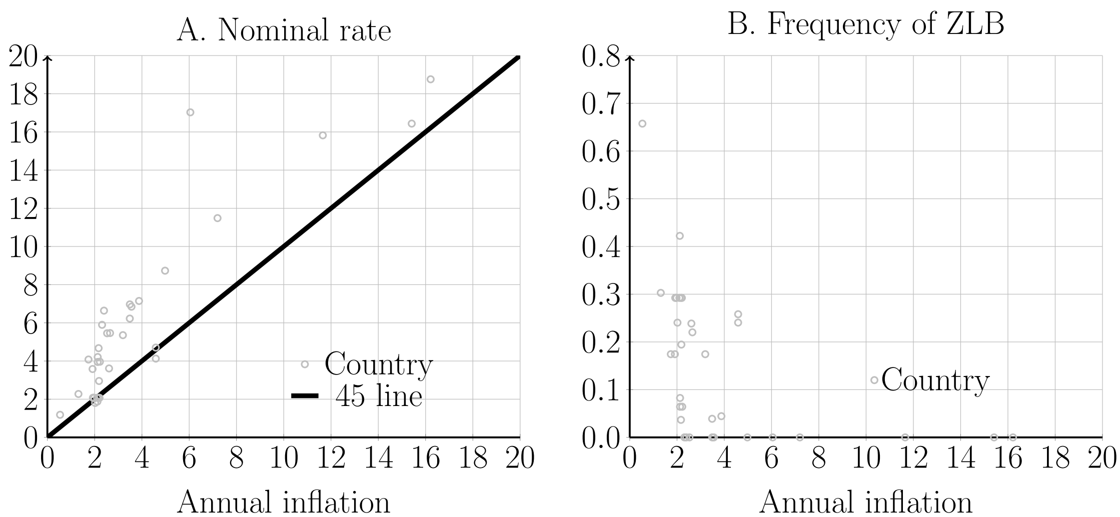
A Frequency of Hitting the Zero Lower Bound Across Countries

I construct a quarterly panel data across countries of inflation and nominal interest rate from the IMF-IFS database and FRED. For all the countries, I use FRED, except for Argentina, Peru and Singapore—for which I use IMF-IFS data. Since my interest is to compute averages over a long period of time, the data is not seasonal adjusted. For interest rates, I use monthly average to compute the quarterly interest rate. For inflation, I use consumer price index (CPI); for nominal interest rate, I use a combination of policy rates—whenever this one is available—and call rates—overnight interbank borrowing rate. The countries from which I use call rates or policy rates are:

- **Policy rate:** Poland, South Africa, Denmark, Finland, Austria, United Kingdom, Czech Republic, Canada, United States, Japan.
- **Call rate:** Sweden, Italy, Australia, Germany, Peru, New Zealand, Portugal, Slovenia, Brazil, Israel, Switzerland, Belgium, France, Mexico, Iceland, Slovakia, Norway, Luxembourg, Netherlands, Chile, Korea, Republic of., Turkey, Ireland, Spain, Poland.

Table II in section B of online appendix describes the data for each country, where I compute the average probability of hitting the ZLB and the average inflation for each country—historically and in the time period 1988-present. I define the event of hitting the ZLB whenever the nominal rate is less than 0.51% for annual rate, I drop countries with average inflation in the period between 1988-2016 more than 4% or countries with less than 20 years of data—see last column. The last two rows describe the average of inflation and frequency of ZLB taking Europe as different countries—average 1—or taking Europe as a country—average 2.

Figure VII – Average Nominal Rate, Inflation and Frequency of Zero Lower Bound



Panel A describes the mean nominal interest rate and inflation over the time period 1988-present. Panel B describes the mean frequency of binding zero lower bound and inflation over the time period 1988-present.

Figure VII plots average interest, average inflation and average frequency of ZLB bound using the same filters as before in the case of average 1.

B Micro-Price Statistics Data-Model

I apply several filters to render the data compatible with the model in the computation of price statistics. Table VI in the online appendix describes the micro-price statistics with the different filters. Columns 1 and 2 of table II describe the price statistics with only filter I and with filter IV described next.

Filter I: I drop outliers in the price change distribution ($\pm 1\%$) to compute the price change distribution. I drop price changes with sale flag, missing flag, and non-comparable product substitution. I drop 3 months in the sample: December 2008, January 2010 and January 2011. In all these months there were changes in VAT rate. I also drop an outlier month, 2005m6.

Filter II: I repeat the same filter as before with an additional filter on sales. The algorithm to detect sale is as follows.

I identify the upper and lower bound of periods with sales of length κ, by

$$\mathcal{F}_\kappa = \left\{ i, t : \sum_{j=0}^{\kappa} \left(p^i_{t+j} - p^i_{t-1+j} \right) = 0 \right\} \quad ; \quad \mathcal{L}_\kappa = \left\{ i, t : \sum_{j=0}^{\kappa} \left(p^i_{t-j} - p^i_{t-1-j} \right) = 0 \right\} \tag{B.33}$$

where p^i_t is the price quote of item i at time t. Importantly, $(i, t) \in \mathcal{F}_\kappa$, if and only if $(i, t + \kappa) \in \mathcal{L}_\kappa$. For any given $(i, t^*) \in \mathcal{F}_\kappa$, I drop all price changes between t^* and $t^* + \kappa$. For the implementation of this algorithm I choose $\kappa = 3$ at monthly frequency.

Filter III: I repeat the same filters as before with an additional filter on heterogeneity at sector level. I assign each id to a sector. If we denote each id with the index i, then each id is in only one sector C_j with $j = 1, 2, ..., n_C$. I redefine the normalized price change by item as

$$\Delta \tilde{p}^i_t = \frac{\Delta p^i_t - \mathbb{E}[\Delta p^i_t | i \in C_j]}{\mathbb{S}td[\Delta p^i_t | i \in C_j]} \mathbb{S}td[\Delta p^i_t] + \mathbb{E}[\Delta p^i_t] \tag{B.34}$$

I computed all price statistics with $\left\{ \Delta \tilde{p}^i_t \right\}_{t,i}$.

Filter IV: I repeat the same filters as before with an additional filter on heterogeneity at item level. I drop all items such that the sum of all price changes with respect to all the ids is less than 100. Then, I assign each id to an item. If we denote each id with the index i, then each id is in only one item C_j with $j = 1, 2, ..., n_C$. I redefine the normalized price change by item as

$$\Delta \tilde{p}^i_t = \frac{\Delta p^i_t - \mathbb{E}[\Delta p^i_t | i \in C_j]}{\mathbb{S}td[\Delta p^i_t | i \in C_j]} \mathbb{S}td[\Delta p^i_t] + \mathbb{E}[\Delta p^i_t] \tag{B.35}$$

I computed all price statistics with $\left\{ \Delta \tilde{p}^i_t \right\}_{t,i}$.

C Numerical Appendix

This numerical appendix describes the equilibrium conditions of the CalvoPlus Model together with the evaluation of Krusell-Smith.

C.1 Recursive Equilibrium conditions

I divide the equilibrium condition into four blocks: household optimality conditions, firm's optimality conditions, monetary policy and aggregate feasibility, Krusell-Smith cross-equation approximation and exogenous shocks. For simplicity, I denote with X the detrended variable of $\tilde{X}$ and S the aggregate state of the economy given by $S = \left(mc_-, \Delta_-, \tilde{R}_-, \eta_z, \eta_g, \eta_q \right)$.

- **Household optimality conditions:**

$$mu(S) = \beta (1+g)^{-\sigma_n} \eta_{q-}(S) R(S) \mathbb{E}_{S'} \left[\left(\frac{U(S')/U_{ss}}{\Sigma(S)^{\frac{1}{1-\sigma_{ez}}}} \right)^{-\sigma_{ez}} \frac{mu(S')}{\Pi(S')} \middle| S \right] \tag{C.36}$$

$$\Sigma(S) = \mathbb{E}_{S'} \left[\left(\frac{U(S')}{U_{ss}} \right)^{1-\sigma_{ez}} \middle| S \right] \tag{C.37}$$

$$mu(S) = \bar{u} \left(C(S) - \kappa \frac{L(S)^{1+\chi}}{1+\chi} \right)^{-\sigma_{np}} \tag{C.38}$$

$$\kappa L(S)^\chi = w(S) \tag{C.39}$$

$$u(S) = \bar{u} \frac{\left(C(S) - \kappa \frac{L(S)^{1+\chi}}{1+\chi} \right)^{1-\sigma_{np}}}{1-\sigma_{np}} \tag{C.40}$$

$$U(S) = (1-\beta) u(S) + \beta (1+g)^{1-\sigma_{np}} U_{ss} \Sigma(S)^{\frac{1}{1-\sigma_{ez}}} \tag{C.41}$$

– Where the original variable can be obtained as:

$$\tilde{C}_t = C(S_t)(1+g)^t \quad ; \quad \tilde{mu}_t = mu(S_t)(1+g)^{-\sigma_{np}t} \quad ; \quad \tilde{L}_t = L_t \quad ; \quad \tilde{\Pi}_t = \Pi(S)$$
$$\tilde{u}_t = u(S_t)(1+g)^{(1-\sigma_{np})t} \quad ; \quad \tilde{U}_t = U(S_t)(1+g)^{(1-\sigma_{np})t} \quad ; \quad \tilde{w}_t = w(S_t)(1+g)^t \tag{C.42}$$

- **Firm's optimality conditions:**

$$v(\tilde{p},S) = \mathbb{E}_{S,x}\left[\left(\frac{U(S')}{U_{ss}\Sigma(S)^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}}\left((1-hz)\max_{c,nc}\left\{v^c(S)-\bar{\theta}(S'),v(\tilde{p}'(\tilde{p}),S')\right\}+hzv^c(S)\right)\right] \tag{C.43}$$

$$P^*(S) = \arg\max_{\tilde{p}}\left\{\Phi(\tilde{p},S)+\beta(1+g)^{1-\sigma_{np}}v(\tilde{p},S)\right\} \tag{C.44}$$

$$v^c(S) = \Phi\left(P^*(S),S\right)+\beta(1+g)^{1-\sigma_{np}}v\left(P^*(S),S\right) \tag{C.45}$$

$$v^{nc}(\tilde{p},S) = \Phi\left(\tilde{p},S\right)+\beta(1+g)^{1-\sigma_{np}}v\left(\tilde{p},S\right) \tag{C.46}$$

$$\Phi(\tilde{p},S) = mu(S)Y(S)\tilde{p}^{-\gamma}\left(\tilde{p}-mc(S)\right) \tag{C.47}$$

$$\tilde{p}'(\tilde{p}) = \begin{cases} \frac{\tilde{p}e^{\Delta a_l}}{\Pi(S')} & \text{with prob. } p \\ \frac{\tilde{p}e^{\Delta a_h}}{\Pi(S')} & \text{with prob. } 1-p \end{cases} \quad ; \quad \bar{\theta}(S) = \theta w(S')mu(S') \quad ; \quad \Delta a \sim N(0,\sigma_a) \tag{C.48}$$

- **Monetary policy and aggregate feasibility:**

$$\begin{aligned}
\Pi(S) &= \left(\frac{1-\Omega(S)}{1-\Omega(S)(P^*(S))^{1-\gamma}}\right)^{\frac{1}{1-\gamma}}\varphi(S) \\
mc(S) &= \iota(1-\tau_{mc})\frac{w(S)^{1-\alpha}}{\eta_{z-}(S)} && \text{(C.49)} \\
R^*(S) &= \tilde{R}_-(S)\left(\frac{1+\bar{\pi}}{\beta g^{-\sigma_{np}}}\right)\left(\left(\frac{\Pi(S)}{(1+\bar{\pi})}\right)^{\phi_\pi}\left(\frac{mc(S)}{mc_{ss}}\right)^{\phi_y}\right)^{1-\phi_r}\left(\frac{mc(S)}{mc_-(S)}\right)^{\phi_{dy}} && \text{(C.50)} \\
\tilde{R}(S') &= \left(\left(\frac{\beta(1+g)^{-\sigma_{np}}}{1+\bar{\pi}}\right)R^*(S)\right)^{\phi_r}\eta_r' && \text{(C.51)} \\
R(S) &= \max\{1,R^*(S)\} && \text{(C.52)} \\
(L(S)-\theta(\Omega(S)-hz)) &= Y(S)\left(\frac{(1-\alpha)}{\alpha w(S)}\right)^{\alpha}\frac{\Delta(S)}{\eta_{z-}(S)} && \text{(C.53)} \\
\eta_{g-}(S)+C(S) &= Y(S)\left(1-\left(\frac{w(S)\alpha}{1-\alpha}\right)^{1-\alpha}\frac{\Delta(S)}{\eta_{z-}(S)}\right) && \text{(C.54)}
\end{aligned}$$

– Where the original variable can be obtained as:

$$\tilde{mc}_t = mc(S_t) \quad ; \quad \tilde{Y}_t = Y(S_t)(1+g)^t \tag{C.55}$$

- **Krusell-Smith cross-equation approximation:** let $P^2(S)$ be a second order polynomial with respect to the aggregate state

$$\log\left(\Delta(S)\right) = P^2(\log(S)) \quad ; \quad \log\left(\Omega(S)\right) = P^2(\log(S)) \quad ; \quad \log\left(\varphi(S)\right) = P^2(\log(S)) \tag{C.56}$$

- **Exogenous shocks:**

$$\begin{aligned}
\log\left(\eta_g(S')\right) &= (1-\rho_g)\log(\eta_g^*)+\rho_g\log(\eta_g(S))+\sigma_g\epsilon_g' && \text{(C.57)} \\
\log\left(\eta_z(S')\right) &= (1-\rho_Z)\log(\eta_z^*)+\rho_z\log(\eta_z(S))+\sigma_z\epsilon_z' && \text{(C.58)} \\
\log\left(\eta_r)S')\right) &= (1-\rho_r)\log(\eta_r^*)+\rho_r\log(\eta_r(S))+\sigma_r\epsilon_r' && \text{(C.59)} \\
\log\left(\eta_q(S')\right) &= (1-\rho_q)\log(\eta_q^*)+\rho_q\log(\eta_{q(S)})+\sigma_q\epsilon_q && \text{(C.60)} \\
& && \text{(C.61)}
\end{aligned}$$

C.2 Krusell-Smith Evaluation

I evaluate Krusell-Smith in the simulation. First, I construct a time series of simulated inflation, price dispersion and frequency of price change, together with simulated marginal cost, interest rate and structural shocks—the states in the economy. Let X_t be the simulated state given by

$$X_t = \left\{mc_-(S_t^s), \Delta_{t-1}^s, \tilde{R}_-(S_t^s), \eta_z(S_t^s), \eta_g(S_t^s), \eta_q(S_t^s), \Pi_t^s, \Omega_t^s\right\}_t \tag{C.62}$$

Y_t be the model implied equilibrium functions that comes from solving the equilibrium conditions

$$Y_t = \{mc(S_t^s), mu(S_t^s), R(S_t^s), R^*(S_t^s), Y(S_t^s), C(S_t^s), L(S_t^s), P^*(S_t^s, w(S_t^s))\}_t \tag{C.63}$$

and Y_t^s be the solution of the equilibrium equations in the simulation given by

$$\hat{Y}_t^s = \left\{\hat{mc}_t^s, \hat{mu}_t^s, \hat{R}_t^s, \hat{R}_t^{*s}, \hat{Y}_t^s, \hat{C}_t^s, \hat{L}_t^s, \hat{P}_t^{*,s}, w_t^s\right\}_t \tag{C.64}$$

Next, I describe the construction of each function. Table XX shows $\sigma_y^{error} = \frac{\mathbb{S}td[\log(|\hat{Y}_t^s/Y_t|)]}{\mathbb{S}td(\log(Y_t))}100$. Figure YY shows a time series for $\hat{Y}_t^s, Y_t$ with and without the zero lower bound constraint.

- **Construction of $\hat{Y}_t^s$ using static equations and $\mu(S_t^s)$:** Given $\mu(S_t^s)$ I construct $\hat{Y}_t$ solving the static system of equations

$$\begin{aligned}
\hat{R}_t^{*,s} &= \tilde{R}_-(S_t^s)\left(\frac{1+\bar{\pi}}{\beta(1+g)^{-\sigma_{np}}}\right)\left(\left(\frac{\Pi_t^s}{(1+\bar{\pi})}\right)^{\phi_\pi}\left(\frac{mc_-(S_{t+1}^s))}{mc_{ss}}\right)^{\phi_y}\right)^{1-\phi_r}\left(\frac{mc_-(S_{t+1}^s}{mc_-(S_t^s)}\right)^{\phi_{dy}} && \text{(C.65)}\\
\hat{R}_t^s &= \max\{1, \hat{R}_t^{*,s}\} && \text{(C.66)}\\
mu(S_t^s) &= (\hat{C}_t^s - \kappa\frac{(\hat{L}_t^s)^{1+\chi}}{1+\chi})^{-\sigma_{np}} && \text{(C.67)}\\
\kappa(\hat{L}_t^s)^\chi &= \hat{w}_t^s && \text{(C.68)}\\
\eta_z(S_t^s)(\hat{L}_t^s - \theta(\Omega_t^s - hz)) &= \hat{Y}_t^s\left(\frac{(1-\alpha)}{\alpha\hat{w}_t^s}\right)^\alpha \Delta_t^s && \text{(C.69)}\\
\eta_{g-}(S) + \hat{C}_t^s &= \hat{Y}_t^s\left(1 - \left(\frac{\hat{w}_t^s\alpha}{1-\alpha}\right)^{1-\alpha}\frac{\Delta_t^s}{\eta_z(S_t^s)}\right) && \text{(C.70)}\\
\hat{mc}_t^s &= \iota(1-\tau_{mc})\frac{(\hat{w}_t^s)^{1-\alpha}}{\eta_z(S_t^s)} && \text{(C.71)}
\end{aligned}$$

- **Construction of $\hat{mu}_t^s$ using forward looking equations:** I construct $\hat{mu}_t^s$ using household Euler equation. First, I project the realized inflation with respect to the state. Let $\tilde{\Pi}(S)$ be this function. Then, I construct marginal utility in the simulation as

$$\hat{mu}_t^s = \beta(1+g)^{-\sigma_n}\eta_{q-}(S_t^s)\hat{R}_t^s\mathbb{E}_{S'}\left[\left(\frac{U(S')/U_{ss}}{\Sigma(S_t^s)^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}}\frac{mu(S')}{\tilde{\Pi}(S')}\middle| S_t^s\right] \tag{C.72}$$

where $S_t^s = (mc_-(S_t^s), \Delta_{t-1}^s, \tilde{R}_-(S_t^s), \eta_z(S_t^s), \eta_g(S_t^s), \eta_q(S_t^s))$.

Table IV – Krusell-Smith Evaluation

	Annual Inflation Target (σ_y^{error} no ZLB , σ_y^{error} ZLB)			
variable	0	2	4	6
Nominal rate	(0.024,0.637)	(0.029,0.590)	(0.055,0.093)	(0.080,0.090)
Gross Output	(0.131,0.579)	(0.133,0.147)	(0.311,0.359)	(0.481,0.437)
Real wage	(0.419,1.614)	(0.408,0.335)	(0.943,1.069)	(1.442,1.312)
Consumption	(0.148,0.758)	(0.151,0.177)	(0.344,0.401)	(0.533,0.485)
Labor supply	(0.418,1.663)	(0.408,0.340)	(0.944,1.071)	(1.442,1.312)
Utility	(0.003,0.274)	(0.002,0.016)	(0.003,0.004)	(0.005,0.005)

This table describes the ratio between the variance of the predicted error and the total variance for a set of control variables.See appendix C for the method to construct these statistics.

Online Appendix: Not for Publication

Optimal Inflation Target in an Economy with Menu Costs and Zero Lower Bound

Andres Blanco

A US Macroeconomic Time Series Data Description

Table I describes the data sourse for US macroeconomic time series. I follow Smets and Wouters (2003) for the construction of each variable. These are the variables used to compute business cycle moment in the data:

- **Real Gross Domestic product** : $ln(GDP/N)$.
- **Labor Supply**:: $ln(E * H/N)$.
- **Real wage**: $ln(W/GDPDEF)$.
- **Inflation**: $ln(GDPDEF/GDPDEF(-1))$.
- **Nominal Interest Rate**: $DFF^{1/4}$.
- **TFP**: $ln(TFP)$
- **G**: $ln(GCEC1/N)$
- **C**: $ln(PCEC/(GDPDEF * N))$

Table I – Macroeconomic Time Series — Description and Sources –

Label	Short description	Source	Frequency-Adjustment	Link
GDP	Real Gross Domestic Product	FRED	Quarterly, Seasonally Adjusted Annual Rate	(1)[a]
GDPDEF	Gross Domestic Product: Implicit Price Deflator	FRED	Quarterly, Seasonally	(1)
DFF	Effective Federal Funds Rate, Percent	FRED	Quarterly, Not Seasonally Adjusted	(1)
E	Civilian Employment Level, Thousands of Persons	FRED	Quarterly, Seasonally Adjusted	(1)
GCEC1	Real Gov. Cons. and Gross Investment	FRED	Quarterly, Seasonally Adjusted	(1)
PCEC	Personal Consumption Expenditures, Billions of Dollars	FRED	Quarterly, Seasonally Adjusted	(1)
W	Business hourly compensation	BLS	Quarterly,Seasonally Adjusted (PRS85006103)	(2)[b]
N	POPULATION LEVEL (1976-2016)	BLS	Monthly (LFU800000000)	(2)
N	POPULATION LEVEL (1948-1976)	BLS	Monthly (LNS10000000)	(2)
H	Average weekly hours worked	BLS	Quarterly, Seasonally Adjusted(PRS85006023)	(2)
TFP	Labor productivity (output per hour)	Cociuba et. al.	Quarterly, Seasonally Adjusted	(3)[c]

[a] https://research.stlouisfed.org/fred2
[b] http://data.bls.gov/cgi-bin/srgate
[c] See Cociuba *et al.* (2009); https://sites.google.com/site/alexanderueberfeldt/research

B International Evidence of Frequency of Hitting the Zero Lower Bound

Table II – Frequency of binding ZLB and Inflation

Country	Part of EU	Start Date	End Date	Historical Freq. ZLB	Historical Mean Inf.	After 1988 ZLB Freq.	After 1988 Mean Inf.	in/out
Argentina	NOT EU	2014q1	2015q2	0	16.21	0	16.21	out
Australia	NOT EU	1990q3	2015q1	0	2.53	0	2.53	in
Austria	EU	1958q1	2014q4	.09	3.27	.19	2.18	in
Belgium	EU	1999q1	2015q1	.29	1.95	.29	1.95	out
Brazil	NOT EU	1996q4	2014q4	0	6.05	0	6.05	out
Canada	NOT EU	1955q1	2015q1	.02	3.64	.04	2.17	in
Chile	NOT EU	1996q1	2015q1	.04	3.49	.04	3.49	out
Czech Republic	EU	1992q1	2015q1	.26	4.59	.26	4.59	out
Denmark	EU	1967q1	2015q1	.05	4.61	.08	2.13	in
Finland	EU	1958q4	2015q1	.03	4.68	.06	2.13	in
France	EU	1955q1	2015q1	.08	4.33	.17	1.74	in
Germany	EU	1960q1	2015q1	.09	2.67	.17	1.91	in
Iceland	NOT EU	1998q2	2015q1	0	4.98	0	4.98	out
Ireland	EU	1999q1	2015q1	.29	2.21	.29	2.21	out
Israel	NOT EU	1992q4	2015q1	.04	3.88	.04	3.88	in
Italy	EU	1999q1	2015q1	.29	1.98	.29	1.98	out
Japan	NOT EU	1955q1	2014q4	.3	2.96	.66	.54	in
Korea	NOT EU	1991q1	2015q1	0	3.49	0	3.49	in
Luxembourg	EU	1999q1	2015q1	.29	2.17	.29	2.17	out
Mexico	NOT EU	1976q1	2015q1	0	21.06	0	11.65	out
Netherlands	EU	1999q1	2015q1	.29	1.93	.29	1.93	out
New Zealand	NOT EU	1985q1	2015q1	0	3.37	0	2.4	in
Norway	NOT EU	1982q1	2015q1	0	3.14	0	2.31	in
Peru	NOT EU	1995q4	2016q1	0	3.56	0	3.56	in
Poland	EU	1989q4	2015q1	0	15.41	0	15.41	out
Portugal	EU	1999q1	2015q1	.29	2.1	.29	2.1	out
Singapore	NOT EU	1987q3	2013q4	.24	2.01	.24	2.02	in
Slovakia	EU	1995q3	2015q1	.24	4.59	.24	4.59	out
Slovenia	EU	2004q1	2015q1	.42	2.12	.42	2.12	out
South Africa	NOT EU	1957q1	2014q4	0	7.52	0	7.2	out
Spain	EU	1973q2	2015q1	.11	6.65	.17	3.2	in
Sweden	EU	1955q1	2015q1	.03	4.37	.06	2.22	in
Switzerland	NOT EU	1972q1	2015q1	.24	2.27	.3	1.31	in
Turkey	NOT EU	1986q2	2014q4	0	33.1	0	32.97	out
United Kingdom	NOT EU	1955q1	2015q1	.1	4.98	.22	2.65	in
United States	NOT EU	1955q1	2015q1	.11	3.62	.24	2.61	in
Europe		1955q1	2015q1	.07	4.37	.13	2.22	
Average 1		1955q1	2016q1	.08	3.69	.14	2.37	
Average 2		1955q1	2016q1	.09	3.37	.14	2.44	

Part of EU describes wheter if the country is part of the European union; **Start and End Date** denotes the initial and final quarter; **Freq. ZLB** denotes the frequency of hitting the zero lower bound defined as the rate less than 0.75 ; **Mean Inf.** denotes mean inflation over the sample; **Historical** denotes the average over the entire sample; **After 1988** denotes the average after 1988.

C Computation and Description of Micro-Price Statistics

C.1 UK Data Description

To construct a panel data of price quotes I merge several databases in a single database. The following list describes the databases used to construct the master file with UK price quotes information together with the links to download the databases. Table III describes the variables from the price quotes main database and table IV describes the rest of the variables from the other sources.

- **Consumer Price Indices Price Quotes Links:** This database contains information about price quotes. Table III describes each variable.

  1. Data for years 1996M1-2010M12: CPI and RPI item indices and price quotes 1996-2010
  2. Data for years 2010M1-2015M8: CPI and RPI item indicces and price quotes 2011-2015
  3. Data for years 2015M9-2016M3-: CPI and RPI item indices and price quotes 2015-

- **Consumer Price Item ID and COICOP classification:** This database contains item id together with COICOP item classification.

  1. 1996M1-2011M12: Classification Framework 1996-2011
  2. 2012M1-2015M8: Classification Frameworks 2012-2015
  3. 2015M9-2016M8: Classification Frameworks 2012-2015

- **Consumer Price Indices by item:** This database contains item id together with CPI index at item level.

  1. 1996M1-2011M12: Classification Framework 1996-2011
  2. 2012M1-2015M8: Classification Frameworks 2012-2015
  3. 2015M9-2016M8: CPI and RPI item indices and price quotes 2015-

- **CPI by Item, Class and Sector Links:** This database contains price indexes at different levels of disaggregation, from item to sector.

  1. 1996-2016 (please refer to Table 63): Consumer Price Inflation Detailed Reference Tables

- **CPI Weights Links:** This database contains yearly weights at item level

  1. 1996-2011: Weighting Framework 1996-2011
  2. 2012-2015: Weighting Frameworks 2012-2015

Table III – Price Quote Data

Name	Label	Source	Frequency	Link
quote-date	date of the quote	Office for National Statistics (ONS)	monthly	(1)
item-id	Product number for ONS UK	ONS	monthly	(1)
shop code	Store number	ONS	monthly	(1)
region	location of store	ONS	monthly	(1)
item-desc	item description	ONS	monthly	(1)
validity	validity of the quote for CPI	ONS	monthly	(1)
price	price of item	ONS	monthly	(1)
indicator box	item or price change	ONS	monthly	(1)
price relative	price divided by base price	ONS	monthly	(1)
log price relative	log of relative price	ONS	monthly	(1)
start date	start date of sampling for CPI	ONS	monthly	(1)
end date	end date of sampling for CPI	ONS	monthly	(1)
base price	price at January	ONS	monthly	(1)
base validity	validity in base month	ONS	monthly	(1)
shop weight	number of specific stores within region	ONS	monthly	(1)
shop type	type of store	ONS	monthly	(1)
stratum cell	stratification	ONS	monthly	(1)
stratum weight	weight of stratum	ONS	monthly	(1)
stratum type	type of stratification	ONS	monthly	(1)

Table IV – Weights, COICOP ID and CPI weights at Different Levels of Disaggregation

Name	Label	Source	Frequency	Link
Consumer Price Indices Sectors and Classes Definition				
coicop id	Classification of Individual Consumption by Purpose	Office for National Statistics (ONS)	-	(2)
coicop type	hierarchy within CPI	ONS	-	(2)
CPI by Item, Class and Sector				
quote date	date of index	Office for National Statistics (ONS)	monthly	(3)
CPI item	price index at item level	ONS	monthly	(3)
CPI class	price index at class level	ONS	monthly	(3)
CPI sector	price index at sector level	ONS	monthly	(3)
item index	RPI index	ONS	monthly	(3)
all gm index	CPI index	ONS	monthly	(3)
gm ra index	index calculated using ratio of averages	ONS	monthly	(3)
index algorithm	algorithm used to calculate index	ONS	monthly	(3)
stratum ind	stratum indicator	ONS	monthly	(3)
CPI Weights				
coicop weight	weight given to class	ONS	yearly	(4)
start date	start date of weight	ONS	-	(4)
end date	end date of weight	ONS	-	(4)

Some of the variables in tables III and IV are easy to understand; therefore, I will omit further explanations. Next I describe some of the variables that require further explication.

- **Validity:** This variable takes values 3; or 4. 3 if the price quote was validated by system, 4 if accepted internally by ONS staff.
- **Shop Code:** Denotes the shop number within region and location. Shops may have same code but be located in different regions or locations.
- **Stratum Weight:** This variable describes the weights across shops and location for constructing item CPI, whenever available.
- **Indicator box:** This variable takes values S, R, M, C, N, T, X, Z. S denotes item on sale, R is sale recovery, M denotes missing in the store, C denotes comparable substitution, N denotes non-comparable substitution, T denotes temporarily out of stock, X denotes comparable substitution item on sale, Z denotes non-comparable item substitution on sale.
- **Stratum type:** This variable denotes if stratified by region, shop, or both. 0 denotes if not stratified, 1 denotes if stratified by region only, 2 denotes if stratified by region and shop, 3 denotes if stratified by shop only.
- **Start Date:** Marks the first date that the price contributed to the index.
- **End Date:** If end date = 99999, then the price contributed to the index. Otherwise, the price was omitted.
- **Region:**

1	Catalog collections	8	Yorks and Humber
2	London	9	NW
3	SE	10	North
4	SW	11	Wales
5	East Anglia	12	Scotland
6	East Midlands	13	NI
7	West Midlands		

- **Shop Type:** The variable denotes if the store is a multiple or an independent. 1 denotes if there are 10 or more outlets (multiple), 2 denotes if fewer than 10 outlets (independents).
- **Stratum cell:** This variable also codes for stratification. 0 denotes non-stratified quote. If the quote is stratified by shop only, then stratum cell takes the corresponding value of shop type. If it is stratified by region only, stratum cell takes the corresponding value of region. If the quote is stratified by both region and shop: if store is a multiple then stratum cell is equal to region; if store is an independent then stratum cell is equal to multiple code + 13.

C.2 Constructing and Merging UK Price Quotes Database

I constructed micro-price statistics in three steps:

- **Step 1**: This step downloads the data and generates an unique database. The output of this step is "pricequotes.dta".
  1. I download monthly price quotes to generate a panel-data from 1996m1-2016-m3 of the variables described in table III.
  2. I drop from the database zero price quotes—temporary out of stock price quotes. I drop also price quotes with validity different from 3 or 4. Additionally, there are two variables that denote indicator box depending on the date (for example in 1996m1, there are orig-indicator-box and indicator-box). In general, these variables are equal, but if they differ, they only differ in the type of substitution—X and S, X and R, and C and N. I only keep with indicator-box in the case it is not empty, or the other variable in the case is empty.
  3. The Office for National Statistics doesn't reveal the highest level of shop disaggregation for each product/region whenever there could be any problem with disclosure. For the same item, location and shop at a given month, there could be two or

more price observations. Hence, this database is not a panel data at the highest level of disaggregation—item, shop and location. In general this is not a problem since most of the repeated price quotes are equal across stores. Nevertheless, I develop a method to recover a panel data of price quotes with the old price quotes. This method is robust as long as two conditions holds: there is no reshuffling of shops with same id and the order across prices holds across stores.

To analyze the magnitude of this problem, table V describes the number of observations with repetitions. For example, around 20 million price quotes are not repeated; therefore, they are at the highest level of disaggregation. Two million observations are repeated twice—two price quotes for the same item, location, shop and month—and so on. For some months, items, shop and locations, there are 4 cases with 49 shops with potentially different quotes.

The majority of price quote repetitions are equal prices (up to the last cent). Table V shows that there aren't more than 7 different repetition. Thus, in general, repeated price quotes are of the same values.

Table V – UK CPI Division in Sectors, Sub-sectors and Classes

Number of Repetition	**Number of Observations** with repetitions	**Number of Observations** with no repetitions
1	23392896	23472914
2	2261938	2147038
3	298812	254346
4	53224	32404
5	10170	4025
6	2238	438
7	637	70
8	432	
9	4536	
10	810	
20	360	
30	60	
42	210	
49	196	

I developed an algorithm to recover a panel data with item-store-location price quotes at the highest level of disaggregation to deal with this problem. The main idea of this algorithm is to order the repeated prices from lowest to higher value whenever there are repetitions and to keep the same ordering across time to minimize the number of price changes. Next, I describe the algorithm whenever there are n repetitions at an item/item/shop/time.

Filtering algorithm: Let i denote an identifier at item-location-shop level and t denote time at monthly frequency. Let $S_{i,t} = \{p_{1ti}, p_{2ti}, ..., p_{n_{t,i}}\}$ denote the set of different prices at i, t level and $n^{\max}(i) = \max_t \#S_{i,t}$ the maximum number of repetitions for item-location-shop i. Assume that for each (t, i), the elements of $S_{i,t}$ are ordered from the lower to highest value. Next I describe the algorithm.

(a) Set $K = n^{\max}(i)$

(b) Generate $\hat{p}^K_{ti} = p_{Kti}$ for the first non-missing value over t. Set $\hat{p}^K_{ti} = p_{Kti}$ whenever p_{Kti} is not empty. If this not possible, generate $\hat{p}^K_{ti} = \max\{p_{1ti}, p_{2ti}, ..., p_{K-1ti}\}$ if $\max\{p_{1ti}, p_{2ti}, ..., p_{K-1ti}\} \geq \hat{p}^K_{t-Hi}$; where H is the minimum non empty value of $\hat{p}^K_{t-Hi}$.

(c) Delete all values p_{Jti} s.t. $\hat{p}^K_{ti} = p_{Jti}$ for $J < K$.

(d) Set $K = K - 1$ and go to (b) if $K = 2$.

After applying this filter, a good is a different item, shop and location for the price quotes $\{\{\hat{p}^k_{ti}\}^{n(i)}_{k=1}\}_{ti}$. At the time of applying the algorithm, it is important to keep track of the indicator box to filter the data with sales, comparable substitutions, etc. Additionally, for each price quote I generate a weight with the amount of repetitions across id—this is important to generate micro-price statistics. For example, if for $id = 1$ and date=1997m1 there are three price 1, 1 and 1.5, then the weight for 1 is 2/3 and the weight for 1.5 is 1/3.

4. The last problem I deal with is missing observations of price quotes. To complete different missing observations I follow the next criteria: if more than 12 months of observations are missing, then the first price quote is a new item. Otherwise, I completed missing observations with the last available price.

5. The database "pricequotes.dta" has 6 variables: id, item-id, price, indicator-box, date and weights. The last variable helps to construct the weights at id level.

- **Step 2**: This step downloads the additional data. The output of this step is "CPI-weights.dta" with weights data at class, group and section levels, "weights.dta" with items weights, "CPI-item.dta" with items CPI, "CPI-Class" " with class level CPI and "Coicop.dta" with the mapping between item id and COICOP id.

1. I download the classification file with item-id together with COICOP classification number and definition. Notice that the items are not fixed to a COICOP category, since the COICOPs are updating whenever there is new information. For example, the same item can have two COICOPs at two different moments of time. For this reason, I generate a panel data of item id and monthly date for the merge.

2. Download aggregate, sector, group and class CPI.

3. Download item weights. In general weights are annual, but sometimes there are monthly or seasonal changes in weights. For this reason, I generate a panel data of item id and monthly dates before the merge.

4. Download item CPI. I keep $index - algorithm$, $stratum - ind$, $item - index$, $all - gm - index$ and $gm - ra - index$, to keep track of item CPI's construction method.

5. Finally, I download sector, group, and class weights to generate aggregate statistics, since I don't have price quotes of all the classes in the economy.

6. For merging between price quotes to COICOP classification, I complete missing COICOP with the mode in the COICOP classifications by item.

- **Step 3**: Merging the data. The output of this step is "UK1.dta".

1. First I merge CPI-class with CPI-weights and save this database as "CPI-ClassWeights.dta". Then I merge this database with the price quotes.

2. For item id 212301, 212302, 212305, 212307, 212308, 212309,212310, 212311 and 630106, there seem to be some problems with the weighting due to some errors in the data base. I assign weights for each case.

3. For item id 212301, 212302, 212305, 212307, 212309, 212310, 212311, 440111, 520309, 520313 and 630106 there are some problems with their respective COICOP. I fix each case.

4. For merging between price quotes to COICOP classification, I complete missing COICOP with the mode in the COICOP classifications by item.

- **Step 4**: Generate weights, price change distribution and stopping time distribution. The output is "UK2.dta".

1. Let j denote the index for item j with weight z_{jt}. I generate the weight of each id denoted with i with the following formula

$$\tilde{\omega}_{it}^{1} = z_{j(i)i} \frac{w_{ti}}{\sum_{i \in \mathcal{I}(j(i))} w_{ti}} \quad \tilde{\omega}_{it}^{1} = \frac{\tilde{\omega}_{it}^{1}}{\sum_{i} \tilde{\omega}_{it}^{1}} \tag{C.1}$$

where $J(i)$ denote the item of id i and $\mathcal{I}(j)$ all the ids of item j. I generate also the total weight by class in the COICOP classification to compute the median of price changes across classes.

2. For each id I, I construct the stopping times and price changes associated to the prices changes of the id i. Hence, the output of this step is the weights, the price changes and the stopping time distribution.

C.3 Computation of Micro-Price Statistics

Table VI describes the price statistics with each filter using CPI weights.

Table VI – Micro-Price Statistics for UK

	Filter			
moments	I	II	III	IV
Absolute Value of Price Change				
mean	0.105	0.105	0.101	0.122
Standar Deviation	0.125	0.126	0.109	0.111
Skewness	2.305	2.340	1.916	1.360
Kurtosis	9.196	9.332	6.995	4.546
5th percentile	0.007	0.007	0.004	0.008
10th percentile	0.013	0.013	0.008	0.015
25th percentile	0.024	0.025	0.025	0.039
50th percentile	0.057	0.057	0.063	0.090
75th percentile	0.134	0.134	0.134	0.172
90th percentile	0.263	0.263	0.245	0.284
95th percentile	0.368	0.375	0.340	0.359
Price Change				
mean	0.027	0.031	0.011	0.011
Standar Deviation	0.161	0.161	0.148	0.165
Skewness	0.228	0.252	0.163	-0.007
Kurtosis	7.577	7.756	5.787	3.941
5th percentile	-0.228	-0.224	-0.233	-0.270
10th percentile	-0.136	-0.131	-0.152	-0.190
25th percentile	-0.025	-0.018	-0.055	-0.081
50th percentile	0.024	0.027	0.010	0.014
75th percentile	0.077	0.080	0.073	0.098
90th percentile	0.184	0.186	0.166	0.211
95th percentile	0.288	0.288	0.260	0.295
Frequency	0.119	0.102	0.102	0.102
Expected Time	8.179	8.936	8.936	8.935

D GMM Estimation With Price Statistics under Different Filters

Table VII describes the estimated parameters with micro-price statistics with different filters. The quantitative results are the following:

- **Filter I:** With this filter idiosyncratic shocks are twice as large than with the final micro-price statistics—filter 3. Additionally, menu cost are larger than filter 2 and 4, since the model needs to generate large prices. The ratio of free price adjustments over total price adjustment is 45% larger than filter 3.
- **Filter II:** With this filter idiosyncratic shocks are twice as large than with the final micro-price statistics—filter 3. Additionally, menu cost are larger than filter 2 and the ratio of free price adjustments over total price adjustment is 71% larger than filter 3.
- **Filter IIIb:** This filter puts zero weight to the physical cost of menu cost and it finds similar parameters with positive weight to this moment.

Table VII – GMM Estimation with Different Filters at 2 Percent Inflation Target

moments	Filter I	(Data,model) Filter II	Filter IVa	Filter IVb
Absolute Value of Price Change				
mean	(0.105,0.098)	(0.105,0.104)	(0.122,0.129)	(0.122,0.121)
Standar Deviation	(0.125,0.113)	(0.126,0.117)	(0.111,0.120)	(0.111,0.114)
Skewness	(2.305,2.372)	(2.340,2.354)	(1.360,1.207)	(1.360,1.243)
Kurtosis	(9.196,9.505)	(9.332,9.638)	(4.546,4.128)	(4.546,4.245)
5th percentile	(0.007,0.003)	(0.007,0.003)	(0.008,0.004)	(0.008,0.003)
10th percentile	(0.013,0.008)	(0.013,0.008)	(0.015,0.009)	(0.015,0.008)
25th percentile	(0.024,0.023)	(0.025,0.027)	(0.039,0.029)	(0.039,0.028)
50th percentile	(0.057,0.059)	(0.057,0.064)	(0.090,0.106)	(0.090,0.109)
75th percentile	(0.134,0.146)	(0.134,0.140)	(0.172,0.193)	(0.172,0.179)
90th percentile	(0.263,0.192)	(0.263,0.224)	(0.284,0.308)	(0.284,0.291)
95th percentile	(0.368,0.354)	(0.375,0.363)	(0.359,0.374)	(0.359,0.356)
Price Change				
mean	(0.027,0.016)	(0.031,0.011)	(0.011,0.016)	(0.011,0.014)
Standar Deviation	(0.161,0.149)	(0.161,0.156)	(0.165,0.176)	(0.165,0.166)
Skewness	(0.228,-0.111)	(0.252,-0.120)	(-0.007,-0.103)	(-0.007,-0.083)
Kurtosis	(7.577,7.582)	(7.756,7.416)	(3.941,3.830)	(3.941,3.911)
5th percentile	(-0.228,-0.192)	(-0.224,-0.219)	(-0.270,-0.294)	(-0.270,-0.282)
10th percentile	(-0.136,-0.148)	(-0.131,-0.149)	(-0.190,-0.212)	(-0.190,-0.199)
25th percentile	(-0.025,-0.038)	(-0.018,-0.048)	(-0.081,-0.075)	(-0.081,-0.066)
50th percentile	(0.024,0.013)	(0.027,0.012)	(0.014,0.014)	(0.014,0.013)
75th percentile	(0.077,0.079)	(0.080,0.081)	(0.098,0.112)	(0.098,0.111)
90th percentile	(0.184,0.152)	(0.186,0.152)	(0.211,0.234)	(0.211,0.212)
95th percentile	(0.288,0.199)	(0.288,0.225)	(0.295,0.316)	(0.295,0.300)
Frequency of Price Change	(0.119,0.102)	(0.102,0.119)	(0.102,0.100)	(0.102,0.102)
Time of Price Change	(8.179,9.765)	(8.936,8.381)	(8.935,9.955)	(8.935,9.781)
Cost of Price Adjustment*100	(0.600,0.573)	(0.600,0.563)	(0.600,0.604)	(0.600,0.585)
Ratio Free/Costly Price Adjustment	(-,0.704)	(-,0.689)	(-,0.478)	(-,0.504)
Estimated Parameters				
σ_{ah}	0.362	0.299	0.208	0.199
σ_{al}	0.026	0.028	0.015	0.016
p	0.018	0.026	0.070	0.066
θ	0.496	0.399	0.305	0.305
hz	0.072	0.082	0.048	0.052

Columns 1 to 3 describes GMM price statistics using target micro-price statistic with the filter 1 to 3. Column 4 describes the GMM estimation with zero weight on the physical cost of price changes.

E Heterogeneity and Aggregation in the Calvo, Menu Cost and Taylor Models

In this section of the online appendix I show in form close solution how to aggregate heterogeneous firms whenever their policy is described by Menu Cost, Calvo or Taylor pricing models. Assume a finite amount of firms denoted with i, with $i = 1, 2, ..., N$. Each firm produces α_i share of the output. For simplicity, I assume continuous time. See Baley and Blanco (2013) for similar ideas in the computation of the aggregate hazard rates.

- **Menu Cost:** Each firm is described by a set of parameters $\{\sigma_i, \bar{p}_i\}$. The price gap follows a Brownian motion $dp_t^i = \sigma_i dW_t^i$ and the inaction region $\mathcal{I}$ is described by the set $[-\bar{p}_i, \bar{p}_i]$. The distribution of price changes is given by

$$\Delta p_i = \begin{cases} \bar{p}_i & \text{with probability } 1/2 \\ -\bar{p}_i & \text{with probability } 1/2 \end{cases} \tag{E.2}$$

  with a kurtosis of 1. The aggregate price change distribution is given by

$$\Delta p = \begin{cases} \bar{p}_i & \text{with probability } \alpha_i 1/2 \\ -\bar{p}_i & \text{with probability } \alpha_i 1/2 \end{cases} \tag{E.3}$$

  with a kurtosis higher than 1. The normalized price change is defined as $\Delta p_i = \frac{\Delta p_i - \mathbb{E}[\Delta p_i]}{\mathbb{S}td[\Delta p_i]} \mathbb{S}td[\Delta p] + \mathbb{E}[\Delta p]$ is given by

$$\Delta p = \begin{cases} \mathbb{S}td[\Delta p] & \text{with probability } 1/2 \\ -\mathbb{S}td[\Delta p] & \text{with probability } 1/2 \end{cases} \tag{E.4}$$

  with kurtosis equal to 1.

- **Calvo:** Each firm is described by a set of parameters $\{\sigma_i, \lambda_i\}$. The price gap follows a Brownian motion $dp_t^i = \sigma_i dW_t^i$ and firm's stopping time distribution follows an exponential distribution given by $f(\tau) = \lambda_i e^{-\tau/\lambda_i}$. The distribution of price changes is given by

$$f(\Delta p_i) = \frac{\sqrt{\lambda_i 0.5}}{\sigma_i} \exp\left(-\frac{\sigma_i}{\sqrt{\lambda_i 0.5}} |\Delta p|\right) \tag{E.5}$$

  with a kurtosis of 6. The aggregate price change distribution is given by

$$f(\Delta p) = \begin{cases} f(\Delta p_i) & \text{with probability } \alpha_i \end{cases} \tag{E.6}$$

  with a kurtosis higher than 6. The normalized price change defined as $\Delta p_i = \frac{\Delta p_i - \mathbb{E}[\Delta p_i]}{\mathbb{S}td[\Delta p_i]} \mathbb{S}td[\Delta p] + \mathbb{E}[\Delta p]$ is given by

$$f(\Delta p) = \exp\left(-|\Delta p|\right) \tag{E.7}$$

  with kurtosis equal to 6. The normalized price change is defined as $\Delta p_i = \frac{\Delta p_i - \mathbb{E}[\Delta p_i]}{\mathbb{S}td[\Delta p_i]} \mathbb{S}td[\Delta p] + \mathbb{E}[\Delta p]$ is given by

$$f(\Delta p) = \frac{1}{\sqrt{2}\mathbb{S}td[\Delta p]} \exp\left(-\frac{\sqrt{2}|\Delta p|}{\mathbb{S}td[\Delta p]}\right) \tag{E.8}$$

  with kurtosis equal to 6.

- **Taylor:** Each firm is described by a set of parameters $\{\sigma_i, T_i\}$. The price gap follows a Brownian motion $dp_t^i = \sigma_i dW_t^i$ and firm's stopping time distribution is a constant $Pr(\tau = T_i) = 1$. The distribution of price changes is given by

$$f(\Delta p_i) = \mathcal{N}(\Delta p, 0, \sigma_i) \tag{E.9}$$

with a kurtosis of 3. The aggregate price change distribution is mixed normal distribution

$$f(\Delta p) = \Big\{ \ \mathcal{N}(\Delta p, 0, \sigma_i) \quad \text{with probability } \alpha_i \tag{E.10}$$

with a kurtosis higher than 3. The normalized price change is defined as $\Delta p_i = \frac{\Delta p_i - \mathbb{E}[\Delta p_i]}{\mathbb{S}td[\Delta p_i]} \mathbb{S}td[\Delta p] + \mathbb{E}[\Delta p]$ is given by

$$f(\Delta p) = \mathcal{N}(\Delta p, 0, \mathbb{S}td[\Delta p]) \tag{E.11}$$

with kurtosis equal to 3.

F Calvo Model

An important problem in the Calvo model with positive trend inflation is that the model does not satisfy the Blanchard and Khan conditions. For this reason, I add business cycle indexation: nominal price adjust automatically to business cycle fluctuations of inflation. If p_{ti} is the nominal price of the firm i at time t, then

$$p_{it+1} = \begin{cases} \left(\frac{\Pi_{t+1}}{(1+\bar{\pi})}\right)^{\lambda} p_{it} & \text{if no price change with prob } 1-\Omega \\ P_t^* & \text{if price change with prob } \Omega \end{cases} \tag{F.12}$$

Notice that this form of indexation does not affect the steady state of the Calvo economy.

F.1 Derivation of Equilibrium Conditions

- Price dispersion with no idiosyncratic shocks:

$$\Delta_t = \left[\int \tilde{p}_{ti}^{-\gamma} di\right] \tag{F.13}$$

$$= [\Omega(P_t^*)^{-\gamma} + \int_{1-\Omega} \tilde{p}_{ti}^{-\gamma} di] \tag{F.14}$$

$$= [\Omega(P_t^*)^{-\gamma} + \int_{1-\Omega} \left(\frac{\tilde{p}_{t-1,i}}{\Pi_t}\left(\frac{\Pi_t}{(1+\bar{\pi})}\right)^{\lambda}\right)^{-\gamma} di] \tag{F.15}$$

$$= [\Omega(p_t^*)^{-\gamma} + (1+\bar{\pi})^{\gamma\lambda}\Pi_t^{\gamma(1-\lambda)}(1-\Omega)\Delta_{t-1} \tag{F.16}$$

- Cost function and feasibility: The cost function is given by

$$\min(1-\tau_{mc})\left[W_t l_t + P_t N_t\right] \qquad s.t. \qquad \eta_{zt} N_{ti}^{\alpha}\left(\Gamma_t l_{ti}\right)^{1-\alpha} = y_{ti} \tag{F.17}$$

Define w_t as the real wage. The demand functions, the total cost and the marginal cost are given by

$$N_{ti} = \frac{y_{ti}}{\eta_{zt}\Gamma_t^{1-\alpha}} w_t^{1-\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \tag{F.18}$$

$$l_{ti} = \frac{y_{ti}}{\eta_{zt}\Gamma_t^{1-\alpha}} w_t^{-\alpha}\left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \tag{F.19}$$

$$totalcost_t = (1-\tau_{mc})\frac{y_{ti}}{\eta_{zt}\Gamma_t^{1-\alpha}} w_t^{1-\alpha}\left(\left(\frac{1-\alpha}{\alpha}\right)^{\alpha} + \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\right) \tag{F.20}$$

$$mc_t = \frac{1-\tau_{mc}}{\eta_{zt}\Gamma_t^{1-\alpha}} w_t^{1-\alpha}\left(\left(\frac{1-\alpha}{\alpha}\right)^{\alpha} + \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\right) \tag{F.21}$$

Let define $\iota = \left((\frac{1-\alpha}{\alpha})^{\alpha} + \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\right)$ then

$$mc_t = \frac{1-\tau_{mc}}{\eta_{zt}\Gamma_t^{1-\alpha}} w_t^{1-\alpha} \tag{F.22}$$

To obtain total labor demand, note that

$$\int_0^1 l_{t,i} di = \int_0^1 \left(\frac{y_{ti}}{\eta_{zt}\Gamma_t^{1-\alpha}} w_t^{-\alpha}\left(\frac{1-\alpha}{\alpha}\right)^{\alpha}\right) di \tag{F.23}$$

$$= \left(\frac{(1-\alpha)}{\alpha w_t}\right)^{\alpha} \frac{1}{\Gamma_t^{1-\alpha}\eta_{zt}} \int_0^1 (y_{t,i})\, di \tag{F.24}$$

$$= \frac{Y_t}{\eta_{zt}\Gamma_t^{1-\alpha}}\left(\frac{(1-\alpha)}{\alpha w_t}\right)^{\alpha} \int_0^1 \tilde{p}_{t,i}^{-\gamma} di \tag{F.25}$$

$$= \frac{Y_t}{\eta_{zt}\Gamma_t^{1-\alpha}}\left(\frac{(1-\alpha)}{\alpha w_t}\right)^{\alpha} \Delta_t \tag{F.26}$$

To obtain feasibility

$$
\begin{aligned}
\eta_{gt} + C_t &= Y_t - \int N_{t,i} dt && \text{(F.27)}\\
&= Y_t \left(1 - \left(\frac{w_t \alpha}{1-\alpha}\right)^{1-\alpha} \frac{\Delta_t}{\Gamma_t^{1-\alpha}\eta_{zt}}\right) && \text{(F.28)}
\end{aligned}
$$

F.2 Calvo Model Equilibrium Conditions

Let X denote the detrended variable and $\tilde{X}$ be the original variable.

- Stationary household optimality conditions

$$mu(S) = \beta(1+g)^{-\sigma_{np}} \eta_{q-}(S) R(S) \mathbb{E}_{S'} \left[\left(\frac{U(S')/U_{ss}}{\Sigma(S)^{\frac{1}{1-\sigma_{ez}}}} \right)^{-\sigma_{ez}} \frac{mu(S')}{\Pi(S')} \middle| S \right] \tag{F.29}$$

$$\Sigma(S) = \mathbb{E}_{S'} \left[\left(\frac{U(S')}{U_{ss}} \right)^{1-\sigma_{ez}} \middle| S \right] \tag{F.30}$$

$$mu(S) = \bar{u}(C(S) - \kappa \frac{L(S)^{1+\chi}}{1+\chi})^{-\sigma_{np}} \tag{F.31}$$

$$\kappa L(S)^{\chi} = w(S) \tag{F.32}$$

$$u(S) = \bar{u} \frac{\left(C(S) - \kappa \frac{L(S)^{1+\chi}}{1+\chi}\right)^{1-\sigma_{np}}}{1-\sigma_{np}} \tag{F.33}$$

$$U(S) = (1-\beta) u(S) + \beta g^{1-\sigma_{np}} U_{ss} \Sigma(S)^{\frac{1}{1-\sigma_{ez}}} \tag{F.34}$$

  - Where the original variable can be obtain as:

$$
\begin{gathered}
\tilde{C}_t = C(S_t)(1+g)^t \quad ; \quad \tilde{mu}_t = mu(S_t)(1+g)^{-\sigma_{np} t} \quad ; \quad \tilde{L}_t = L_t \quad ; \quad \tilde{\Pi}_t = \Pi(S) \\
\tilde{u}_t = u(S_t)(1+g)^{(1-\sigma_{np})t} \quad ; \quad \tilde{U}_t = U(S_t)(1+g)^{(1-\sigma_{np})t} \quad ; \quad \tilde{w}_t = w(S_t)(1+g)^t
\end{gathered} \tag{F.35}
$$

- Stationary firms optimality condition, inflation and price dispersion

$$
\begin{aligned}
H(S) &= \frac{\gamma}{\gamma-1}(1-\beta) mu(S) Y(S) mc(S) + \ldots \\
&\ldots + \beta(1+g)^{1-\sigma_{np}}(1-\Omega)\mathbb{E}_{S'} \left[\left(\Pi(S')^{(1-\lambda)\gamma}(1+\bar{\pi})^{\lambda\gamma}\right) \left(\frac{\tilde{U}(S')/U_{ss}}{\tilde{\Sigma}(S)^{\frac{1}{1-\sigma_{ez}}}} \right)^{-\sigma_{ez}} H(S') \middle| S \right] \\
F(S) &= mu(S) Y(S)(1-\beta) + \ldots \\
&\ldots + \beta(1+g)^{1-\sigma_{np}}(1-\Omega)\mathbb{E}_{S'} \left[\left(\Pi(S')^{(1-\lambda)(\gamma-1)}(1+\bar{\pi})^{\lambda(\gamma-1)}\right) \left(\frac{U(S')/U_{ss}}{\Sigma(S)^{\frac{1}{1-\sigma_{ez}}}} \right)^{-\sigma_{ez}} F(S') \middle| S \right] \\
P^*(S) &= \frac{H(S)}{F(S)} \\
P^*(S) &= \left[\frac{1-(1-\Omega)(1+\bar{\pi})^{\lambda(\gamma-1)} \Pi(S)^{(1-\lambda)(\gamma-1)}}{\Omega} \right]^{\frac{1}{1-\gamma}} \\
\Delta(S) &= \left(\Omega \left(P(S)^*\right)^{-\gamma} + (1-\Omega) \left(\Pi(S)^{(1-\lambda)\gamma} (1+\bar{\pi})^{\lambda\gamma}\right) \Delta_-(S) \right)
\end{aligned}
$$

  - $\tilde{mc}_t = mc(S_t), \tilde{Y}_t = Y_t(1+g)^t, \tilde{P}_t^* = P_t^*(S_t), \tilde{H}_t = H(S_t)(1+g)^{(1-\sigma_{np})t}, \tilde{F}_t = F(S_t)(1+g)^{(1-\sigma_{np})t}$

- Stationary monetary policy and aggregate feasibility with change of variable

$$mc(S) = (1-\tau_{mc})\iota \frac{w(S)^{1-\alpha}}{\eta_{z-}(S)} \tag{F.36}$$

$$\mathcal{M}(S) = \frac{1-\tau_{mc}}{mc(S)} \tag{F.37}$$

$$R_t^*(S) = \tilde{R}_-(S)\left(\frac{1+\bar{\pi}}{\beta g^{-\sigma_{np}}}\right)\left(\left(\frac{\Pi(S)}{(1+\bar{\pi})}\right)^{\phi_\pi}\left(\frac{mc(S)}{mc_{ss}}\right)^{\phi_y}\right)^{1-\phi_r}\left(\frac{mc(S)}{mc_-(S)}\right)^{\phi_{dy}} \tag{F.38}$$

$$\tilde{R}(S') = \left(\left(\frac{\beta g^{-\sigma_{np}}}{1+\bar{\pi}}\right)R^*(S)\right)^{\phi_r}\eta_r' \tag{F.39}$$

$$R(S) = \max\{1, R^*(S)\} \tag{F.40}$$

$$\eta_{z-}(S)L(S) = Y(S)\left(\frac{(1-\alpha)}{\alpha w(S)}\right)^{\alpha}\Delta(S) \tag{F.41}$$

$$\eta_{g-}(S)+C(S) = Y(S)\left(1-\left(\frac{w(S)\alpha}{1-\alpha}\right)^{1-\alpha}\frac{\Delta(S)}{\eta_{z-}(S)}\right) \tag{F.42}$$

- Exogenous shocks

$$\begin{aligned}
\log(\eta_g)(S') &= (1-\rho_g)\log(\eta_g^*)+\rho_g\log(\eta_g(S))+\sigma_g\epsilon_g' && \text{(F.43)}\\
\log(\eta_z)(S') &= (1-\rho_Z)\log(\eta_z^*)+\rho_z\log(\eta_z(S))+\sigma_z\epsilon_z' && \text{(F.44)}\\
\log(\eta_r)(S') &= \sigma_r\epsilon_r' && \text{(F.45)}\\
\log(\eta_q)(S') &= (1-\rho_q)\log(\eta_q^*)+\rho_q\log(\eta_{q(S)})+\sigma_q\epsilon_q && \text{(F.46)}\\
& && \text{(F.47)}
\end{aligned}$$

F.3 Solution Method for the Calvo Model

The solution method has in 3 steps:

1. Solve the model using 2-order order perturbation methods.
   - Compute the steady state.
   - Set the hypercube limits for the solution with projection methods.
   - Get initial conditions for the coefficients in the policies function using linear regression over the simulation.
2. Solve the model using global methods without the zero lower bound.
   - In this step I use an iterative method. The initial condition is given by the previous step.
3. Solve the model using global methods with the zero lower bound.
   - In this step I use an iterative method. The initial condition is given by the previous step.

F.3.1 Step 1: Perturbation Method Around the Steady State

First, I compute the steady state of the model

$$\eta_{q,ss} = 1 \quad ; \quad \eta_{z,ss} = \eta_z$$

$$\eta_{g,ss} = \eta_g \quad ; \quad \eta_{r,ss} = 1$$

$$P^*_{ss} = \left(\frac{1-(1-\Omega)(1+\bar{\pi})^{\gamma-1}}{\Omega}\right)^{1/(1-\gamma)} \quad ; \quad mc_{ss} = \frac{P^*_{ss}(\gamma-1)}{\gamma}\frac{1-\beta\eta_{q,ss}(1-\Omega)g^{1-\sigma_n}(1+\bar{\pi})^{\gamma}}{1-\beta\eta_{q,ss}g^{1-\sigma_n}(1-\Omega)(1+\bar{\pi})^{\gamma-1}}$$

$$w_{ss} = \left(\frac{mc_{ss}\eta_{z,ss}}{(1-\tau_{mc})\iota}\right)^{1/(1-\alpha)} \quad ; \quad L_{ss} = \left(\frac{w_{ss}}{\kappa}\right)^{\frac{1}{\chi}}$$

$$\Delta_{ss} = \frac{\Omega(P^*_{ss})^{-\gamma}}{1-(1-\Omega)\Pi^{\gamma}_{ss}} \quad ; \quad Y_{ss} = \eta_{z,ss}\frac{L_{ss}}{\left(\frac{1-\alpha}{\alpha w_{ss}}\right)^{\alpha}\Delta_{ss}}$$

$$C_{ss} = Y_{ss}\left(1-\left(\frac{w_{ss}\alpha}{1-\alpha}\right)^{1-\alpha}\frac{\Delta_{ss}}{\eta_{z,ss}}\right) - \eta_{g,ss} \quad ; \quad mu_{ss} = \bar{u}\left(C_{ss} - \kappa\frac{L^{1+\chi}_{ss}}{1+\chi}\right)^{-\sigma_{np}}$$

$$u_{ss} = \bar{u}\frac{(C_{ss}-\kappa\frac{L^{1+\chi}_{ss}}{1+\chi})^{1-\sigma_{np}}}{1-\sigma_{np}} \quad ; \quad U_{ss} = u_{ss}\frac{1-\beta}{1-\beta g^{1-\sigma_{np}}}$$

$$H_{ss} = \frac{\gamma}{\gamma-1}\frac{(1-\beta)mu_{ss}Y_{ss}mc_{ss}}{1-\beta g^{1-\sigma_n}(1-\Omega)(1+\bar{\pi})^{\gamma}} \quad ; \quad F_{ss} = (1-\beta)\frac{mu_{ss}Y_{ss}}{1-\beta g^{1-\sigma_n}(1-\Omega)(1+\bar{\pi})^{\gamma-1}}$$

$$R_{ss} = \frac{1+\bar{\pi}}{\beta g^{-\sigma_{np}}} \quad ; \quad \Pi_{ss} = (1+\bar{\pi})$$

$$\Sigma_{ss} = 1 \quad ; \quad \Sigma^2_{ss} = 1 \quad ; \quad \mathcal{M}_{ss} = (1-\tau_L)^{1-\alpha}/mc$$

Then, I solve the system of equation described in the previous section without ZLB with second order perturbation methods. For all the positive variables I apply the log transformation $X_t = X_{ss}e^{\hat{X}_t}$ and I didn't apply log transformation for variables that are negative.

- Positive variables:

$$\left\{mu, \Pi, \Sigma, \Sigma^2, C, L, w, S, Y, P^*, F, mc, \Delta, , R^*, R, \eta_G, \eta_Z, \eta_R, \eta_Q\right\}$$

- Non-positive variables: $\{u, U\}$

Household equilibrium conditions:

$$mu_{ss}e^{\hat{mu}_t} = \beta g^{-\sigma_n}\eta_{Qss}e^{\hat{\eta}_{Qt}}R_{ss}e^{\hat{R}_t}\mathbb{E}_t\left[\left(\frac{U_{t+1}/U_{ss}}{\Sigma^2_{ss}e^{\hat{\Sigma}^2_t}}\right)^{-\sigma_{ez}}\frac{mu_{ss}}{\Pi_{ss}}e^{\hat{mu}_{t+1}-\hat{\Pi}_{t+1}}\right] \tag{F.48}$$

$$\Sigma_{ss}e^{\hat{\Sigma}_t} = \left(\frac{U_{t+1}}{U_{ss}}\right)^{\frac{1}{1-\sigma_{ez}}} \tag{F.49}$$

$$\Sigma^2_{ss}e^{\hat{\Sigma}^2_t} = \left(\Sigma_{ss}e^{\hat{\Sigma}_t}\right)^{1/(1-\sigma_{ez})} \tag{F.50}$$

$$mu_{ss}e^{\hat{mu}_t} = \bar{u}\left(C_{ss}e^{\hat{C}_t} - \kappa\frac{(L_{ss}e^{\hat{L}_t})^{1+\chi}}{1+\chi}\right)^{-\sigma_{np}} \tag{F.51}$$

$$\kappa L^{\chi}_{ss}(e^{\hat{L}_t})^{\chi} = w_{ss}e^{\hat{w}_t} \tag{F.52}$$

$$\tilde{u}_t = \bar{u}\frac{\left(C_{ss}e^{\hat{C}_t} - \kappa\frac{(L_{ss}e^{\hat{L}_t})^{1+\chi}}{1+\chi}\right)^{1-\sigma_{np}}}{1-\sigma_{np}} \tag{F.53}$$

$$\tilde{U}_t = (1-\beta)\tilde{u}_t + \beta g^{1-\sigma_{np}}U_{ss}\Sigma^2_{ss}e^{\hat{\Sigma}^2_t} \tag{F.54}$$

$$\tag{F.55}$$

Firm equilibrium conditions

$$H_{ss}e^{\hat{H}}_t = \frac{\gamma}{\gamma-1}(1-\beta)mu_{ss}e^{\hat{mu}_t}Y_{ss}e^{\hat{Y}_t}mc_{ss}e^{\hat{mc}_t}+\dots \tag{F.56}$$

$$\dots+\beta g^{1-\sigma}(1-\Omega)\mathbb{E}_t\left[\Pi_{ss}^{\gamma}e^{\hat{\Pi}_{t+1}(1-\lambda)\gamma}\left(\frac{U_{t+1}/U_{ss}}{\Sigma_{ss}^2e^{\hat{\Sigma}_t^2}}\right)^{-\sigma_e z}H_{ss}e^{\hat{H}_{t+1}}\right] \tag{F.57}$$

$$F_{ss}e^{\hat{F}_t} = (1-\beta)mu_{ss}e^{\hat{mu}_t}Y_{ss}e^{\hat{Y}_t}+\dots \tag{F.58}$$

$$\dots+\beta g^{1-\sigma_{np}}(1-\Omega)\mathbb{E}_t\left[\Pi_{ss}^{(\gamma-1)}\left(e^{\hat{\Pi}_{t+1}(1-\lambda)(\gamma-1)}\right)\left(\frac{U_{t+1}/U_{ss}}{\Sigma_{ss}^2e^{\hat{\Sigma}_t^2}}\right)^{-\sigma_e z}F_{ss}e^{\hat{F}_{t+1}}\right] \tag{F.59}$$

$$P_{ss}^*e^{\hat{P}_t^*} = \frac{H_{ss}}{F_{ss}}e^{\hat{H}_t-\hat{F}_t} \tag{F.60}$$

$$P_{ss}^*e^{\hat{P}_t^*} = \left[\frac{1-(1-\Omega)\Pi_{ss}^{(\gamma-1)}e^{\hat{\Pi}_t(1-\lambda)(\gamma-1)}}{\Omega}\right]^{\frac{1}{1-\gamma}} \tag{F.61}$$

Aggregate equilibrium condition:

$$mc_{ss}e^{\hat{mc}_t} = \iota(1-\tau_{mc})\frac{(w_{ss})^{1-\alpha}e^{\hat{w}_t(1-\alpha)}}{\eta_{Zss}e^{\eta_{Zt}}} \tag{F.62}$$

$$\mathcal{M}_{ss}e^{\hat{M}_t} = \frac{1-\tau_{mc}}{mc_{ss}e^{\hat{mc}_t}} \tag{F.63}$$

$$\Delta_{ss}e^{\hat{\Delta}_t} = \left(\Omega\left(P_{ss}^*e^{\hat{P}_t^*}\right)^{-\gamma}+(1-\Omega)\Pi_{ss}^{\gamma}\left(e^{\hat{\Pi}_t(1-\lambda)\gamma}\right)\Delta_{ss}e^{\hat{\Delta}_{t-1}}\right) \tag{F.64}$$

$$R_{ss}e^{\hat{R}_t^*} = \left(\frac{1+\bar{\pi}}{\beta g^{-\sigma_{np}}}\right)R_{ss}e^{\hat{R}_t}\left(\left(e^{\hat{\Pi}_t}\right)^{\phi_\pi}\left(e^{\hat{mc}_t}\right)^{\phi_y}\right)^{1-\phi_r}\left(e^{\hat{mc}_t-\hat{mc}_{t-1}}\right)^{\phi_{dy}} \tag{F.65}$$

$$R_{ss}e^{\hat{R}_t} = \left(\frac{R_{ss}^*e^{R_{t-1}^*}\beta g^{-\sigma_{np}}}{1+\bar{\pi}}\right)^{\phi_r}\eta_{Rss}e^{\hat{\eta}_{Rt}} \tag{F.66}$$

$$\eta Zsse^{\hat{\eta}_{Zt}}L_{ss}e^{\hat{L}_t} = \hat{Y}_{ss}e^{\hat{Y}_t}\left(\frac{(1-\alpha)}{\alpha w_{ss}e^{\hat{w}_t}}\right)^{\alpha}\Delta_{ss}e^{\hat{\Delta}_t} \tag{F.67}$$

$$\eta_{Gss}e^{\hat{\eta}_{Gt}}+e^{\hat{C}_t}C_{ss} = \hat{Y}_{ss}e^{\hat{Y}_t}\left(1-\left(\frac{w_{ss}e^{\hat{w}_t}\alpha}{1-\alpha}\right)^{1-\alpha}\frac{\Delta_{ss}e^{\hat{\Delta}_t}}{\eta_{Zss}e^{\hat{\eta}_{Zt}}}\right) \tag{F.68}$$

$$\tag{F.69}$$

Structural shocks

$$\hat{\eta}_{Gt} = \rho_G\hat{\eta}_{Gt-1}+\sigma_G\epsilon_{Gt} \tag{F.70}$$

$$\hat{\eta}_{Zt} = \rho_Z\hat{\eta}_{Zt-1}+\sigma_Z\epsilon_{Zt} \tag{F.71}$$

$$\hat{\eta}_{Rt} = \sigma_R\epsilon_{Rt} \tag{F.72}$$

$$\hat{\eta}_{Qt} = \rho_Q\hat{\eta}_{Qt-1}+\bar{\sigma}_Q\epsilon_{Qt} \tag{F.73}$$

Remark 1 *It is important to take perturbation at log steady state for high persistence shocks .*

Remark 2 *It is important to normalize the derivative with respect to $U_s s$ the discount factor. If not, there are large numerical errors in the first and second derivatives.*

F.3.2 Step 2: Solving the model without ZLB

I use an iterative method to find the equilibrium.

- **Step 2.1**: Initiate

$$mu^0(S),R^0(S),\tilde{R}^0(S),\Pi^0(S),U^0(S),H^0(S),F^0(S),\Delta^0(S) \tag{F.74}$$

using the second order perturbation methods.

- **Step 2.2**: Given $mu^i(S), \tilde{R}^i(S), \Pi^i(S), U^i(S), H^i(S), F^i(S), \Delta^i(S)$, I use the Euler equation to get

$$\Sigma^{i+1}(S) = \mathbb{E}_{S'}\left[\left(\frac{U^i(S')}{U_{ss}}\right)^{1-\sigma_{ez}}\middle| S\right] \tag{F.75}$$

$$mu^{i+1}(S) = \beta g^{-\sigma_n}\eta_q(S)R^i(S)\mathbb{E}_{S'}\left[\left(\frac{U^i(S')/U_{ss}}{(\Sigma^{i+1}(S))^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}}\frac{mu^i(S')}{\Pi^i(S')}\middle| S\right] \tag{F.76}$$

- **Step 2.3**: With $mu(S)^{i+1}(S)$ I solve the following system

$$\kappa L^{i+1}(S)^\chi = w^{i+1}(S) \tag{F.77}$$

$$mu^{i+1}(S) = \bar{u}(C^{i+1}(S) - \kappa\frac{L^{i+1}(S)^{1+\chi}}{1+\chi})^{-\sigma_n} \tag{F.78}$$

$$\eta_z(S)L^{i+1}(S) = Y^{i+1}(S)\left(\frac{(1-\alpha)}{\alpha w^{i+1}(S)}\right)^\alpha \Delta^i(S) \tag{F.79}$$

$$\eta_g(S) + C^{i+1}(S) = Y^{i+1}(S)\left(1-\left(\frac{w^{i+1}(S)\alpha}{1-\alpha}\right)^{1-\alpha}\frac{\Delta^i(S)}{\eta_z(S)}\right) \tag{F.80}$$

Compute the marginal cost and period utility given by

$$mc^{i+1}(S) = (1-\tau_{mc})\iota\frac{\left(w^{i+1}(S)\right)^{1-\alpha}}{\eta_z(S)} \qquad u^{i+1}(S) = \bar{u}\frac{(C^{i+1}(S) - \kappa\frac{L^{i+1}(S)^{1+\chi}}{1+\chi})^{1-\sigma_{np}}}{1-\sigma_{np}} \tag{F.81}$$

- **Step 2.4**: Update forward looking policies $U(S), H(S), F(S)$

$$\begin{aligned}
U^{i+1}(S) &= (1-\beta)u^{i+1}(S) + \beta g^{1-\sigma_{np}}U_{ss}\Sigma^{i+1}(S)^{\frac{1}{1-\sigma_{ez}}}\\
H^{i+1}(S) &= \frac{\gamma}{\gamma-1}(1-\beta)mu^{i+1}(S)Y^{i+1}(S)mc^{i+1}(S) + \dots\\
&\dots + \beta g^{1-\sigma_{np}}(1-\Omega)\mathbb{E}_{S'}\left[\left(\frac{U^i(S')/U_{ss}}{(\Sigma^i(S))^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}}\left((\Pi^i(S'))^{(1-\lambda)\gamma}(1+\bar{\pi})^{\lambda\gamma}\right)H^i(S')\middle| S\right]\\
F^{i+1}(S) &= mu^{i+1}(S)Y^{i+1}(S)(1-\beta) + \dots\\
&\dots + \beta g^{1-\sigma_{np}}(1-\Omega)\mathbb{E}_{S'}\left[\left(\frac{(U^i(S'))/U_{ss}}{(\Sigma^i(S))^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}}\left((\Pi^i(S'))^{(1-\lambda)(\gamma-1)}(1+\bar{\pi})^{\lambda(\gamma-1)}\right)F^i(S')\middle| S\right]
\end{aligned}$$

- **Step 2.5**: Update backward looking policies $R(S), \tilde{R}^0(S), \Pi^0(S), \Delta^0(S)$

$$\begin{aligned}
\Pi^{i+1}(S) &= \left(\frac{1-\Omega}{1-\Omega(((P^*)^{i+1}(S)))^{1-\gamma}}\right)^{1/((1-\gamma)(1-\lambda))}(1+\bar{\pi})^{\frac{-\lambda}{1-\lambda}}\\
\Delta^{i+1}(S) &= \left(\Omega\left((P^*)^{i+1}(S))\right)^{-\gamma} + (1-\Omega)\left(\Pi^{i+1}((S))^{(1-\lambda)\gamma}(1+\bar{\pi})^{\lambda\gamma}\right)\Delta^i(S)\right)\\
(R^*)^{i+1}(S) &= \tilde{R}_-(S)\left(\frac{1+\bar{\pi}}{\beta g^{-\sigma_{np}}}\right)\left(\left(\frac{\Pi^{i+1}(S)}{(1+\bar{\pi})}\right)^{\phi_\pi}\left(\frac{\tilde{mc}^{i+1}(S)}{mc_{ss}}\right)^{\phi_y}\right)^{1-\phi_r}\left(\frac{mc^{i+1}(S)}{\tilde{mc}(S)}\right)^{\phi_{dy}}\\
\tilde{R}^{i+1}(S) &= \left(\left(\frac{\beta g^{-\sigma_{np}}}{1+\bar{\pi}}\right)(R^*)^{i+1}(S)\right)^{\phi_r} R^{i+1}(S) = (R^*)^{i+1}(S);
\end{aligned}$$

- **Step 2.6**: If

$$error = mean_S\left(|mu^i(S) - mu^{i+1}(S)| + |\Pi^i(S) - \Pi^{i+1}(S)| + |U^i(S) - U^{i+1}(S)|\right) < \epsilon \tag{F.82}$$

Then stop. Otherwise go back to step 2.2.

Remark 3 *The projection method that I use to solve the equilibrium is Smoliak sparse grid with anisotropic construction.*

Remark 4 *To construct the expectation I use Gauss-legendre quadrature and I use the inverse of the accumulated normal in the nodes to recover the nodes in the normal distribution.*

Remark 5 *In step 3 I use Golden search to obtain the labor supply. Specifically, labor supply in the i iteration satisfies*

$$L^{i+1}(S) = \arg\min_X \left(mu^i(S) - (C(X) - \kappa \frac{X(S)^{1+\chi}}{1+\chi})^{-\sigma_n} \right)^2 \tag{F.83}$$

with

$$C(X) = \frac{\eta_z(S)}{\Delta^i(S)} X \left(\frac{\kappa X^\chi \alpha}{1-\alpha} \right)^\alpha \left(1 - \left(\frac{\kappa X^\chi \alpha}{1-\alpha} \right)^{1-\alpha} \frac{\Delta^i(S)}{\eta_z(S)} \right) - \eta_g(S) \tag{F.84}$$

F.3.3 Step 3: Solving the model with ZLB

I repeat the same algorithm as in step 2 using as the initial condition the policy function of the model without zero lower bound with global solution. The only difference comes in step 2.4 where I solve the following system:

$$\begin{aligned}
\Delta^{i+1}(S) &= \left(\Omega \left((P^*)^{i+1}(S) \right) \right)^{-\gamma} + (1-\Omega) \left(\Pi^{i+1}((S))^{(1-\lambda)\gamma} (1+\bar{\pi})^{\lambda\gamma} \right) \Delta^i(S) \Big) \\
(R^*)^{i+1}(S) &= \tilde{R}_-(S) \left(\frac{1+\bar{\pi}}{\beta g^{-\sigma_{np}}} \right) \left(\left(\frac{\Pi^{i+1}(S)}{(1+\bar{\pi})} \right)^{\phi_\pi} \left(\frac{\tilde{mc}^{i+1}(S)}{mc_{ss}} \right)^{\phi_y} \right)^{1-\phi_r} \left(\frac{mc^{i+1}(S)}{\tilde{mc}(S)} \right)^{\phi_{dy}} \\
\tilde{R}^{i+1}(S) &= \left(\left(\frac{\beta g^{-\sigma_{np}}}{1+\bar{\pi}} \right) (R^*)^{i+1}(S) \right)^{\phi_r} \\
R^{i+1}(S) &= \max\left\{ 1, (R^*)^{i+1}(S) \right\}
\end{aligned}$$

G CalvoPlus Model

This section describes the computation of the CalvoPlus model in the steady state and with business cycle.

G.1 CalvoPlus Model: Steady State

G.1.1 CalvoPlus Model Equilibrium Equations

The equilibrium equations for the steady state are given by

- Household optimality conditions

$$mu_{ss} = K(C_{ss} - \kappa \frac{L_{ss}^{1+\chi}}{1+\chi})^{-\sigma_{np}} \tag{G.85}$$

$$\kappa L_{ss}^{\chi} = w_{ss} \tag{G.86}$$

$$u_{ss} = \kappa \frac{\left(C_{ss} - \kappa \frac{L_{ss}^{1+\chi}}{1+\chi}\right)^{1-\sigma_n}}{1-\sigma_n} \tag{G.87}$$

$$U_{ss} = (1-\beta)u_{ss} + \beta\,(1+g)^{1-\sigma_{np}}\,U_{ss} \tag{G.88}$$

- Stationary idiosyncratic conditions

$$v(\tilde{p}) = \mathbb{E}_{\tilde{p}'}\left[\left((1-hz)\max_{c,nc}\left\{\max_{\tilde{p}} V(\tilde{p}) - \theta w_{ss} mu_{ss}, V(\tilde{p}'(\tilde{p}))\right\} + hz \max_{\tilde{p}} V(\tilde{p})\right)\right] \tag{G.89}$$

$$V(\tilde{p}) = \Phi(\tilde{p}) + \beta g^{1-\sigma_{np}} v(\tilde{p}) \tag{G.90}$$

$$\Phi(\tilde{p}) = mu_{ss} Y_{ss} \tilde{p}^{-\gamma}(\tilde{p} - mc_{ss}) \tag{G.91}$$

$$\tilde{p}'(\tilde{p}) = \begin{cases} \frac{\tilde{p}e^{\Delta a_h}}{1+\bar{\pi}} & \text{with prob. } p \\ \frac{\tilde{p}e^{\Delta a_l}}{1+\bar{\pi}} & \text{with prob. } 1-p \end{cases} \tag{G.92}$$

$$\tilde{p}^* = \arg\max_{\tilde{p}} V(\tilde{p}) \tag{G.93}$$

$$\Psi = \{x : V(\tilde{p}^*) - \theta w_{ss} mu_{ss} \le V(\tilde{p})\} \tag{G.94}$$

$$\mathcal{C} = \left\{(\tilde{p}_-, \Delta a) : \frac{\tilde{p}_- e^{\Delta a}}{1+\bar{\pi}} \in \Psi\right\} \tag{G.95}$$

- Aggregate feasibility and monetary policy

$$mc_{ss} = (1-\tau_{mc})\iota\frac{(w_{ss})^{1-\alpha}}{\eta_{zss}} \tag{G.96}$$

$$\mathcal{M}_{ss} = \frac{(1-\tau_{mc})}{mc_{ss}} \tag{G.97}$$

$$R^*_{ss} = \left(\frac{1+\bar{\pi}}{\beta\,(1+g)^{-\sigma_{np}}}\right) \qquad R_{ss} = \left(\left(\frac{\beta g^{-\sigma_{np}}}{1+\bar{\pi}}\right)R^*_{ss}\right)^{\phi_r} \tag{G.98}$$

$$1+\bar{\pi} = \left(\frac{1-\Omega_{ss}}{1-\Omega_{ss}(\tilde{p}^*_{ss})^{1-\gamma}}\right)^{\frac{1}{1-\gamma}}\varphi_{ss} \tag{G.99}$$

$$\Omega_{ss} = hz + (1-hz)\int_{(\tilde{p}_-,\Delta a)\notin\mathcal{C}} f_-(d\tilde{p}_-)g(d\Delta a) \tag{G.100}$$

$$\varphi_{ss} = \left(\int_{(\tilde{p}_-,\Delta a)\in\mathcal{C}} \frac{\left(\tilde{p}_- e^{\Delta a}\right)^{1-\gamma}}{1-\Omega_{ss}}(1-hz)f_-(d\tilde{p}_-)g(d\Delta a)\right)^{\frac{1}{1-\gamma}} \tag{G.101}$$

$$\Delta_{ss} = \int_{\tilde{p}} \tilde{p}^{-\gamma} f(\tilde{p}) \quad ; \quad f(\tilde{p}) := \text{distribution of posted prices} \tag{G.102}$$

G.1.2 CalvoPlus Model Equilibrium Computation

1. **Step 1**: Guess $(\Delta^1_{ss};\Omega^1_{ss},mc^1_{ss})$.
2. **Step 2**: For $(\Delta^i_{ss};\Omega^i_{ss},mc^i_{ss})$ solve

$$\begin{aligned}
w_{ss} &= \left(\frac{mc_{ss}\eta^Z_{ss}}{(1-\tau_{mc})\iota}\right)^{\frac{1}{1-\alpha}} \\
L_{ss} &= \left(\frac{w_{ss}}{\kappa}\right)^{\frac{1}{\chi}} \\
Y_{ss} &= \eta^Z_{ss}\frac{L_{ss}-(\Omega_{ss}-hz)\,\theta}{\Delta_{ss}}\left(\frac{\alpha w_{ss}}{1-\alpha}\right)^{\alpha} \\
C_{ss} &= Y_{ss}\left(1-\left(\frac{w_{ss}\alpha}{1-\alpha}\right)^{1-\alpha}\frac{\Delta_{ss}}{\eta^Z_{ss}}\right)-\eta^G_{ss} \\
u_{ss} &= \bar{u}\frac{(C_{ss}-\kappa\frac{L_{ss}^{1+\chi}}{1+\chi})^{1-\sigma_n}}{1-\sigma_n} \\
mu_{ss} &= \bar{u}(C_{ss}-\kappa\frac{L_{ss}^{1+\chi}}{1+\chi})^{-\sigma_n} \\
U_{ss} &= \frac{u_{ss}(1-\beta)}{1-\beta g^{1-\sigma_{np}}} \\
\Pi_{ss} &= 1+\bar{\pi} \\
R_{ss} &= \frac{1+\bar{\pi}}{\beta g^{-\sigma_{np}}} \\
\Sigma_{ss} &= 1 \\
\tilde{R}_{ss} &= 1
\end{aligned}$$

3. Given $(Y^i_{ss}, mu^i_{ss}, w^i_{ss}, L^i_{ss})$, solve

$$\begin{aligned}
v^i_1(\tilde{p}) &= (1-hz)\max\left\{\max_x \Phi(x) + \beta g^{1-\sigma_{np}} v^i_1(x) - mu_{ss}w_{ss}\theta, \Phi(\tilde{p}) + \beta g^{1-\sigma_{np}} v^i_1(\tilde{p})\right\} + \dots \\
&\dots + hz\left(\Phi(\tilde{p}) + \beta g^{1-\sigma_{np}} v^i_1(\tilde{p})\right) \\
v^i_2(\tilde{p}) &= \mathbb{E}_{\tilde{p}'}\left[v^i_1(\tilde{p}')^i\right] \\
\Phi(\tilde{p}) &= mu_{ss}Y_{ss}\tilde{p}^{-\gamma}\left(\tilde{p} - mc_{ss}\right) \\
\tilde{p}'(\tilde{p}) &= \begin{cases} \frac{\tilde{p}e^{\sigma^h_a \epsilon^a}}{1+\bar{\pi}} & \text{with pr. } p \\ \frac{\tilde{p}e^{\sigma^l_a \epsilon^a}}{1+\bar{\pi}} & \text{with pr. } 1-p \end{cases}
\end{aligned}$$

and get $\tilde{p}^*, \Psi$ and $\mathcal{C}$.

(a) *Technical 1*: For the firm problem I use spline of 2nd or 3rd order to approximate the value function. It is important to not do 1st order splines, since it generates jump in the reset price in the iteration.

(b) *Technical 2*: I use contraction together with colocation to solve the Bellman equation of the firm.

(c) *Technical 3*: I use Brent optimization method to solve the firm problem.

4. **Step 3**: Fix a grid between $[\tilde{p}_{min,s}, \tilde{p}_{max,s}]$ with n_s points. Construct the three transition matrices $F_{\Delta a}, F_\Pi, F_{p'}$ with $\mathcal{C}$ and Ψ where

(a) $F_{\Delta a}$ is given by the transition probability $\tilde{p}_1 = \tilde{p}\frac{A'}{A}$.

(b) F_Π is given by the transition probability $\tilde{p}_2 = \frac{\tilde{p}_1}{\Pi_{ss}}$.

(c) $F_{p'}$ is given by the transition probability $\tilde{p}_3 = \begin{cases} \tilde{p}_2 I(\tilde{p}\in\mathcal{C}) + I(\tilde{p}\notin\mathcal{C})P^*_{ss} & \text{with prob. } 1-hz \\ P^*_{ss} & \text{with prob. } hz \end{cases}$.

using linear splines over the grid $[\tilde{p}_{min,s}, \tilde{p}_{max,s}]$. To compute the ergodic distribution use the eigenvector of the unit eigenvalue of $F_{\Delta a}F_\Pi F_{p'}$ that gives the ergodic distribution n_{ss}. After having the ergodic distribution compute the reset inflation, price dispersion and frequency of price change.

- $n_{aux} = (n_{ss}F_{\Delta a}F_\Pi)I(p\in C)(1-hz)$.
- $\Omega^*_{ss} = 1 - \sum_i n_{aux}(i)$.
- $\varphi_{ss} = (1+\bar{\pi})\left(\sum_i \tilde{p}(i)^{1-\gamma}\frac{n_{aux}(i)}{\sum_i n_{aux}(i)}\right)^{\frac{1}{1-\gamma}}$.
- $\Pi_{ss} = \left(\frac{1-\Omega_{ss}}{1-\Omega P^{1-\gamma}_{ss}}\right)^{\frac{1}{1-\gamma}}\varphi_{ss}$.
- $\Delta^*_{ss} = \sum_i \tilde{p}(i)^{-\gamma}n_{ss}(i)$

5. Update aggregate state from the simulation.

(a) If $abs(\frac{\Pi_{ss}}{1+\bar{\pi}}) < A$, then

$$mc^{i+1}_{ss} = mc^i_{ss} + adj * \frac{mc^{i+!}_{ss} - mc^i_{ss}}{\Pi^{i+1}_{ss} - \Pi^i_{ss}}\Pi^{i+1}_{ss} \qquad \Omega^{i+1}_{ss} = \Omega^i_{ss} + adj(\Omega^*_{ss} - \Omega^i_{ss}) \qquad \Delta^{i+1}_{ss} = \Delta^i_{ss} + adj(\Delta^*_{ss} - \Delta^i_{ss}) \tag{G.103}$$

(b) If $abs(\frac{\Pi_{ss}}{1+\bar{\pi}}) > A$, then

$$mc^{i+1}_{ss} = mc^i_{ss}\left(\frac{1+\bar{\pi}}{\Pi_{ss}}\right)^{adj} \tag{G.104}$$

6. Check convergence of the policy

$$\max(abs(\frac{C^{i+1}_{ss} - C^i_{ss}}{C^i_{ss}}, \frac{L^{i+1}_{ss} - L^i_{ss}}{L^i_{ss}}, \frac{mc^{i+1}_{ss} - mc^i_{ss}}{mc^i_{ss}}, \Omega^{i+1}_{ss} - \Omega^i_{ss}, \frac{\Delta^{i+1}_{ss} - \Delta^i_{ss}}{\Delta^i_{ss}}, \frac{Y^{i+1}_{ss} - Y^i_{ss}}{Y^i_{ss}})) < tol_{conv} \tag{G.105}$$

Go to step 2 if it doesn't converge.

7. After converge compute micro-price statistics with extended $F^*_{\Delta a}$. I can the distribution of price change as $f(p) = n_{ss} * F_{\Delta a} F_{\Pi} * (p_i \notin C)$.

G.2 CalvoPlus Model: Business Cycle

The algorithm to solve the model consists in 3 steps:

Step 1: Use perturbations methods to approximate the equilibrium dynamics without the ZLB. Use the reset price with Calvo. Project price dispersion, reset inflation and frequency of price change into the state.

Step 2: Solve equilibrium conditions with global methods ignoring the zero lower bound.

Step 3: Solve equilibrium conditions with global methods with the zero lower bound.

G.2.1 Step 1: Approximation of the Equilibrium

1. **Step 1**: Initiate the Krusell-Smith projection

$$\Delta(S) = P_1(\log(S)) \quad ; \quad \Omega(S) = P_1(\log(S)) \quad ; \quad \varphi(S) = P_1(\log(S)) \quad ; \quad \tau_1^C \tag{G.106}$$

   where P^1 denotes a linear projection.

2. **Step 2**: Given

$$\Delta(S) = P_i(\log(S)) \quad ; \quad \Omega(S) = P_i(\log(S)) \quad ; \quad \varphi(S) = P_i(\log(S)) \quad ; \quad \tau_i^C \tag{G.107}$$

solve the aggregate equilibrium equations using second order approximation (with $\Omega^* = p + hz$)

$$
\begin{aligned}
mu_i(S) &= \beta g^{-\sigma_n} \eta_{q-}(S) R_i(S) \mathbb{E}_{S'}\left[\left(\frac{U_i(S')/U_{ss}}{\Sigma_i(S)^{\frac{1}{1-\sigma_{ez}}}} \right)^{-\sigma_{ez}} \frac{mu_i(S)}{\Pi_i(S')} \middle| S \right] \\
\Sigma_i(S) &= \mathbb{E}_{S'}\left[\left(\frac{U_i(S')}{U_{ss}} \right)^{1-\sigma_{ez}} \middle| S \right] \\
mu_i(S) &= \bar{u}(C_i(S) - \kappa \frac{L_i(S)^{1+\chi}}{1+\chi})^{-\sigma_{np}} \\
\kappa L_i(S)^{\chi} &= w_i(S) \\
u_i(S) &= \bar{u} \frac{\left(C_i(S) - \kappa \frac{L_i(S)^{1+\chi}}{1+\chi} \right)^{1-\sigma_{np}}}{1-\sigma_{np}} \\
U_i(S) &= (1-\beta) u_i(S) + \beta g^{1-\sigma_{np}} U_{ss} \Sigma_i(S)^{\frac{1}{1-\sigma_{ez}}} \\
\Pi_i(S) &= \left(\frac{1-\Omega_i(S)}{1-\Omega_i(S)(P_i^*(S))^{1-\gamma}} \right)^{\frac{1}{1-\gamma}} \varphi_i(S) \\
mc_i(S) &= \iota((1-\tau_{MC}) \frac{w_i(S))^{1-\alpha}}{\eta_{z-}(S)} \\
R_i(S) &= \tilde{R}_-(S) \left(\frac{1+\bar{\pi}}{\beta g^{-\sigma_{np}}} \right) \left(\left(\frac{\Pi_i(S)}{(1+\bar{\pi})} \right)^{\phi_\pi} \left(\frac{mc_i(S)}{mc_{ss}} \right)^{\phi_y} \right)^{1-\phi_r} \left(\frac{mc_i(S)}{mc_-(S)} \right)^{\phi_{dy}} \\
\tilde{R}_i(S') &= \left(\left(\frac{\beta g^{-\sigma_{np}}}{1+\bar{\pi}} \right) R_i(S) \right)^{\phi_r} \eta_r' \\
0 &= -\eta_{z-}(S)(L_i(S) - \theta(\Omega_i(S) - hz)) + Y_i(S) \left(\frac{(1-\alpha)}{\alpha w_i(S)} \right)^{\alpha} \Delta_i(S) \\
\eta_{g-}(S) + C_i(S) &= Y_i(S) \left(1 - \left(\frac{w_i(S)\alpha}{1-\alpha} \right)^{1-\alpha} \frac{\Delta_i(S)}{\eta_{z-}(S)} \right) \\
H_i(S) &= \frac{\gamma}{\gamma-1} (1-\beta) mu_i(S) Y_i(S) mc_i(S) \tau_i^C + \ldots \\
&\cdots + \beta g^{1-\sigma_{np}} (1-\Omega^*) \mathbb{E}_{S'}\left[\left(\frac{U_i(S')/U_{ss}}{\Sigma_i(S)^{\frac{1}{1-\sigma_{ez}}}} \right)^{-\sigma_{ez}} \Pi_i(S')^{\gamma} H_i(S') \middle| S \right] \\
F_i(S) &= mu_i(S) Y_i(S)(1-\beta) + \ldots \\
&\cdots + \beta g^{1-\sigma_{np}} (1-\Omega^*) \mathbb{E}_{S'}\left[\left(\frac{U_i(S')/U_{ss}}{\Sigma_i(S)^{\frac{1}{1-\sigma_{ez}}}} \right)^{-\sigma_{ez}} \Pi_i(S')^{\gamma-1} F_i(S') \middle| S \right] \\
P_i^*(S) &= \frac{H_i(S)}{F_i(S)}
\end{aligned}
$$

(a) *Warning*: The order of the projection has to be equal to the order of the order of the perturbation method . Since I'm approximating the equilibrium dynamics, I use first order perturbation for the state variables, but second order perturbation for the control variables. An alternative method is to use pruning. I find that this method generates a good approximation of the equilibrium.

$$
\begin{aligned}
\Delta_i(S) &= P_i(\log(S)) \\
\Omega_i(S) &= P_i(\log(S)) \\
\varphi_i(S) &= P_i(\log(S)) \\
\log(\eta_g(S')) &= (1-\rho_g)\log(\eta_g^*) + \rho_g \log(\eta_g(S)) + \sigma_g \epsilon_g' \\
\log(\eta_z(S')) &= (1-\rho_Z)\log(\eta_z^*) + \rho_z \log(\eta_z(S)) + \sigma_z \epsilon_z' \\
\log(\eta_r(S')) &= \sigma_r \epsilon_r' \\
\log(\eta_q(S')) &= (1-\rho_q)\log(\eta_q^*) + \rho_q \log(\eta_{q(S)}) + \sigma_q \epsilon_q
\end{aligned}
$$

3. **Step 3**: Given

$$
mu_i(S), mc_i(S), U_i(S), Y_i(S), w_i(S), \Pi_i(S) \tag{G.108}
$$

solve the firm's problem

$$
\begin{aligned}
v_i(p,S) &= (1-hz)\max\left\{v_i^c(S) - mu_i(S)w_i(S)\theta, v_i^{nc}(\tilde{p},S)\right\} + hz v_i^c(S) \\
v_i^c(S) &= \max_x \Phi(x,S) + \beta g^{1-\sigma_{np}} v_2^i(x,S) \\
v_i^{nc}(\tilde{p},S) &= \Phi\left(\frac{\tilde{p}}{\Pi_i(S)}, S\right) + \beta g^{1-\sigma_{np}} v_2^i\left(\frac{\tilde{p}}{\Pi_i(S)}, S\right) \\
Ev_i(p,S) &= \mathbb{E}_{\tilde{p}'}\left[\left(\frac{U_i(S')/U_{ss}}{\Sigma_i(S)^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}} v_1^i(\tilde{p}', S')\right] \\
\Phi(\tilde{p},S) &= mu_i(S)Y_i(S)\tilde{p}^{-\gamma}\left(\tilde{p} - (1-\tau_L)mc_i(S)\right) \\
\tilde{p}'(\tilde{p},S') &= \begin{cases} \tilde{p}e^{\sigma_a \epsilon_h^a} & \text{with prob. } p \\ \tilde{p}e^{\sigma_a \epsilon_l^a} & \text{with prob. } 1-p \end{cases}
\end{aligned}
$$

and get $P_i^{mc}(S), \Psi_i(S), \mathcal{C}_i(S)$

$$
\begin{aligned}
P_i^{mc}(S) &= \arg\max_x \Phi(x,S) + \beta g^{1-\sigma_{np}} v_1^i(x,S) \\
\Psi_i(S) &= \{\tilde{p} : v_i^{nc}(\tilde{p},S) \geq v_i^c(S) - mu_i(S)w_i(S)\theta\} \\
\mathcal{C}_i(S) &= \left\{(\tilde{p}_-, \Delta a) : \tilde{p}_- e^{\Delta a} \in \Psi(S)\right\}
\end{aligned}
$$

(a) *Technical 1*: For the firm problem I use spline in the firm's relative price and smoliak polynomials for the aggregate state.

(b) *Technical 2*: I use contraction together with colocation to solve the Bellman equation of the firm.

(c) *Technical 3*: I use Brent optimization method to solve the firm problem.

(d) *Technical 4*: To solve this problem I preallocate the base in the optimization before solving the firm's problem, avoiding the kronnerker product in the expectation

$$
\begin{aligned}
\tilde{\Phi}(s,S) &= \sum_{\epsilon_s,\epsilon_S} w(\epsilon_s)w(\epsilon_S)\left(\frac{U(S(S,\epsilon_S))/U_{ss}}{\Sigma(S)^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}} \Phi(s'(s,\epsilon_s), S'(S,\epsilon_S)) \\
&= \sum_{\epsilon_S}\sum_{\epsilon_s} w(\epsilon_s)w(\epsilon_S)\left(\frac{U(S(S,\epsilon_S))/U_{ss}}{\Sigma(S)^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}} \left(\Phi_s(s'(s,\epsilon_s)) \otimes \Phi_S(S'(S,\epsilon_S)\right) \\
&= \left(\sum_{\epsilon_s} w(\epsilon_s)\Phi_s(s'(s,\epsilon_s))\right) \otimes \left(\sum_{\epsilon_S} w(\epsilon_S)\left(\frac{U(S(S,\epsilon_S))/U_{ss}}{\Sigma(S)^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}} \Phi_S(S'(S,\epsilon_S)\right)
\end{aligned}
$$

Step 4: Given the policy

$$\Pi_i(S), mc_i(S), \tilde{R}_i(S), P_i^{mc}(S), \mathcal{C}_i(S) \tag{G.109}$$

and some initial conditions s_{idio} and S_{agge}

(a) Compute the distribution after repricing decision

$$n^1_{aux,t} = (n_{t-1} F_{\Delta a}) I(\tilde{p}e^{\Delta} \in \mathcal{C}_i(S_{t-1}))(1 - hz) \tag{G.110}$$

(b) Compute frequency of price change $\Omega_t = 1 - \sum_i n_{aux,t}(i)$.

(c) Compute the reset inflation

$$\varphi_t = \left(\sum_i \tilde{p}(i)^{1-\gamma} \frac{n_{aux,t}(i)}{sum_i n_{aux,t}(i)} \right)^{\frac{1}{1-\gamma}} \tag{G.111}$$

(d) Compute inflation

$$\Pi_t = \left(\frac{1 - \Omega_t}{1 - \Omega_t P_t^{1-\gamma}} \right)^{1/(1-\gamma)} \varphi_t \tag{G.112}$$

(e) Update the distribution $n_t = n_{t-1} F_{\Delta A} F_{\Pi_t} F_p$

(f) Compute price dispersion $\varphi_t = \sum_i \tilde{p}^{-\gamma} n_t$

(g) Update state S_t from S_{t-1}

4. **Step 5:** Check convergence of the policy.

$$\max_{S \in S^{erg}} (abs(\frac{C_i(S) - C_{i-1}(S)}{C_{i-1}(S)}, \frac{L_i(S) - L_{i-1}(S)}{L_{i-1}(S)}, \frac{mc_i(S) - mc_{i-1}(S)}{mc_{i-1}(S)}, \dots \tag{G.113}$$

$$\dots, R_i(S) - R_{i-1}(S), \frac{P_i^{mc}(S) - P_{i-1}^{mc}(S)}{P_{i-1}^{mc}(S)}, \frac{\Pi_i(S) - \Pi_{i-1}(S)}{\Pi_{i-1}(S)})) < tol_{conv} \tag{G.114}$$

where $S = \{S_1, S_2, S_3, ...\}$ is obtain from the simulation to evaluate the model in the ergodic set from the previous iteration. Continue the procedure if it doesn't converge.

5. **Step 6:** Update the coefficient in the projection

$$\Delta_t = P_{i+1}(\log(S_{t-1})) \quad ; \quad \Omega_t = P_{i+1}(\log(S_{t-1})) \quad ; \quad \varphi_t = P_{i+1}(\log(S_{t-1})) \tag{G.115}$$

in the case of τ^C_{i+1} update in such a way that the inflation in the simulation is equal to the inflation target. Go to step 2.

G.2.2 Step 2: Equilibrium Ignoring the Zero Lower Bound.

1. **Step 1**: Initiate the Krusell-Smith projection

$$\Delta(S) = P_1^2(\log(S)) \quad ; \quad \Omega(S) = P_1^2(\log(S)) \quad ; \quad \varphi(S) = P_1^2(\log(S)) \tag{G.116}$$

where P^2 denotes a quadratic projection and a policy function.

2. **Step 2**: Fix ξ. Given

$$\Delta(S) = P_i^2(\log(S)) \quad ; \quad \Omega(S) = P_i^2(\log(S)) \quad ; \quad \varphi(S) = P_i^2(\log(S)) \tag{G.117}$$

and

$$\Pi_{1,i}(S) \quad ; \quad U_{1,i}(S) \quad ; \quad \Sigma_{1,i}(S) \quad ; \quad mu_{1,i}(S) \tag{G.118}$$

- **Step 2.1**: Given $mu_{j,i}(S), U_{j,i}(S), \Sigma_{j,i}(S), mu_{j,i}(S)$, use the Euler equation to get

$$\Sigma_{j+1,i}(S) = \mathbb{E}_{S'}\left[\left(\frac{U_{j+1,i}(S')}{U_{ss}}\right)^{1-\sigma_{ez}}\middle| S\right] \tag{G.119}$$

$$mu_{j+1,i}(S) = \beta g^{-\sigma_n}\eta_q(S)R_{j,i}(S)\mathbb{E}_{S'}\left[\left(\frac{U_{j,i}(S')/U_{ss}}{(\Sigma_{j,i}(S))^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}}\frac{mu_{j,i}(S')}{\Pi_{j,i}(S')}\middle| S\right] \tag{G.120}$$

- **Step 2.2**: With $mu(S)_{j+1,i}(S)$ solve the following system

$$\kappa L_{j+1,i}(S)^{\chi} = w_{j+1,i}(S) \tag{G.121}$$

$$mu_{j+1,i}(S) = \bar{u}(C_{j+1,i}(S) - \kappa\frac{L_{j+1,i}(S)^{1+\chi}}{1+\chi})^{-\sigma_n} \tag{G.122}$$

$$\eta_z(S)L_{j+1,i}(S) = Y^{i+1}(S)\left(\frac{(1-\alpha)}{\alpha w_{j+1,i}(S)}\right)^{\alpha}\Delta_i(S) \tag{G.123}$$

$$\eta_g(S) + C_{j+1,i}(S) = Y_{j+1,i}(S)\left(1 - \left(\frac{w_{j+1,i}(S)\alpha}{1-\alpha}\right)^{1-\alpha}\frac{\Delta_i(S)}{\eta_z(S)}\right) \tag{G.124}$$

Compute the marginal cost and period utility given by

$$mc_{j+1,i}(S) = \frac{\iota(1-\tau_{mc})w_{j+1,i}(S)^{1-\alpha}}{\eta_z(S)} \qquad u_{j+1,i}(S) = \bar{u}\frac{(C_{j+1,i}(S) - \kappa\frac{L_{j+1,i}(S)^{1+\chi}}{1+\chi})^{1-\sigma_{np}}}{1-\sigma_{np}} \tag{G.125}$$

- **Step 2.3**: Update forward looking policies $U(S), H(S), F(S)$

$$\begin{aligned}
U_{j+1,i}(S) &= (1-\beta)u_{j+1,i}(S) + \beta g^{1-\sigma_{np}}U_{ss}\Sigma_{j+1,i}(S)^{\frac{1}{1-\sigma_{ez}}} \\
R^*_{j+1,i}(S) &= \tilde{R}_-(S)\left(\frac{1+\bar{\pi}}{\beta g^{-\sigma_{np}}}\right)\left(\left(\frac{\Pi_{j+1,i}(S)}{(1+\bar{\pi})}\right)^{\phi_\pi}\left(\frac{\tilde{mc}_{j+1,i}(S)}{mc_{ss}}\right)^{\phi_y}\right)^{1-\phi_r}\left(\frac{mc_{j+1,i}(S)}{\tilde{mc}(S)}\right)^{\phi_{dy}} \\
\tilde{R}_{j+1,i}(S) &= \left(\left(\frac{\beta g^{-\sigma_{np}}}{1+\bar{\pi}}\right)(R^*)_{j+1,i}(S)\right)^{\phi_r} R_{j+1,i}(S) = (R^*)_{j+1,i}(S);
\end{aligned}$$

- **Step 2.4**: If $j+1 = \xi$ go to step 2.5. If

$$error = mean_S\left(|mu^i(S) - mu^{i+1}(S)| + |\Pi^i(S) - \Pi^{i+1}(S)| + |U^i(S) - U^{i+1}(S)|\right) < \epsilon \tag{G.126}$$

go to step 2.5. Otherwise, go to step 2.1.

- **Step 2.5**: Solve firms Bellman equation

$$\begin{aligned}
v_{j+1,i}(p,S) &= (1-hz)\max\left\{v^c_{j+1,i}(S) - mu_{j+1,i}(S)w_{j+1,i}(S)\theta, v^{nc}_{j+1,i}(\tilde{p},S)\right\} + hzv^c_{j+1,i}(S) \\
v^c_{j+1,i}(S) &= \max_x \Phi(x,S) + \beta g^{1-\sigma_{np}}Ev_{j+1,i}(x,S) \\
v^{nc}_{j+1,i}(\tilde{p},S) &= \Phi\left(\frac{\tilde{p}}{\Pi_{j+1,i}(S)}, S\right) + \beta g^{1-\sigma_{np}}Ev_{j+1,i}\left(\frac{\tilde{p}}{\Pi_i(S)}, S\right) \\
Ev_{j+1,i}(p,S) &= \mathbb{E}_{\tilde{p}'}\left[\left(\frac{U_{j+1,i}(S')/U_{ss}}{\Sigma_i(S)^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}} v_{j+1,i}(\tilde{p}',S')\right] \\
\Phi(\tilde{p},S) &= (1-\beta)mu_{j+1,i}(S)Y_{j+1,i}(S)\tilde{p}^{-\gamma}(\tilde{p} - mc_{j+1,i}(S)) \\
\tilde{p}'(\tilde{p},S') &= \begin{cases} \tilde{p} & \text{with prob. } p \\ \tilde{p}e^{\sigma_a\epsilon^a - \frac{\sigma_a^2}{2}} & \text{with prob. } 1-p \end{cases}
\end{aligned}$$

and get $P_{j+1,i}(S)$

$$\begin{aligned} P^{mc}_{j+1,i}(S) &= \arg\max_x \Phi(x,S) + \beta g^{1-\sigma_{np}} v_1^i(x,S) \\ Pi_{j+1,i}(S) &= \left(\frac{1-\Omega_i(S)}{1-\Omega_i(S)(P^*_{j+1,i}(S))^{1-\gamma}} \right)^{\frac{1}{1-\gamma}} \varphi_i(S) \end{aligned}$$

- **Step 2.6**: If

$$\max_{S \in S^{erg}} (abs(\frac{C_{j+1,i}(S) - C_{j,i}(S)}{C_{j,i}(S)}, \frac{L_{j+1,i}(S) - L_{j,i}(S)}{L_{j,i}(S)}, \frac{mc_{j+1,i}(S) - mc_{j,i}(S)}{mc_{j,i}(S)}, \dots \quad \text{(G.127)}$$

$$\dots, R_{j+1,i}(S) - R_{j,i}(S), \frac{P_{j+1,i}(S) - P_{j,i}(S)}{P_{j,i}(S)}, \frac{\Pi_{j+1,i}(S) - \Pi_{j,i}(S)}{\Pi_{j,i}(S)})) \geq tol_{conv} \quad \text{(G.128)}$$

go to step 2.1. If

$$\max_{S \in S^{erg}} (abs(\frac{C_{j+1,i}(S) - C_{j,i}(S)}{C_{j,i}(S)}, \frac{L_{j+1,i}(S) - L_{j,i}(S)}{L_{j,i}(S)}, \frac{mc_{j+1,i}(S) - mc_{j,i}(S)}{mc_{j,i}(S)}, \dots \quad \text{(G.129)}$$

$$\dots, R_{j+1,i}(S) - R_{j,i}(S), \frac{P_{j+1,i}(S) - P_{j,i}(S)}{P_{j,i}(S)}, \frac{\Pi_{j+1,i}(S) - \Pi_{j,i}(S)}{\Pi_{j,i}(S)})) < tol_{conv} \quad \text{(G.130)}$$

Set

$$C_{1,i+1}(S) = C_{j+1,i}(S) \quad ; \quad L_{1,i+1}(S) = L_{j+1,i}(S) \quad \text{(G.131)}$$

$$mc_{1,i+1}(S) = mc_{j+1,i}(S) \quad ; \quad \Pi_{1,i+1}(S) = \Pi_{j+1,i}(S) \quad \text{(G.132)}$$

and check if

$$\max_{S \in S^{erg}} (abs(\frac{C_{j,i+1}(S) - C_{1,i}(S)}{C_{1,i}(S)}, \frac{L_{1,i+1}(S) - L_{1,i}(S)}{L_{1,i}(S)}, \frac{mc_{1,i+1}(S) - mc_{1,i}(S)}{mc_{1,i}(S)}, \dots \quad \text{(G.133)}$$

$$\dots, R_{1,i+1}(S) - R_{1,i}(S), \frac{P_{1,i+1}(S) - P_{1,i}(S)}{P_{1,i}(S)}, \frac{\Pi_{1,i+1}(S) - \Pi_{1,i}(S)}{\Pi_{1,i}(S)})) < tol_{conv} \quad \text{(G.134)}$$

If this is the case, I find the equilibrium. If previous inequality doesn't hold, go to step 3.

Step 3: Given the policy

$$\Pi_i(S), mc_i(S), \tilde{R}_i(S), P_i^{mc}(S), \mathcal{C}_i(S) \quad \text{(G.135)}$$

and some initial conditions s_{idio} and S_{agge}

(a) Compute the distribution after repricing decision

$$n_{aux,t} = (n_{t-1} F_{\Delta a}) I(\tilde{p}e^{\Delta} \in \mathcal{C}_i(S_{t-1}))(1 - hz) \quad \text{(G.136)}$$

(b) Compute frequency of price change $\Omega_t = 1 - sum_i n_{aux,t}(i)$.

(c) Compute the reset inflation

$$\varphi_t = \left(\sum_i \tilde{p}(i)^{1-\gamma} \frac{n_{aux,t}(i)}{sum_i n_{aux,t}(i)} \right)^{\frac{1}{1-\gamma}} \quad \text{(G.137)}$$

(d) Compute inflation

$$\Pi_t = \left(\frac{1-\Omega_t}{1-\Omega_t P_t^{1-\gamma}} \right)^{1/(1-\gamma)} \varphi_t \quad \text{(G.138)}$$

(e) Update the distribution $n_t = n_{t-1} F_{\Delta A} F_{\Pi_t} F_p$

(f) Compute price dispersion $\varphi_t = \sum_i \tilde{p}^{-\gamma} n_t$

(g) Update state S_t from S_{t-1}

3. **Step 4:** Update the coefficient in the projection

$$\Delta_t = P^2_{i+1}(\log(S_{t-1})) \quad ; \quad \Omega_t = P^2_{i+1}(\log(S_{t-1})) \quad ; \quad \varphi_t = P^2_{i+1}(\log(S_{t-1})) \tag{G.139}$$

Go to step 2.

G.2.3 Step 3: Solving the model with ZLB

I repeat the same algorithm as in step 2 using as initial condition the global solution without zero lower bound. The only difference comes in step 2.3 where I solve the following system:

$$\begin{aligned}
(R^*)^{i+1}(S) &= \tilde{R}_{-}(S)\left(\frac{1+\bar{\pi}}{\beta g^{-\sigma_{np}}}\right)\left(\left(\frac{\Pi^{i+1}(S)}{(1+\bar{\pi})}\right)^{\phi_\pi}\left(\frac{\tilde{mc}^{i+1}(S)}{mc_{ss}}\right)^{\phi_y}\right)^{1-\phi_r}\left(\frac{mc^{i+1}(S)}{\tilde{mc}(S)}\right)^{\phi_{dy}} \\
\tilde{R}^{i+1}(S) &= \left(\left(\frac{\beta g^{-\sigma_{np}}}{1+\bar{\pi}}\right)(R^*)^{i+1}(S)\right)^{\phi_r} \\
R^{i+1}(S) &= \max\left\{1,(R^*)^{i+1}(S)\right\}
\end{aligned}$$

One of the main problem in solving the model with ZLB are explosive dynamics in the ZLB whenever the frequency of the zero lower bound become sufficiently high—bigger than 50%.

To avoid this problem, I follow Fernández-Villaverde *et al.* (2015) and I add a consumption tax that stimulate consumption whenever output gap is sufficiently low.

$$mu(S) = \beta(1-\tau^C(S))(1+g)^{-\sigma_{np}}\eta_{q-}(S)R(S)\mathbb{E}_{S'}\left[\left(\frac{U(S')/U_{ss}}{\Sigma(S)^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}}\frac{mu(S')}{\Pi(S')}\middle| S\right] \tag{G.140}$$

$$\tau^c(S) = \begin{cases} 0 & \text{if } mc(S) \geq \underline{mc} \\ 1 - \dfrac{1}{\beta(1+g)^{-\sigma_{np}}\eta_{q-}(S)R(S)\mathbb{E}_{S'}\left[\left(\frac{U(S')/U_{ss}}{\Sigma(S)^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}}\frac{mu(S')}{\Pi(S')}\middle| S\right]} & \text{otherwise} \end{cases} \tag{G.141}$$

Importantly, this tax to consumption is never active in the optimal inflation target and I only use it for levels of inflation less than the optimal.

H Asset Pricing Computation

This section computes the price and rate of return of a set of assets, since I use asset pricing implications of the model to calibrate the risk-aversion parameter in the Epstein-Zin preferences formulation. Let $\Lambda(S, S')$ be the stochastic discount factor in the model. Given Epstain-Zin preferences, the stochastic discount factor is given by

$$\Lambda(S, S') = \beta(1+g)^{1-\sigma_{np}} \left(\frac{U(S')}{\mathbb{E}_{S'}\left[-U(S')^{1-\sigma_{ez}} \,|\, S\right]^{\frac{1}{1-\sigma_{ez}}}} \right)^{-\sigma_{ez}} \frac{mu(S')}{mu(S)} \tag{H.142}$$

where β is the discount factor; $1+g$ is the growth rate of consumption; U is the representative consumer value function and mu is the marginal utility. I use S to denote the state of the economy. $\Lambda(S, S')$ is the marginal rate of substitution between a unit of consumption in S and $(1+g)$ units of consumption tomorrow S'. It is important to note that I'm adding the growth rate of the economy in the discount factor, since all the assets' payment schedules have growth. The marginal rate of substitution between a dollar in S and S' denoted by $\tilde{\Lambda}(S, S')$ is given

$$\tilde{\Lambda}(S, S') = (1+g)\frac{\Lambda(S, S')}{\Pi(S')} \tag{H.143}$$

where $\Pi(S')$ is the aggregate inflation. Remember that I calibrate the model at monthly frequency; thus I need to convert rate of return at annual frequency to compare them with the data. For all the asset prices, I calculate the de-trended value.

- **Real Short Term Risk Free Asset**: This asset pays $(1+g)$ units of consumption tomorrow in all the states of the world. The price $\nu^{st,rf}(S)$ in state S and the annual rate of return $r^{st,rf}(S)$ of this asset are given by

$$\nu^{st,rf}(S) = \mathbb{E}_{S'}\left[\Lambda(S, S') \middle| S\right] \quad ; \quad r^{st,rf}(S) = \left(\frac{1+g}{\nu^{st,rf}(S)}\right)^{12} \tag{H.144}$$

- **Real Short Term Risky Asset**: This asset pays aggregate consumption tomorrow in all the states of the word. The price $\nu^{st,rr}(S)$ in state S and the annual rate of return $r^{st,rr}(S)$ of this asset are given by

$$\nu^{st,rr}(S) = \mathbb{E}_{S'}\left[\Lambda(S, S')C(S') \middle| S\right] \quad ; \quad r^{st,rr}(S) = \mathbb{E}_{S'}\left[\left(\frac{(1+g)C(S')}{\nu^{st,rr}(S)}\right)^{12} \middle| S\right] \tag{H.145}$$

- **Real Long Term Risk Free Asset**: This asset pays $(1-\beta)(1+g)^t$ units of consumption tomorrow in all the states of the word in all periods. The price $\nu^{lt,rf}(S)$ in state S and the annual rate of return $r^{lt,rf}(S)$ of this asset are given by

$$\nu^{lt,rf}(S) = \mathbb{E}_{S'}\left[\Lambda(S, S')\left(1-\beta+\nu^{lt,rf}(S')\right) \middle| S\right] \tag{H.146}$$

$$r^{lt,rf}(S) = \left((1+g)\frac{1-\beta+\nu^{lt,rf}(S')}{\nu^{lt,rf}(S)}\right)^{12} \tag{H.147}$$

- **Real Long Term Risky Asset**: This asset pays $(1-\beta)$ aggregate consumption tomorrow in all the states of the word in all periods. The price $\nu^{lt,rr}(S)$ in state S and the annual rate of return $r^{lt,rr}(S)$ of this asset are given by

$$\nu^{lt,rr}(S) = \mathbb{E}_{S'}\left[\Lambda(S, S')\left((1-\beta)C(S')+\nu^{lt,rr}(S')\right) \middle| S\right] \tag{H.148}$$

$$r^{lt,rr}(S) = \mathbb{E}_{S'}\left[\left((1+g)\frac{(1-\beta)C(S')+\nu^{lt,rr}(S')}{\nu^{lt,rr}(S)}\right)^{12} \middle| S\right] \tag{H.149}$$

- **Nominal Consol**: This asset pays $(1-\beta)$ dollar tomorrow in all the states of the word with a geometric coupon δ_c. The price $\xi^{nc}(S)$ in state S and the annual rate of return $r^{nc}(S)$ of this asset are given by

$$\xi^{nc}(S) = 1-\beta+\delta_c\mathbb{E}_{S'}\left[\tilde{\Lambda}(S, S')\xi^{nc}(S') \middle| S\right] \tag{H.150}$$

$$r^{nc}(S) = \left(\frac{\delta_c\xi^{nc}(S)}{\xi^{nc}(S)-(1-\beta)}\right)^{12} \tag{H.151}$$

- **Nominal Consol with Risk Free Preferences**: This asset pays $(1-\beta)$ dollar tomorrow in all the states of the word with a geometric coupon δ_c and it is evaluated with risk free preferences. The price $\nu^{lt,rr}(S)$ in state S and the annual rate of return $r^{lt,rr}(S)$ of this asset are given by

$$\xi^{nc}(S) = 1 - \beta + \delta_c \frac{1}{R_t} \mathbb{E}_{S'} \left[\xi^{nc}(S') \middle| S \right] \tag{H.152}$$

$$r^{nc}(S) = \left(\frac{\delta_c \xi^{nc}(S)}{\xi^{nc}(S) - (1-\beta)} \right)^{12} \tag{H.153}$$

Table VIII – Asset Pricing Implication Calvo and CalvoPlus at One Percent IT

	Calvo		CalvoPlus	
Mean Return	No ZLB	ZLB	No ZLB	ZLB
Assets				
Short-Risk Free	3.38	3.48	3.41	3.30
Short-Risky	3.82	3.94	3.69	3.85
Long-Risk Free	4.48	4.53	4.38	4.53
Long-Risky	4.51	4.56	4.41	4.57
Risk Free Nominal Consol-Risky	4.21	3.81	4.29	3.53
Nominal Consol	4.68	4.20	4.79	3.68
Equity Premium	1.13	1.08	1.00	1.27
Consumption Premium	0.03	0.03	0.03	0.04
Bond Premium	0.48	0.39	0.51	0.15

Rows 1 to 6 describe the rate of returns of the real short term risk free asset, the real short term risky asset, the real long term risk free asset, the real long term risky asset, the nominal consol and the nominal consol with risk free preferences. The equity premium is the 4th row minus the 1st row, the consumption premium is the 4th row minus the 3rd row and the bond premium is the 6th row minus the 5th row.

I Business Cycle Statistics With No ZLB and 2% Inflation Target

This section describes the computation of the linear impulse-response for the Calvo and CalvoPlus model, together with the business cycle statistics.

I.1 Business Cycle Statistics

Let $m_x^{T_s,M}$ be the moment m, variable x, sampling length T_s and model M. The business cycle moments are composed by the standard deviation, persistence and correlations with respect to output and inflation. Next, I describe the steps for computing $m_x^{T,m}$ in the two models using Monte Carlo methods.

1. Simulate the model for a large T. Let $\{X_t\}_{t=0}^{T}$ be the time-series of the aggregate variables.
2. Generate a random sequence of dates $\{t_i\}_{i=1}^{N}$ and draw $\{\{X_t\}_{t_i}^{t_i+T_s}\}_{i=1}^{N}$ samples.
3. For each each random sample $i = 1, 2, ..., N$ compute $m_x^{T_s,m}$.

I apply a linear trend to all the variables in the data to construct the business cycle component. Since the focus of this paper is periods of stable trend inflation, I drop the time period before 1984Q1. Since I'm ignoring the ZLB, I also drop periods after 2007Q2. Table IX describes the business cycle statistics in the data, in the CalvoPlus model, in the Calvo model and the [5,95] interval confidence of the difference of each statistic in the Calvo and CalvoPlus model.

I.2 Linear Impulse Response Functions

Let IR_{tx}^{My} be the linear impulse-response in the model M, after t periods of the structural shock $x \in \{z, g, r, q\}$ of the y variable. Let $\hat{IR}_{tx}^{MyT_s}$ be the estimate in a sample of length T_s. Next, I describe the steps to generate the random variable $\hat{IR}_{tx}^{MT}$ using Monte Carlo methods.

1. Simulate the model for a large T. Let $\{X_t\}_{t=0}^{T}$ be the time-series of the aggregate variables, S_t^X the vector that includes real marginal cost, price dispersion, nominal interest rate and the exogenous variables, and let S_t^Y the vector that includes output, inflation and consumption.
2. Generate a random i.i.d. sequence of dates $\{t_i\}_{i=1}^{N}$ and draw $\{\{X_t\}_{t_i}^{t_i+T_s}\}_{i=1}^{N}$ samples.
3. For each random sample $i = 1, 2, ..., N$:

   i. Estimate the state space model:

$$S_{t+1}^X = \beta_x S_t^X + \Omega_x \epsilon_{t+1}^x \quad ; \quad S_t^Y = \beta_y S_t^X + \Omega_y \epsilon_t^y \tag{I.154}$$

   ii. Compute the impulse-response with respect to σ_x aggregate shock, where x denotes a variable. Compute the impulse-response $IR_x(t,i)^{MyT_s}$ using the model I.154 from the simulated data.

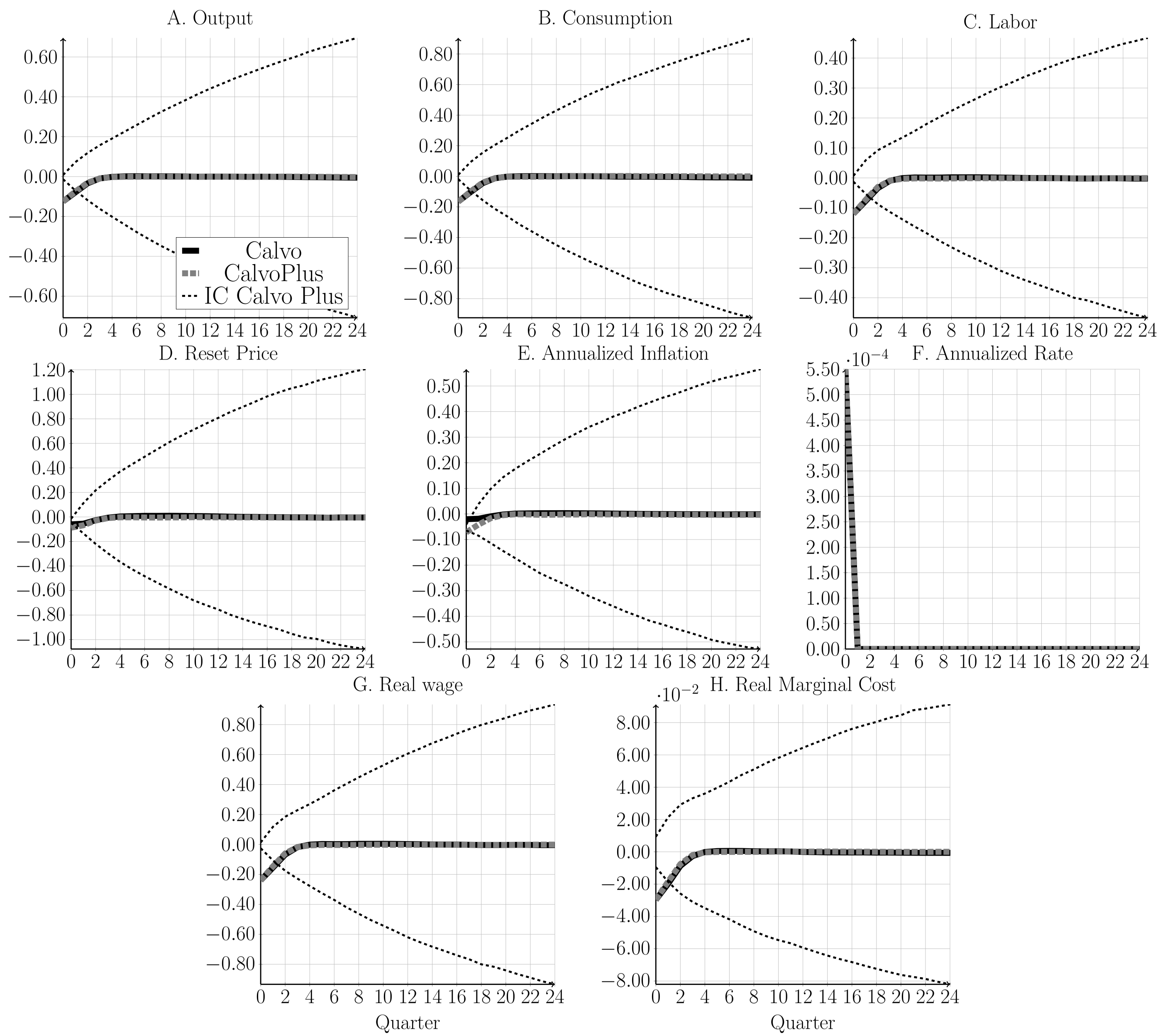
4. $\{IR_x(t,i)^{MyT_s}\}_{i=1}^{N}$ is a random sample from $\hat{IR}_{tx}^{MyT_s}$.

Table IX – Business Cycle Moments at 2 Percent Inflation and no ZLB

moment	GDP	C	L	R	w	Π
Data						
Standard Dev.	1.608	1.493	1.979	0.385	2.445	0.214
Autocorrelation	0.948	0.942	0.971	0.947	0.957	0.641
Corr. with GDP	1.000	0.897	0.746	0.553	0.138	0.032
Corr. with inflation	0.138	0.092	-0.199	0.255	0.293	1.000
CalvoPlus Business Cycle Statistics—Median						
Standard Dev.	1.237	1.627	0.936	0.280	0.468	0.114
Autocorrelation	0.844	0.845	0.778	0.846	0.778	0.863
Corr. with GDP	1.000	0.993	0.966	-0.288	0.966	0.060
Corr. with inflation	0.966	0.040	0.313	0.642	0.313	1.000
Calvo Business Cycle Statistics—Median						
Standard Dev.	1.333	1.742	1.036	0.261	0.518	0.068
Autocorrelation	0.857	0.858	0.811	0.823	0.811	0.904
Corr. with GDP	1.000	0.994	0.963	-0.270	0.963	0.069
Corr. with inflation	0.963	0.054	0.326	0.649	0.326	1.000
Difference Between Calvo and CalvoPlus Models IC(15,85)						
Standard Dev.	(-0.67,0.55)	(-0.89,0.71)	(-0.52,0.31)	(-0.11,0.14)	(-0.26,0.15)	(-0.00,0.10)
Autocorrelation	(-0.15,0.13)	(-0.15,0.13)	(-0.20,0.14)	(-0.12,0.19)	(-0.20,0.14)	(-0.17,0.09)
Corr. with GDP	(-0.00,0.00)	(-0.01,0.01)	(-0.04,0.07)	(-0.62,0.55)	(-0.04,0.07)	(-0.64,0.67)
Corr. with inflation	(-0.04,0.07)	(-0.65,0.68)	(-0.58,0.56)	(-0.34,0.37)	(-0.58,0.56)	(-0.00,0.00)

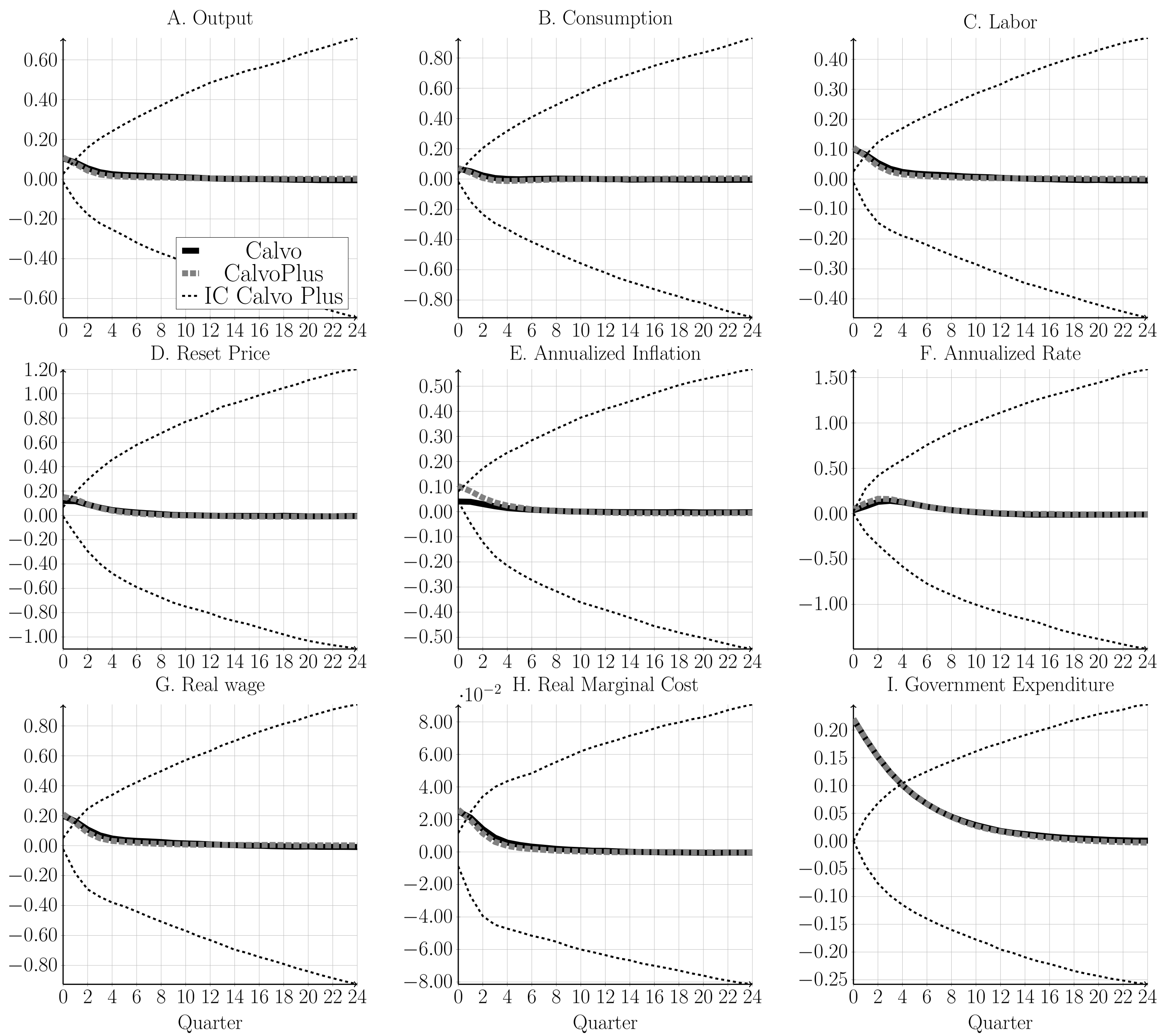
GDP=Gross Domestic Output Per Capital; C=Consumption; L=Total Labor Supply; R= quarterly Interest Rate; w= Real Wage; Π= Quarterly Inflation. The description of the variables in the data are in the Online Appendix, see section A. The variables in the data are linear detrended from 1984:Q1 to 2007:Q1. The moments in the Calvo and CalvoPlus models are the 50 percentiles of each statistics. The difference are the 5 and 95 percentile. All statistics are over 5000 simulation over 22 years.

Figure I – Impulse-Response to a Monetary Shock



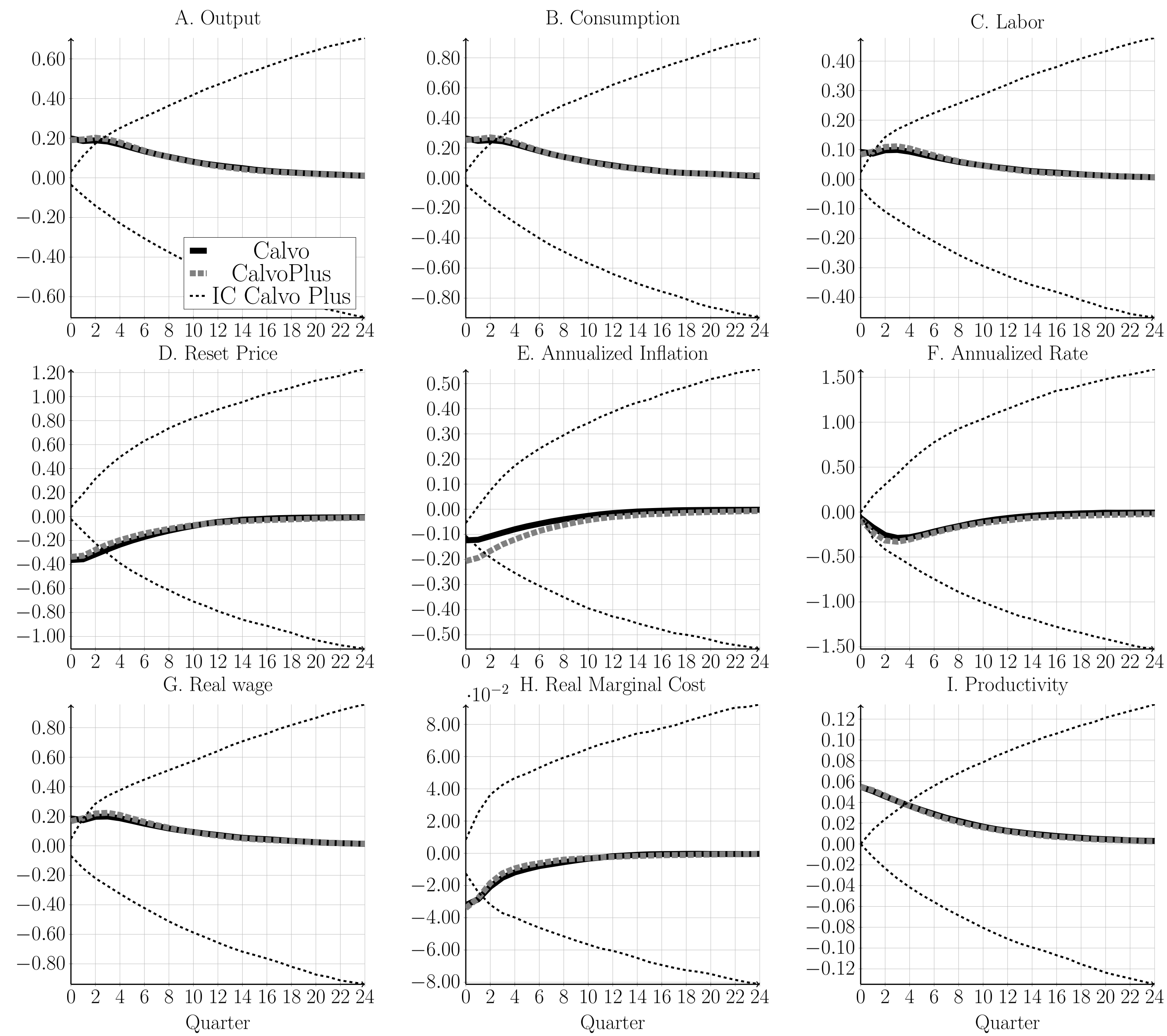
Panel A to I describe the impulse response functions of the output, consumption, labor, reset price, annualized inflation, annualized nominal rate, real wage and real marginal cost to a monetary shock at 2% inflation target in the models without zero lower bound constraint to the nominal interest rate. The black solid line describes the median impulse-response functions of the Calvo model; the gray dotted line describes the median impulse-response functions of the CalvoPlus model; the black dashed lines describe the 5 and 95 percentiles of the difference in the impulse-response functions between the two models over 5000 simulations of 22 years.

Figure II – Impulse-Response to a Government Expenditure Shock



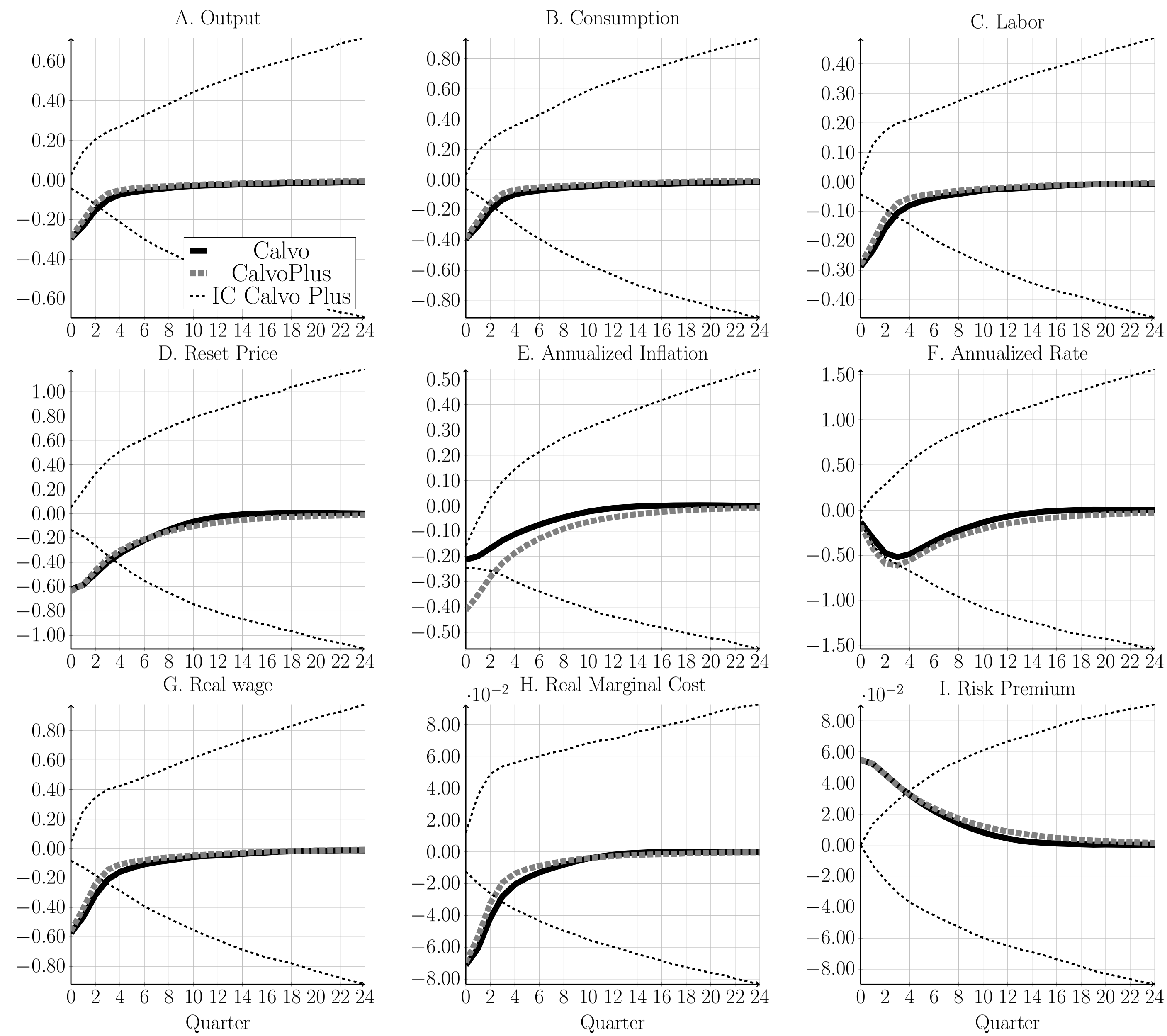
Panel A to I describe the impulse response functions of the output, consumption, labor, reset price, annualized inflation, annualized nominal rate, real wage and real marginal cost to a government expenditure shock at 2% inflation target in the models without zero lower bound constraint to the nominal interest rate. The black solid line describes the median impulse-response functions of the Calvo model; the gray dotted line describes the median impulse-response functions of the CalvoPlus model; the black dashed lines describe the 5 and 95 percentiles of the difference in the impulse-response functions between the two models over 5000 simulations of 22 years.

Figure III – Impulse-Response to a Productivity Shock



Panel A to I describe the impulse response functions of the output, consumption, labor, reset price, annualized inflation, annualized nominal rate, real wage and real marginal cost to a productivity shock at 2% inflation target in the models without zero lower bound constraint to the nominal interest rate. The black solid line describes the median impulse-response functions of the Calvo model; the gray dotted line describes the median impulse-response functions of the CalvoPlus model; the black dashed lines describe the 5 and 95 percentiles of the difference in the impulse-response functions between the two models over 5000 simulations of 22 years.

Figure IV – Impulse-Response to a Risk Premium Shock

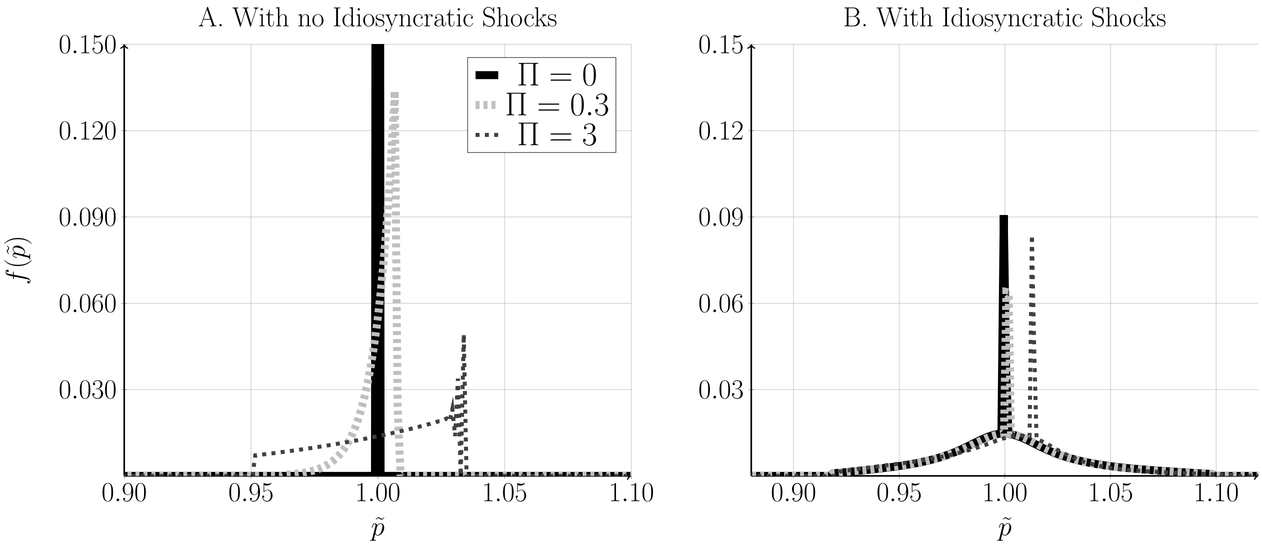


Panel A to I describe the impulse response functions of the output, consumption, labor, reset price, annualized inflation, annualized nominal rate, real wage and real marginal cost to a risk premium shock at 2% inflation target in the models without zero lower bound constraint to the nominal interest rate. The black solid line describes the median impulse-response functions of the Calvo model; the gray dotted line describes the median impulse-response functions of the CalvoPlus model; the black dashed lines describe the 5 and 95 percentiles of the difference in the impulse-response functions between the two models over 5000 simulations of 22 years.

J Understanding the Interaction Between Idiosyncratic Shocks and Menu Cost: Additional Figures and Details of the Computation

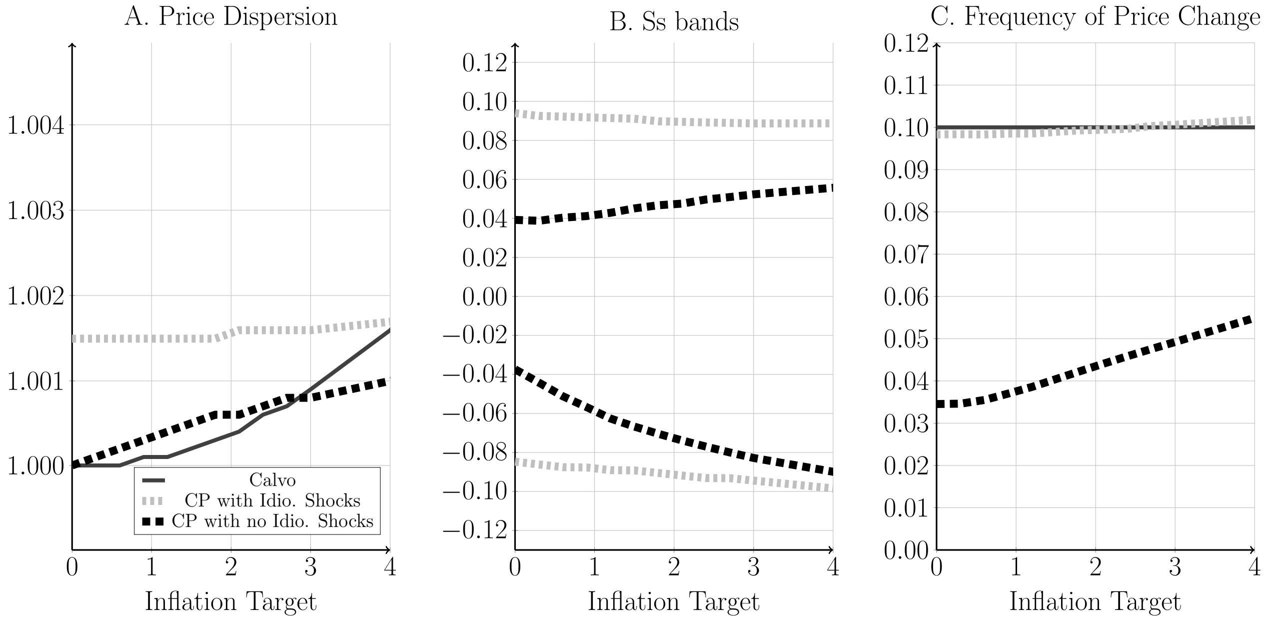
In this section, I compute the equilibrium in the steady state economy with and without idiosyncratic shocks. To solve the model without idiosyncratic shocks, I use $hz = 0.0001$, since if $hz = 0$ we don't have mixed conditions and there are infinite ergodic steady state distributions. Additionally, I implemented a constant real wage with a subsidy to real marginal cost as a function of steady state inflation.

Figure V – Steady State Distribution of Relative Prices



Panel A and B describe the distribution of relative price at 0%, 0.3% and 4% inflation inflation target.

Figure VI – Steady State Statistics in the Calvo, and Menu Cost with and without Idiosyncratic Shocks



Panel A describes the price dispersion in the Calvo, and in the CalvoPlus model with and without idiosyncratic shocks. Panel B describes the log-distance between the upper and lower Ss bands with respect to the reset price in the menu cost models. Panel C describes the frequency of price in the three models..

K Auxiliar Theorems

Proposition 3 (Welfare and Wedges) *Let $X = (\Delta, mc, FC, \eta_z, \eta_g)$ be the sufficiency statistics for consumption and labor and S the aggregate state of the economy. Then the welfare satisfies*

$$\begin{aligned} KK(X) &= \frac{\alpha}{1-\alpha}\left(\frac{mc\ \eta_z}{\iota(1-\tau_{mc})}\right)^{1/(1-\alpha)} \qquad L(X) = \left(\frac{(1-\alpha)}{\kappa\alpha}KK(X)\right)^{1/\chi} \\ C(X) &= \frac{\eta_z}{\Delta}(L-FC)KK(X)^{\alpha}\left(1-KK(X)^{1-\alpha}\frac{\Delta}{\eta_z}\right)-\eta_g(S) \\ U(S) &= \frac{\left(C(X(S))-\kappa\,(1+\chi)^{-1}\,L(X(S))^{1+\chi}\right)^{1-\sigma_{np}}}{1-\sigma_{np}}+\beta g^{1-\sigma_{np}}\mathbb{E}_{S'}\left[U(S)^{1-\sigma_{ez}}\,\middle|\,S\right]^{\frac{1}{1-\sigma_{ez}}} \end{aligned} \tag{29}$$

where $FC(S)$ in the menu cost model is $\theta(\Omega(S)-hz)$ and in the Calvo model $FC(S) = 0$.

Proof. The equilibrium conditions are

$$w_t = \kappa L_t^{\chi} \tag{K.155}$$

$$mc_t = \iota(1-\tau_{mc})\frac{w_t^{1-\alpha}}{\eta_{tz}} \tag{K.156}$$

$$(L_t - FC_t) = Y_t\left(\frac{1-\alpha}{\alpha w_t}\right)^{\alpha}\frac{\Delta_t}{\eta_{tz}} \tag{K.157}$$

$$C_t = Y_t\left(1-\left(\frac{\alpha w_t}{1-\alpha}\right)^{1-\alpha}\frac{\Delta_t}{\eta_{tz}}\right)-\eta_{tg} \tag{K.158}$$

If we define $KK_t := \frac{\alpha w_t}{1-\alpha}$, then we can rewrite KK_t

$$KK_t = \frac{\alpha}{1-\alpha}\left(\frac{mc_t\eta_{tz}}{\iota(1-\tau_{mc})}\right)^{1/(1-\alpha)} \tag{K.159}$$

In the same way

$$\kappa L_t^{\chi} = \frac{(1-\alpha)}{\alpha} KK_t \tag{K.160}$$

Finally, the feasibility constraint

$$C_t = \frac{\eta_{tz}}{\Delta_t}(L_t - FC_t)KK_t^{\alpha}(1 - KK_t^{1-\alpha}\frac{\Delta_t}{\eta_{tz}}) - \eta_{tg} \tag{K.161}$$

■

Proof. Define $x_{ti} = \log\left(\frac{p_{ti}A_{ti}}{P_t}\right)$, then using a first order on the price index

$$0 \approx \int xg(x,\Pi)dx \tag{K.162}$$

Since a first order approximation of the $\Delta(\Pi) = \int \left(\frac{p_{ti}A_{ti}}{P_t}\right)^{-\gamma} di$ is zero, a second order approximation gives

$$\log\left(\Delta(\Pi)\right) \approx \gamma \int_{s(\Pi)}^{S(\pi)} x^2 g(x,\Pi)dx = \propto \int x^2 g(x,\Pi)dx \tag{K.163}$$

■

Proposition 1 *Let $\Delta_{ss}(\Pi_{ss})$ be the price dispersion in the Calvo model at a level of inflation Π. Then $\Delta_{ss}(\Pi_{ss})$ is continuous, with* $\left.\frac{d\Delta_{ss}(\Pi_{ss})}{d\Pi_{ss}}\right|_{\Pi_{ss}=1} = 0$ *and* $\left.\frac{d^2\Delta_{ss}(\Pi_{ss})}{d\Pi_{ss}^2}\right|_{\Pi_{ss}=1}$.

Proof. The price dispersion in steady state is given

$$\Delta_{ss} = \Omega \frac{P_{ss}^{-\gamma}}{1-(1-\Omega)\Pi_{ss}^{\gamma}} \tag{K.164}$$

Using the steady state reset price equations

$$\Delta_{ss}(\Pi_{ss}) = \Omega^{\frac{1}{1-\gamma}} \frac{\left(1-(1-\Omega)\Pi_{ss}^{\gamma-1}\right)^{-\frac{\gamma}{1-\gamma}}}{1-(1-\Omega)\Pi_{ss}^{\gamma}} \tag{K.165}$$

It is easy to see that $\Delta_{ss}(\Pi_{ss})$ is continuous, since it is division of continuous functions. For the first order effect, let $N(\Pi) = \left(1-(1-\Omega)\Pi_{ss}^{\gamma-1}\right)^{-\frac{\gamma}{1-\gamma}}$ and $D(\Pi) = 1-(1-\Omega)\Pi_{ss}^{\gamma}$, then we have that

$$\begin{aligned}
\frac{dN(\Pi_{ss})}{d\Pi_{ss}} &= \frac{\gamma}{1-\gamma}\left(1-(1-\Omega)\Pi_{ss}^{\gamma-1}\right)^{\frac{1}{\gamma-1}}(1-\Omega)(\gamma-1)\Pi_{ss}^{\gamma-2} \\
\frac{dD(\Pi_{ss})}{d\Pi_{ss}} &= (1-\Omega)\gamma\Pi_{ss}^{\gamma-1}
\end{aligned}$$

With the first derivative given by

$$\left.\frac{d\Delta_{ss}(\Pi_{ss})}{d\Pi_{ss}}\right|_{\Pi_{ss}=1} = \Omega^{\frac{1}{1-\gamma}} \frac{\Omega(\frac{\gamma}{1-\gamma}\Omega^{\frac{-1}{1-\gamma}}(1-\Omega)(\gamma-1)) - (1-\Omega)\gamma\Omega^{-\frac{\gamma}{1-\gamma}}}{\Omega^2} \tag{K.166}$$

$$= \Omega^{\frac{1}{1-\gamma}-2}\gamma(1-\Omega)(\Omega^{1-\frac{1}{1-\gamma}} - \Omega^{-\frac{\gamma}{1-\gamma}}) \tag{K.167}$$

$$= 0 \tag{K.168}$$

To show convexity notice that $\Delta(\Pi_{ss})$ has a minimum value of 1 at $\Pi_{ss} = 1$. Therefore, at 1 must be convex. ■

L Projection Method Used in the CalvoPlus Model

This section discusses briefly the projection method used in the CalvoPlus model to approximate the firm's value function. This is important, since the optimal reset price and Ss bands come from numerical approximation of the value function; hence, it is important to have a reliable method to approximate the firm's value function. The main idea of this method is to use 2rd order splines on the firm's state variable and Smoliak sparse grid on the aggregate state variables. First, I will describe the construction of grid and then the approximation methods.

- **Grid:** Let $s = [\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_{n_s}]' \in R^{n_s \times 1}$ be the grid on the firm's idiosyncratic state variables and let $S = [S_1, S_2, \ldots, S_{n_S}]' \in R^{n_S \times 6}$ be the grid on the firm's aggregate state variables with $n_s - 1$ breakpoints. I follow Judd *et al.* (2014) and Krueger and Kubler (2004) to construct a sparse grid in S. If we order the firm's state (S, s), then I construct the grid for the firm's state as

$$(S, s) := [\mathcal{I}_{n_s \times 1} \otimes S, s \otimes \mathcal{I}_{n_S \otimes 1}] \in R^{(n_s * n_S) \times 7} \tag{L.169}$$

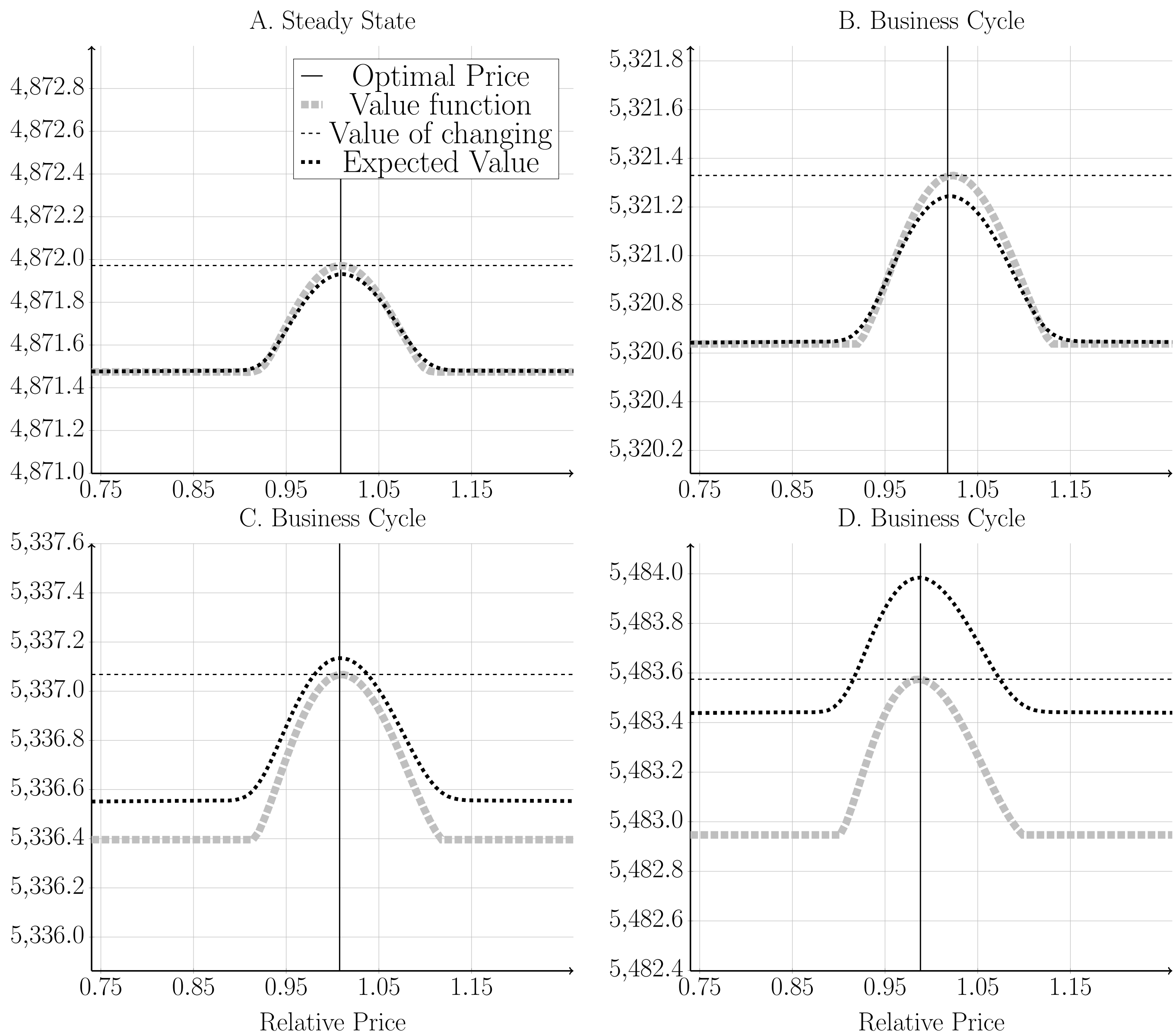
- **Function bases:** let $Z \in R^{H \times 7}$ be an arbitrary point in the grid. To generate the function base for Z I generate the base for the idiosyncratic state variable $\Phi_s(Z_s) \in R^{H \times n_s}$ using 2nd order splines, I generate the base for the aggregate state variable $\Phi_S(Z_S) \in R^{H \times n_S}$ using Smoliak polinomial as in Judd *et al.* (2014) and then I take the Kronequer product

$$\Phi(Z) = \Phi_s(Z_s) \otimes \Phi_S(Z_S) \in R^{H \times (n_s * n_S)} \tag{L.170}$$

Figure VIII describes the value of changing the price, no changing the price, the optimal price and the expected continuation value in the steady state and in the model with business cycle fluctuation. In the steady state, I'm using 3rd order splines and in the model with business cycle I'm using Smoliak sparse grid in the idiosyncratic and aggregate state variables. We can see three properties: 1) there is a large difference in the value of not changing the price in the steady state and in the model with business cycle, 2) the expected continuation value with business cycle has oscillations that the steady state expected value function doesn't have, and 3) the Ss bands in the model with business cycle has bigger Ss bands since it cannot capture the concave-convex shape of the value function. The main reason for this numerical error comes from the sparsity in the grid of the idiosyncratic state and the Chebyshev polynomial cannot capture the shape of the value function.

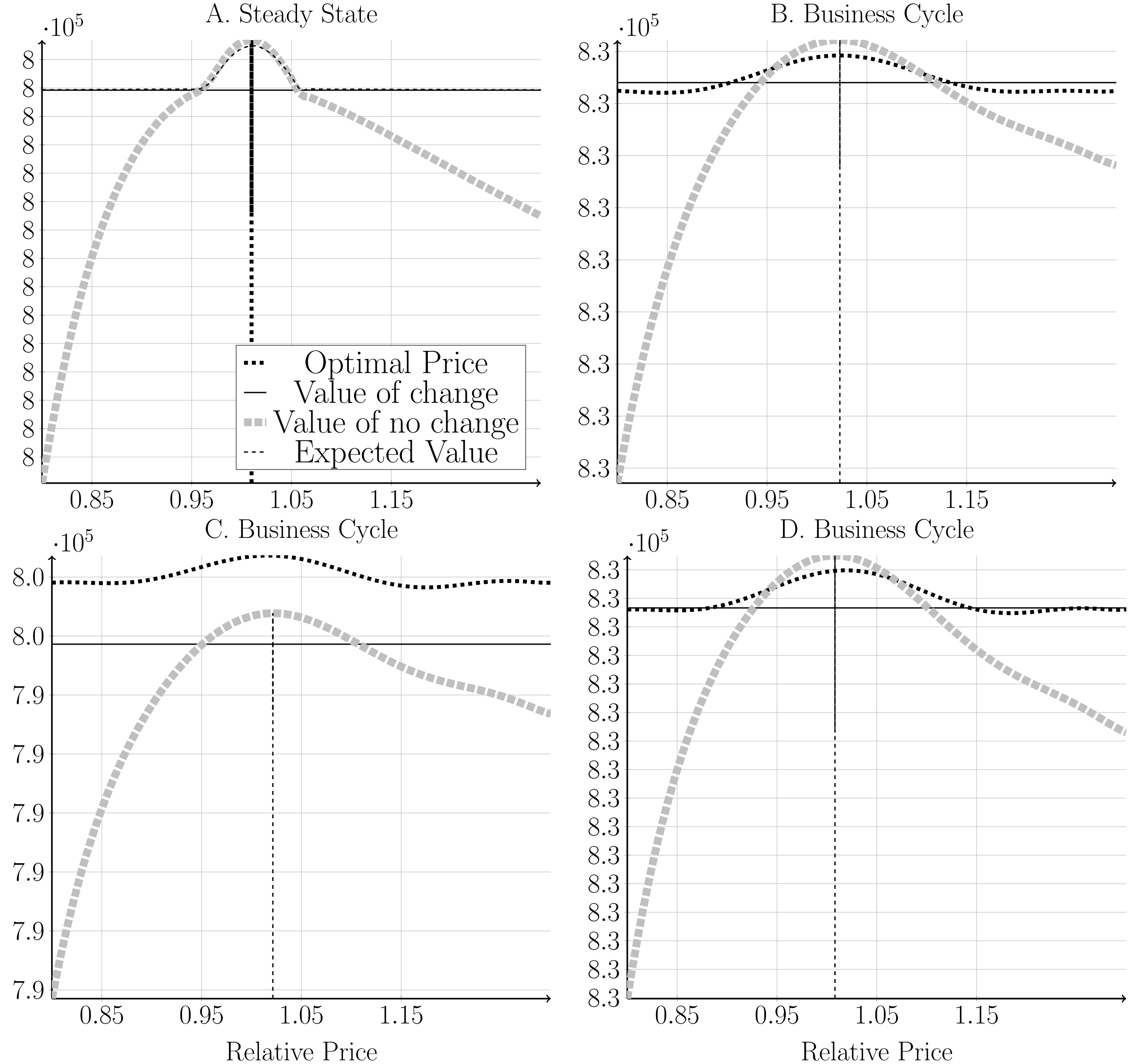
Figure VII describes the value function, the value of changing, the optimal price and the expected value in the steady state and in the model with business cycle fluctuation. In the steady state, I'm using 3rd order splines and in the model with business cycle I'm using Completed Smoliak sparse grid in the state variables. As we can see, none of the previous problems hold.

Figure VII – Value Functions with Completed Smoliak sparse-grid method



Panel A describes the value function, the optimal price, the value of changing the price—without the menu cost—and the expected continuation value in the steady state. Panel B to D describe the value function, the optimal price, the value of changing the price—without the menu cost—and the expected continuation value with business cycle.

Figure VIII – Value Functions with Smoliak sparse-grid method



Panel A describes the value function of no changing the price, the optimal price, the value of changing the price—without the menu cost—and the expected continuation value in the steady state. Panel B to D describe the value function, the optimal price, the value of changing the price—without the menu cost—and the expected continuation value with business cycle.

M Analytical Example of Krusell-Smith

To provide an analytical example, I simplify the model in several dimensions in this section: preferences are given by expected utility $\sigma_{ez} = 0$ with period utility $u(C, L) = \log(C) - L$; the only input of production is labor $y_i = A_i l_i$ and menu costs are constant; the risk premium shock is the only structural shock in the economy and it is i.i.d.; the Taylor rule is given by $R_t = \frac{1}{\beta}\left(\frac{\Pi_t}{1}\right)^{\phi_\pi}$. I abstract from growth rate; thus $g = 0$ and I set the $1 - \tau_{mc} = \frac{\gamma-1}{\gamma}$. Additional the stochastic process for the quality shock is given by

$$\Delta \log(A_{ti}) = \begin{cases} U_{t+1i} & \text{with prob. } p \\ 0 & \text{with prob. } 1-p \end{cases} \quad ; \quad U_{ti} \sim_{i.i.d} U(-\phi/2, \phi/2) \tag{M.171}$$

The final assumption is the one used in Gertler and Leahy (2008): firms can only change the price is they receive an idiosyncratic shock. Under these assumptions, it is possible to obtain all the equilibrium equations.

Proposition 2 *If $\phi/2 > \sqrt{\theta 2 \frac{1-p\beta}{\gamma-1}}$. Then equilibrium is given by*

$$-\hat{C}_t = \hat{R}_t + \epsilon_{qt} - \mathbb{E}_t[\hat{C}_{t+1} + \hat{\Pi}_{t+1}] \quad ; \quad \hat{R}_t = \phi_\pi \hat{\Pi}_t \quad ; \quad \hat{L}_t + \hat{\Delta}_t = \hat{C}_t \tag{M.172}$$

$$\hat{\Pi} = \frac{(1-p)(1-\beta p)}{p}\hat{C}_t + \beta \mathbb{E}_t[\hat{\Pi}_{t+1}] \quad ; \quad \hat{\Delta}_t = (1-p)\hat{\Delta}_t + \frac{p}{1-p}\hat{\Pi}_t^2 \tag{M.173}$$

where $\hat{X} = \log(X_t/X_{ss})$ and X_{ss} is the steady state of the variable X.

Proof. The proof is a simple extension of Gertler and Leahy (2008) for the Phillips curve and Woodford and Walsh (2005) for the dynamic law of price dispersions. ■

Under the approximation of the Phillips curve and the law of price dispersion, an equilibrium is a stochastic process for inflation, consumption, labor, price dispersion and nominal interest rate that satisfies the system of equations (M.175) together with (M.176). Since inflation and consumption are forward looking variables, there exists an unique equilibrium if this economy satisfies the Taylor rule principle, i.e. the interest rate respond strongly to inflation.

Given that the equilibrium system is triangular—first I can solve for inflation, consumption and nominal rate and then for labor and price dispersion—the equilibrium consumption, inflation and nominal interest rate are given by

$$[\hat{C}_t \quad \hat{\Pi}_t \quad \hat{R}_t] = [K_c \quad K_\pi \quad K_R]\epsilon_{qt} \tag{M.174}$$

Krusell-Smith fails in this economy, since if I replace the Phillips curve by the equilibrium policy of inflation $\hat{\Pi}_t = K_\pi \epsilon_{qt}$ there are infinite solutions in the following system

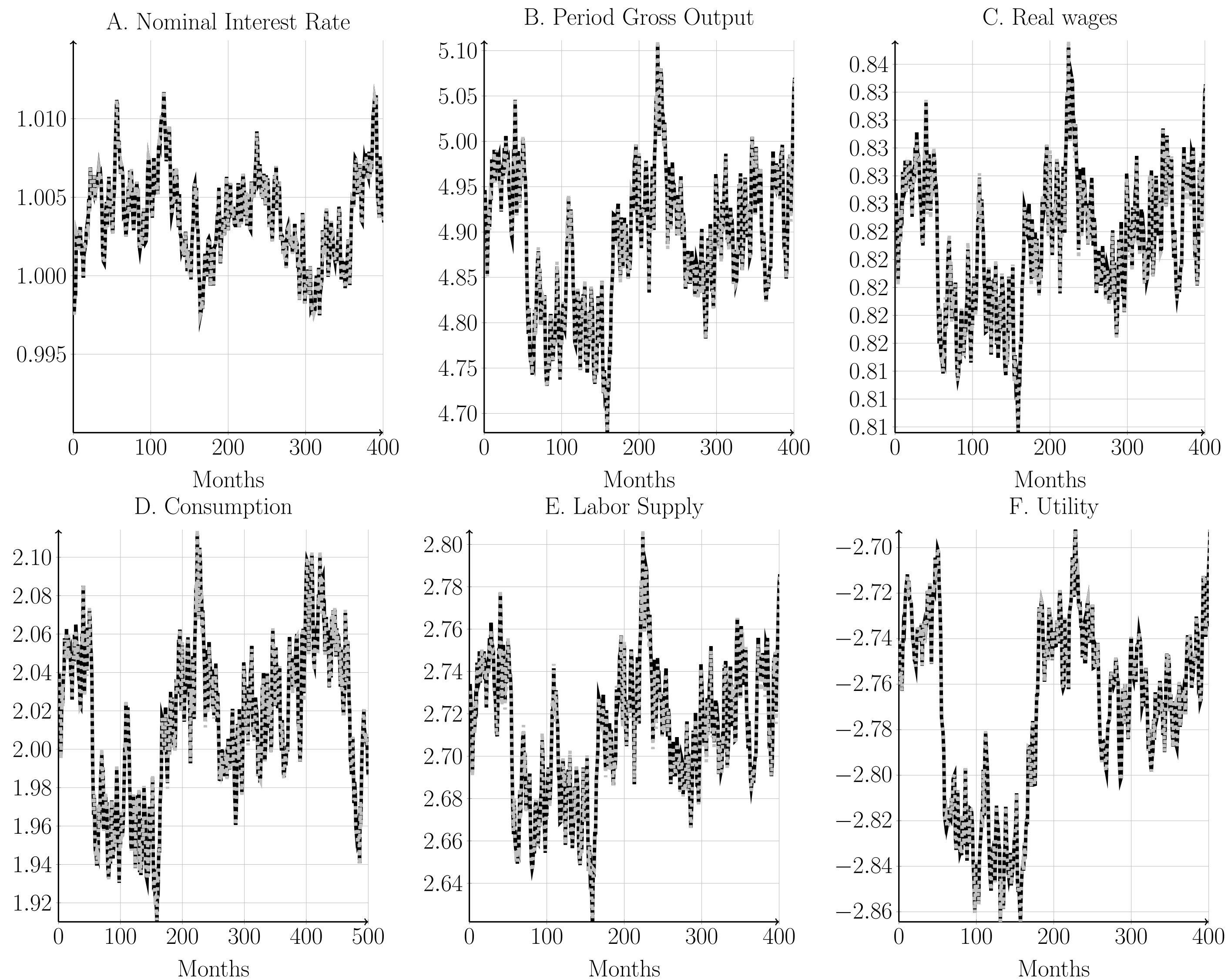
$$-\hat{C}_t = \hat{R}_t + \epsilon_{qt} - \mathbb{E}_t[\hat{C}_{t+1} + \hat{\Pi}_{t+1}] \quad ; \quad \hat{R}_t = \phi_\pi \hat{\Pi}_t \quad ; \quad \hat{L}_t + \hat{\Delta}_t = \hat{C}_t \tag{M.175}$$

$$\hat{\Pi}_t = K_\pi \epsilon_{qt} \quad ; \quad \hat{\Delta}_t = (1-p)\hat{\Delta}_t + \frac{p}{1-p}\hat{\Pi}_t^2 \tag{M.176}$$

The main reason why nominal interest rate responds to real wage is because there is an equilibrium equation given by the Phillips curve that relates inflation to real wage. *This is the critical property that Krusell-Smith breaks.*

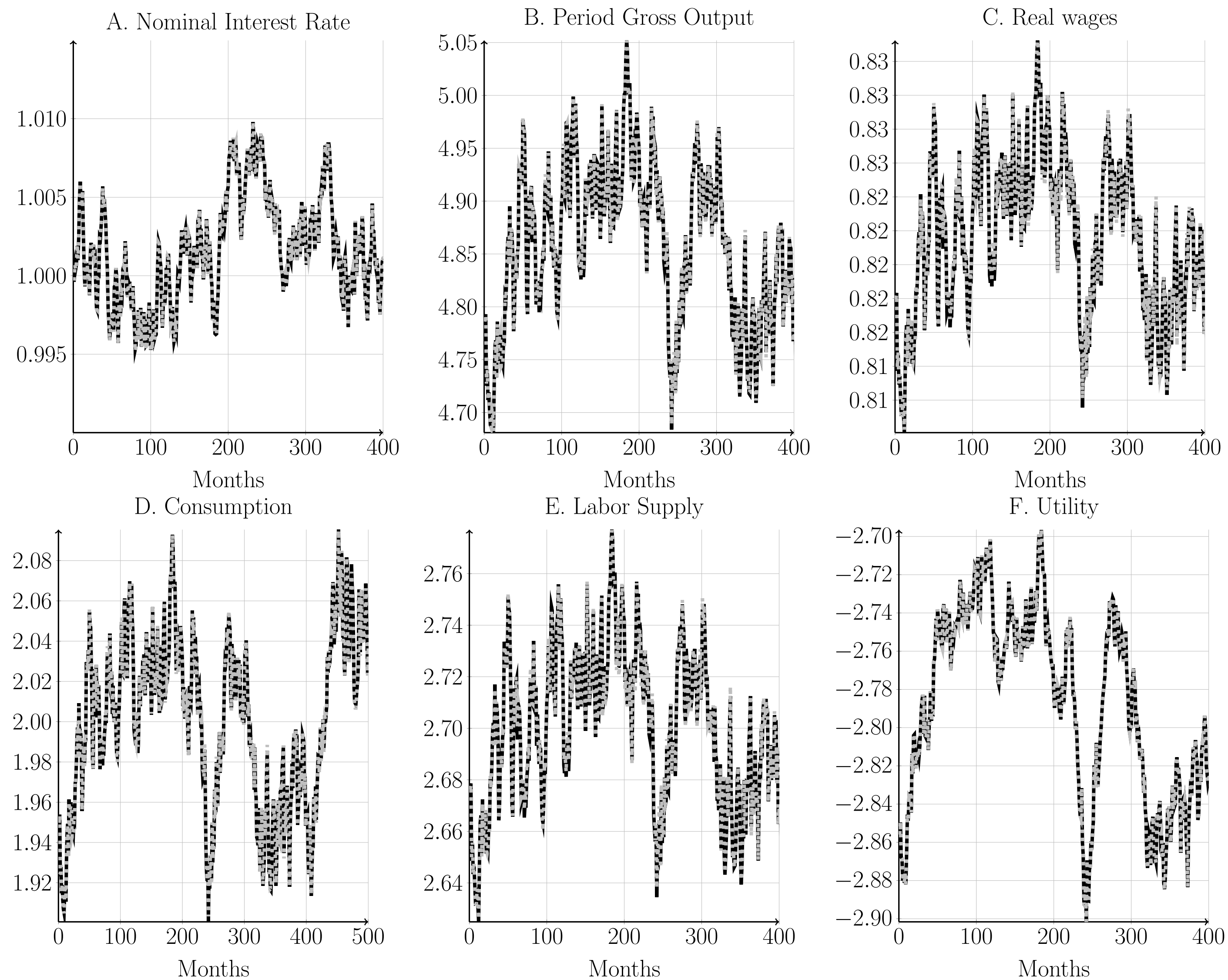
N Figures for Evaluation of Krusell-Smith

Figure IX – Predicted and Simulated Aggregate Time Series with no ZLB at 0% Inflation Target



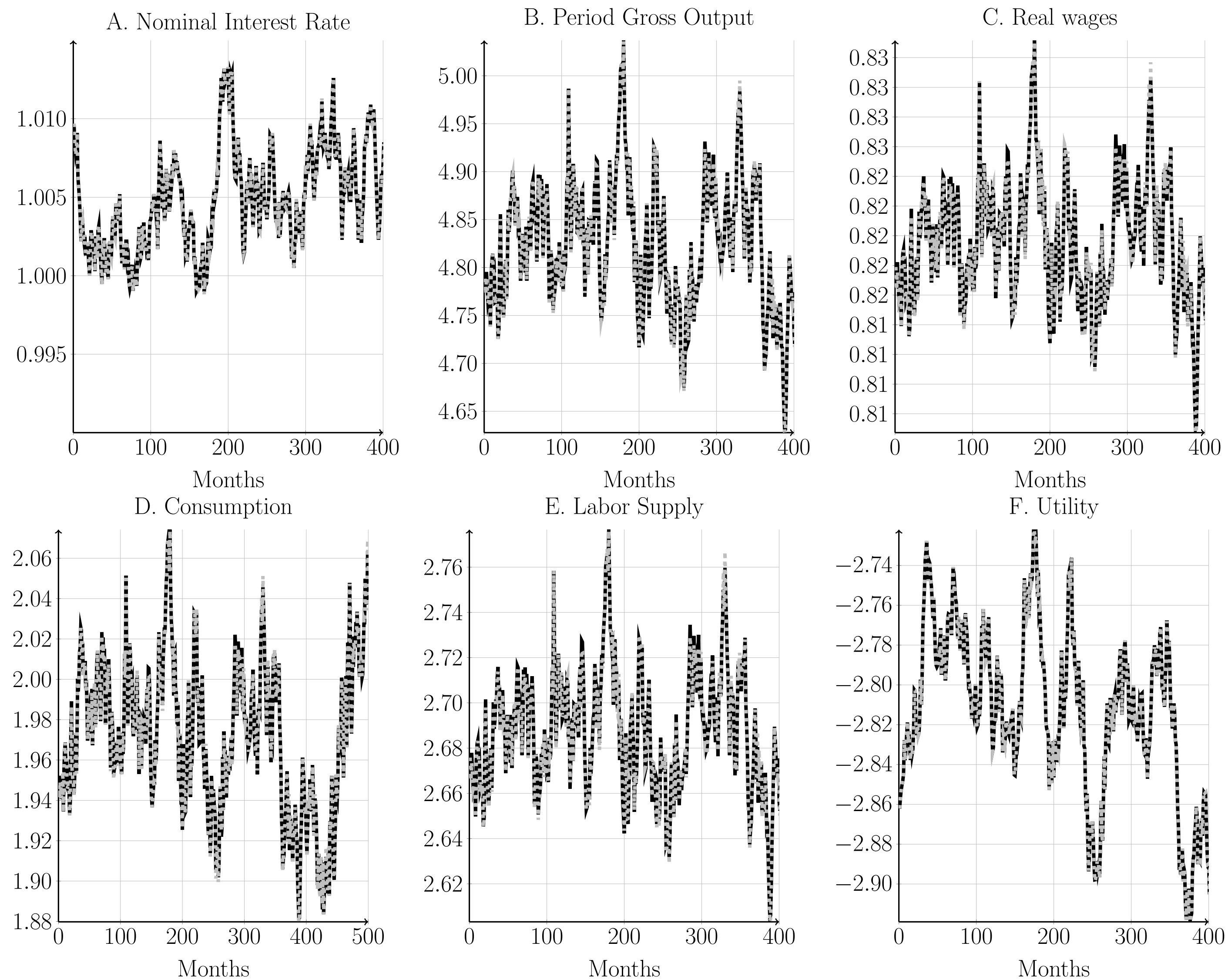
Panel A to F describe the macroeconomic time series for one simulation. The black line describes the model implied aggregate variable with the Krusell-Smith projections and the light-gray line describes the implied macroeconomic time series with simulated inflation, frequency of price change and price dispersion.

Figure X – Predicted and Simulated Aggregate Time Series with no ZLB at 2% Inflation Target



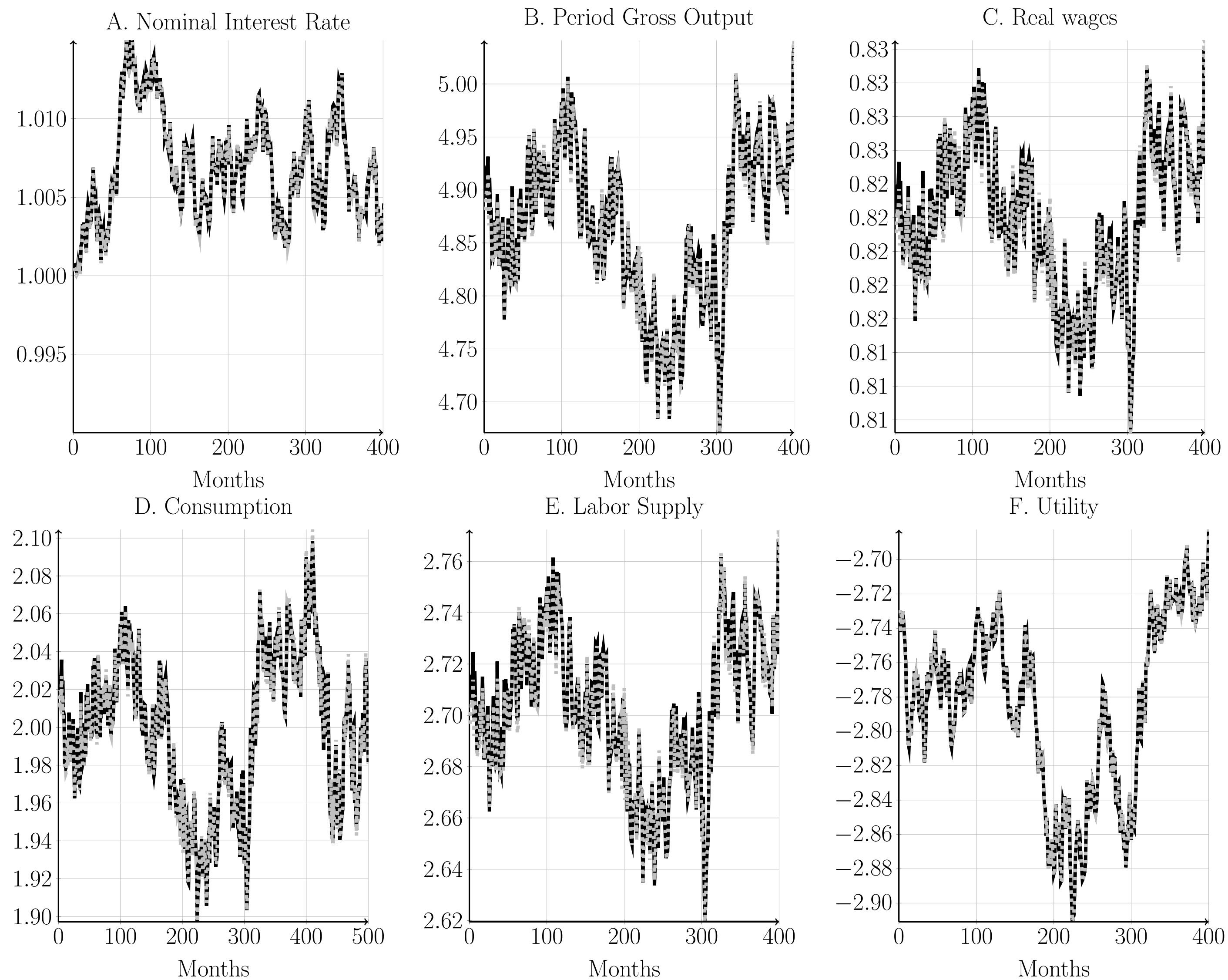
Panel A to F describe the macroeconomic time series for one simulation. The black line describes the model implied aggregate variable with the Krusell-Smith projections and the light-gray line describes the implied macroeconomic time series with simulated inflation, frequency of price change and price dispersion.

Figure XI – Predicted and Simulated Aggregate Time Series with no ZLB at 4% Inflation Target



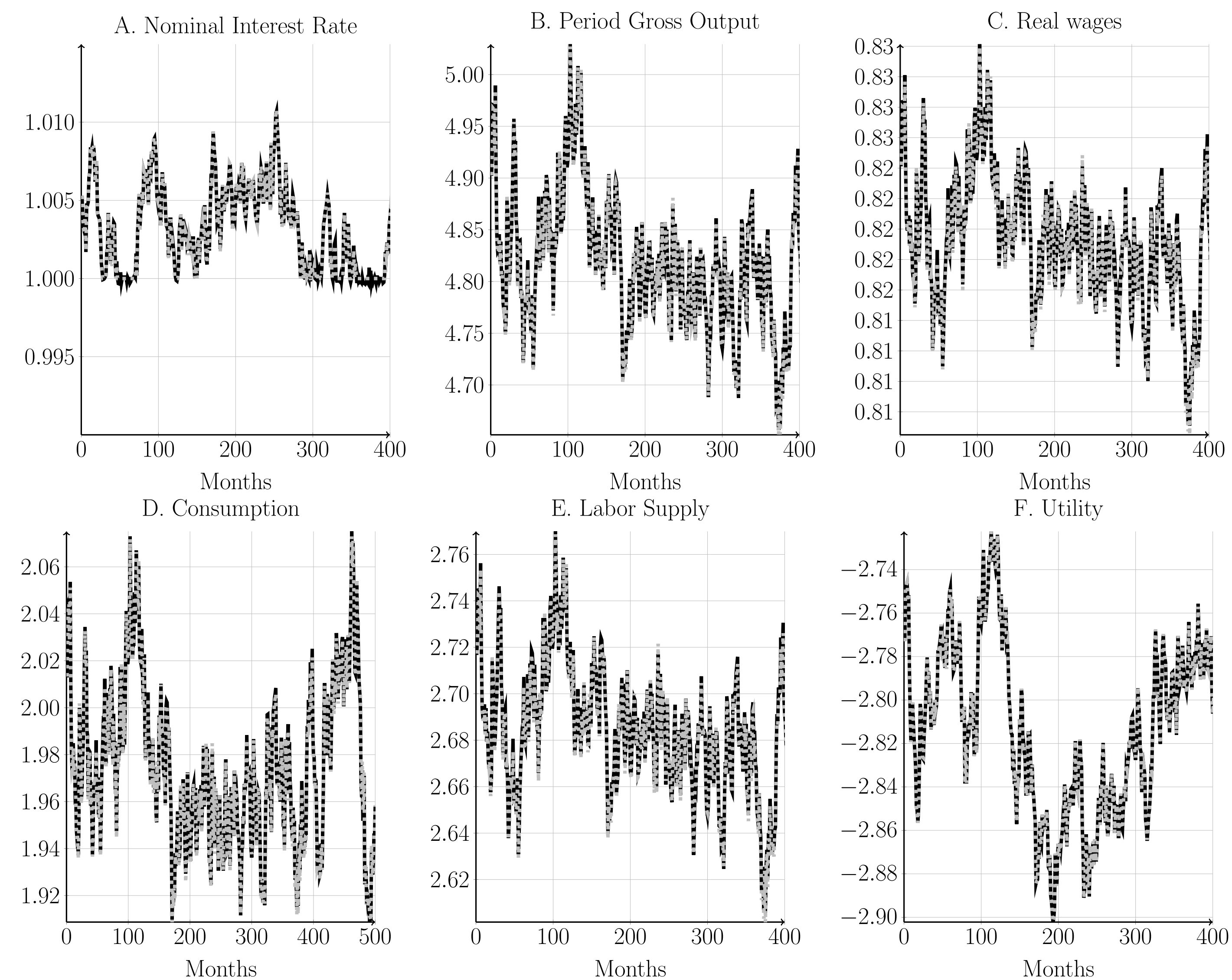
Panel A to F describe the macroeconomic time series for one simulation. The black line describes the model implied aggregate variable with the Krusell-Smith projections and the light-gray line describes the implied macroeconomic time series with simulated inflation, frequency of price change and price dispersion.

Figure XII – Predicted and Simulated Aggregate Time Series with no ZLB at 6% Inflation Target



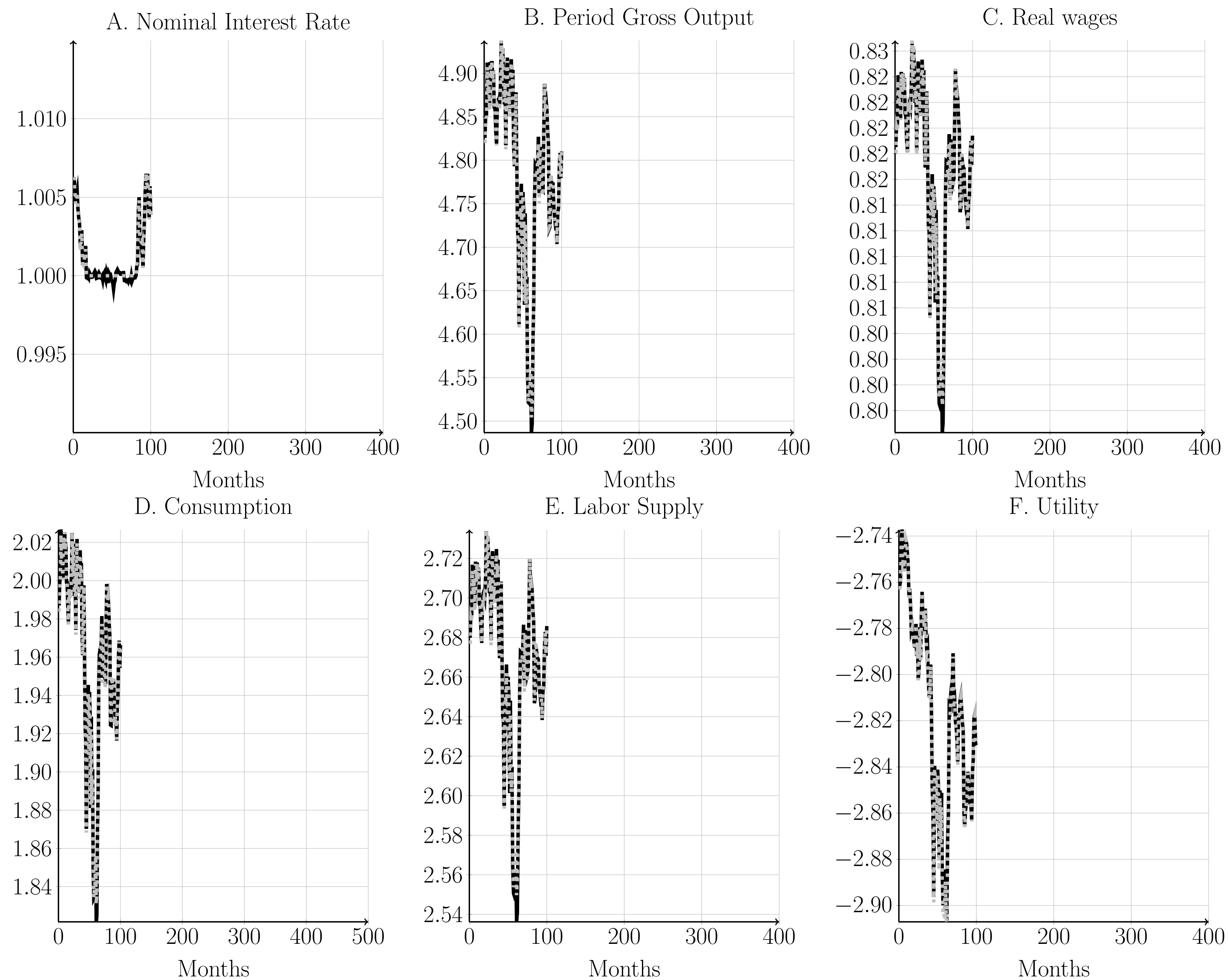
Panel A to F describe the macroeconomic time series for one simulation. The black line describes the model implied aggregate variable with the Krusell-Smith projections and the light-gray line describes the implied macroeconomic time series with simulated inflation, frequency of price change and price dispersion.

Figure XIII – Predicted and Simulated Aggregate Time Series with ZLB at 0% Inflation Target



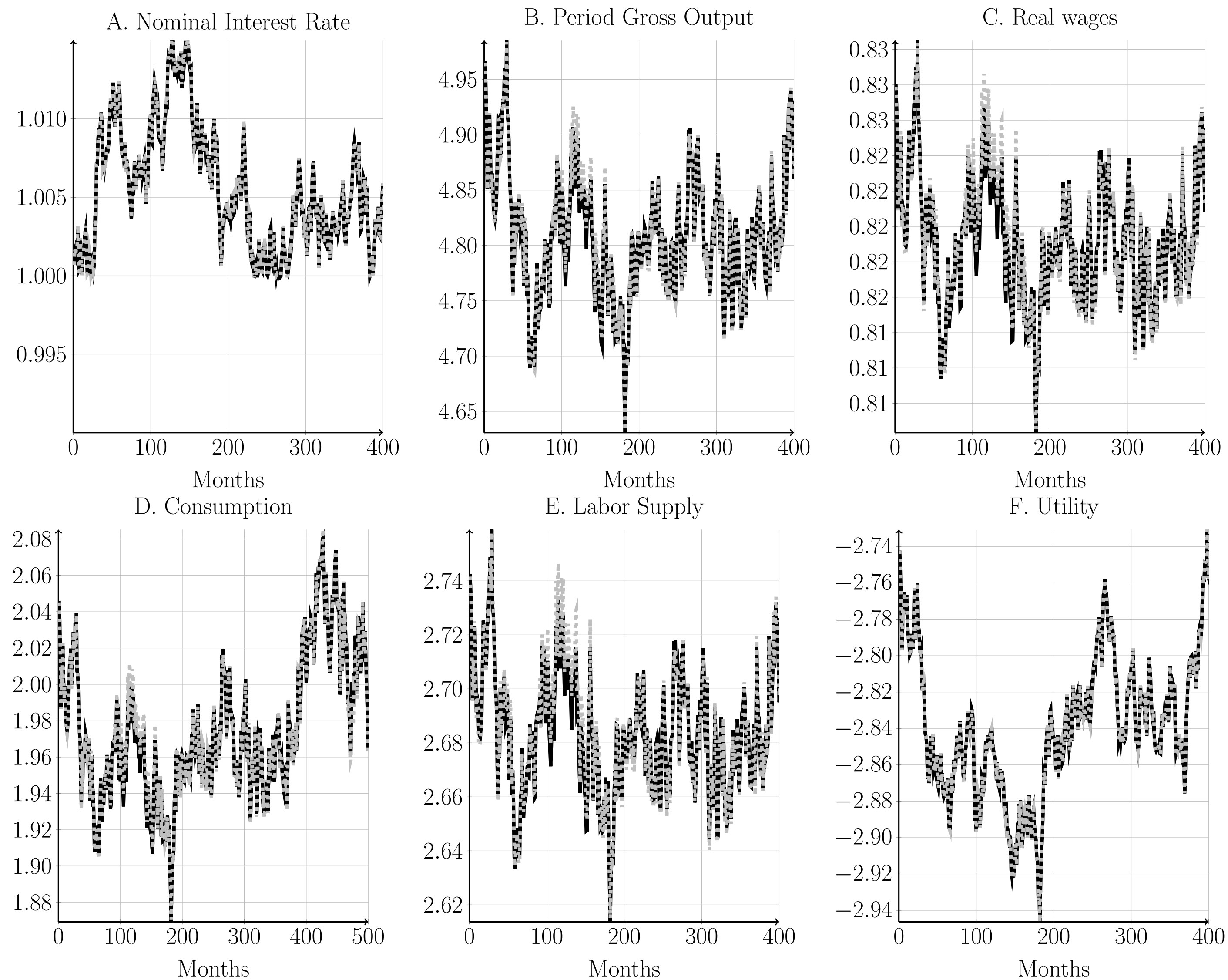
Panel A to F describe the macroeconomic time series for one simulation. The black line describes the model implied aggregate variable with the Krusell-Smith projections and the light-gray line describes the implied macroeconomic time series with simulated inflation, frequency of price change and price dispersion.

Figure XIV – Predicted and Simulated Aggregate Time Series with ZLB at 2% Inflation Target



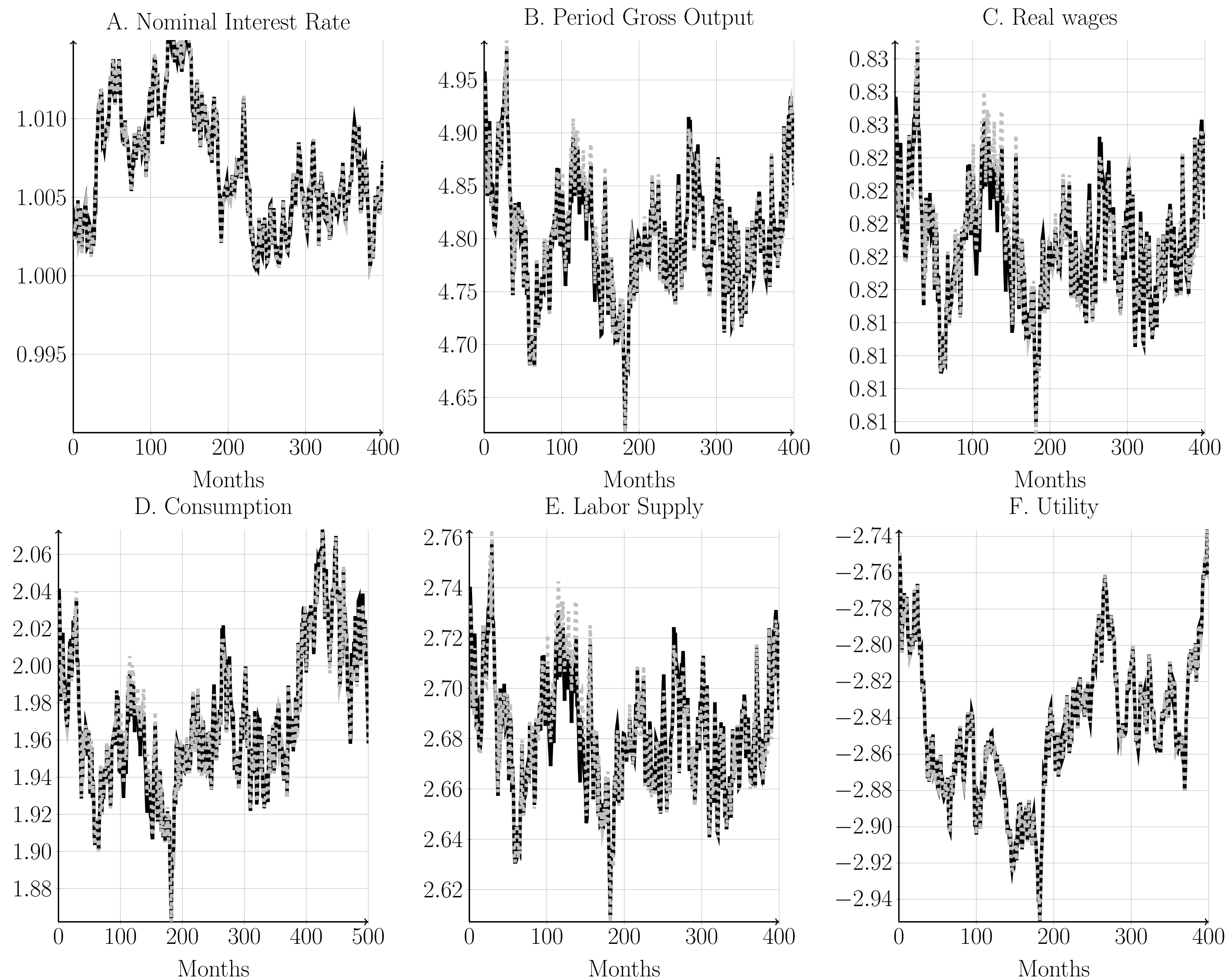
Panel A to F describe the macroeconomic time series for one simulation. The black line describes the model implied aggregate variable with the Krusell-Smith projections and the light-gray line describes the implied macroeconomic time series with simulated inflation, frequency of price change and price dispersion.

Figure XV – Predicted and Simulated Aggregate Time Series with ZLB at 4% Inflation Target



Panel A to F describe the macroeconomic time series for one simulation. The black line describes the model implied aggregate variable with the Krusell-Smith projections and the light-gray line describes the implied macroeconomic time series with simulated inflation, frequency of price change and price dispersion.

Figure XVI – Predicted and Simulated Aggregate Time Series with ZLB at 6% Inflation Target



Panel A to F describe the macroeconomic time series for one simulation. The black line describes the model implied aggregate variable with the Krusell-Smith projections and the light-gray line describes the implied macroeconomic time series with simulated inflation, frequency of price change and price dispersion.

O ZLB dynamics in the Calvo and CalvoPlus Model

This section describes the computation of the conditional impulse-response for the Calvo and CalvoPlus model, together with the business cycle statistics.

O.1 Conditional Impulse Response Functions

Let $IR_{tx}^{My\mathcal{F}K}$ be the conditional impulse-response in the model M, after t periods—with $t \leq K$—of the structural shock $x \in \{z, g, r, q\}$ in the y variable, with initial state in $\mathcal{F}$. Next, I describe the steps to generate the random variable $IR_{tx}^{My\mathcal{F}K}$ using Monte Carlo methods.

1. Simulate the model for a large T. Let $\{S_t^X\}_{t=0}^T$ be the time-series of the aggregate state—in the case of the CalvoPlus model, it includes the distribution of relative prices. Let $X_{T,N} = \{X_t\}_{t=0}^{T_N} = \{S_{Nt}^X\}_{t=0}^T$ be a sample of length N. The main idea of only taking the state every N periods is that it saves memory to do the i.i.d. sampling. If N is large enough, It would not be necessary to do an i.i.d. random sampling.
2. Keep with $\{X_t\}$ in $\mathcal{F}$. Call this new vector $\tilde{X}_{T,N}$.
3. Generate a random i.i.d. sequence of dates $\{t_i\}_{i=1}^N$ and draw $\{\tilde{X}_{t_i}\}_{i=1}^N$ samples.
4. For each random sample $i = 1, 2, ..., N$:
   i. Set $Z_0 = \tilde{X}_{t_i}$, shock the variable x with $h\sigma_x$ standard deviations in period 1 and no other shocks in this period.
   ii. Continue simulating the model for K periods for every variable y.
   iii. Record $IR_x(t, i)^{My\mathcal{F}K}$ for all y variables.
5. $\{IR_x(t, i)^{My\mathcal{F}K}\}_{i=1}^N$ is a random sample from $IR_{tx}^{My\mathcal{F}K}$.

O.2 Additional Statistics

Figures XVII and XVIII describe the kernel estimate of the structural shocks conditional on being at the zero lower bound and unconditional. To compute each graph I apply a kernel density estimate to η_{zt} and $\eta_{zt}|R_t^* < 1$.

O.3 Impulse-Response Functions Calvo-CalvoPlus Models

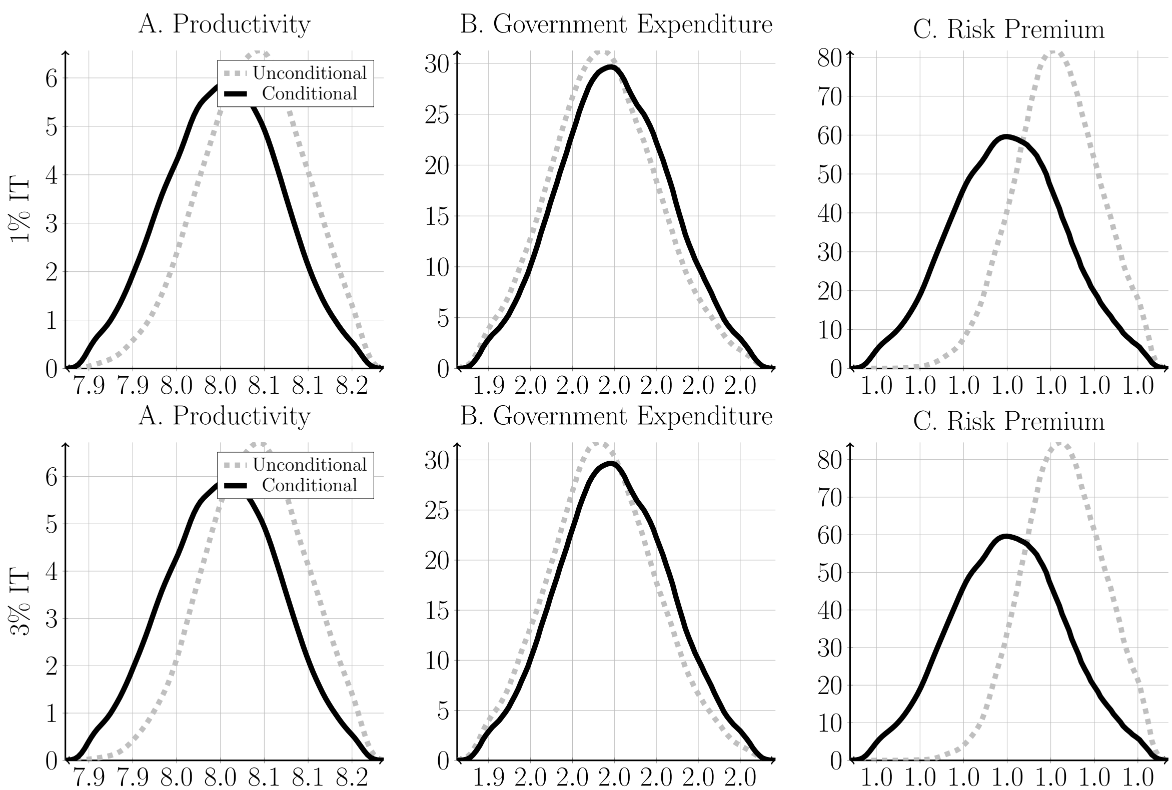
O.4 Computations of the statistics in Table III

O.5 Computations of the statistics in Table III

For a given simulation $\{R_t^*, mc_t, \Delta_t, C_t, L_t, \Pi_t, P_t, \varphi_t, \Omega_t\}_{t=0}^T$ I compute the statistics in table III in the following way:

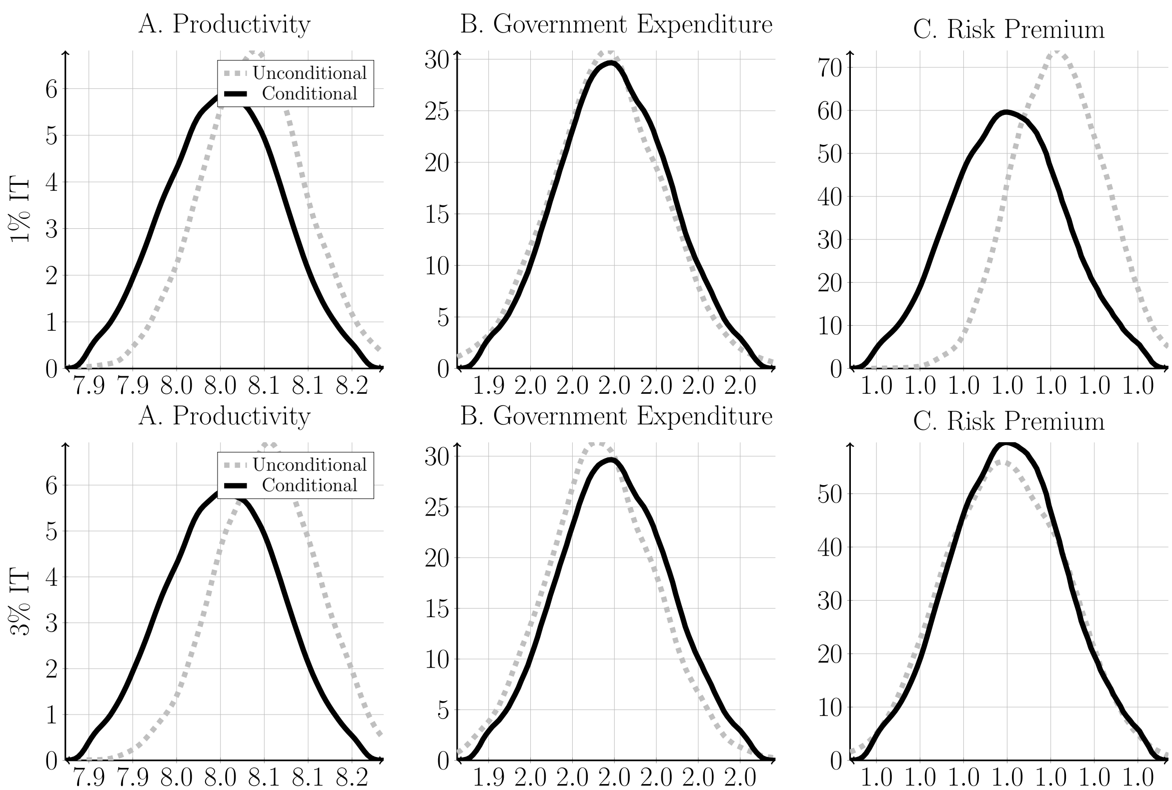
- Frequency: $\frac{\sum_t I(R_t^*<1)}{T}$.
- Duration: Let $\mathcal{T}^0$ be the set of first date of ZLB given by $\mathcal{T}^0 = \{t \leq T : R_{t-1}^* > 1 \ \& \ R_t^* \leq 1\}$ and $\tau(x)$ the duration for the a spell x given by $\tau(x) = \min_j\{j : R_{t-1+j}^* > 1\}$. Then for all $\tau_i \in \tau([\mathcal{T}^0])$, I compute the mean, standard deviation, skewness and the 10,50 and 95 percentiles.
- $(\mathbb{E}[\tilde{H}_t] = T^{-1}\sum_t log(H_t)$ with $H \in \{mc, \Delta, C, L, \Pi, P, \varphi, \Omega\}$.
- $,\mathbb{E}^{zlb}[\hat{X}_t] = \frac{\sum_t \log(H_t/(T^{-1}\sum_t H_t))I(R_t^*<1)}{\sum_t I(R_t^*<1)}$ with $H \in \{mc, \Delta, C, L, \Pi, P, \varphi, \Omega\}$.
- $\mathbb{E}^{no\ zlb}[\hat{X}_t]) = \frac{\sum_t \log(H_t/(T^{-1}\sum_t H_t))I(R_t^*>1)}{\sum_t I(R_t^*>1)}$ with $H \in \{mc, \Delta, C, L, \Pi, P, \varphi, \Omega\}$.
- $(\mathbb{S}td[\tilde{X}_t] = T^{-1}\sum_t log(H_t)^2 - \left(T^{-1}\sum_t log(H_t)\right)^2$ with $H \in \{mc, \Delta, C, L, \Pi, P, \varphi, \Omega\}$.

Figure XVII – Conditional and Unconditional Distribution of Aggregate Shocks in the Calvo-Plus model



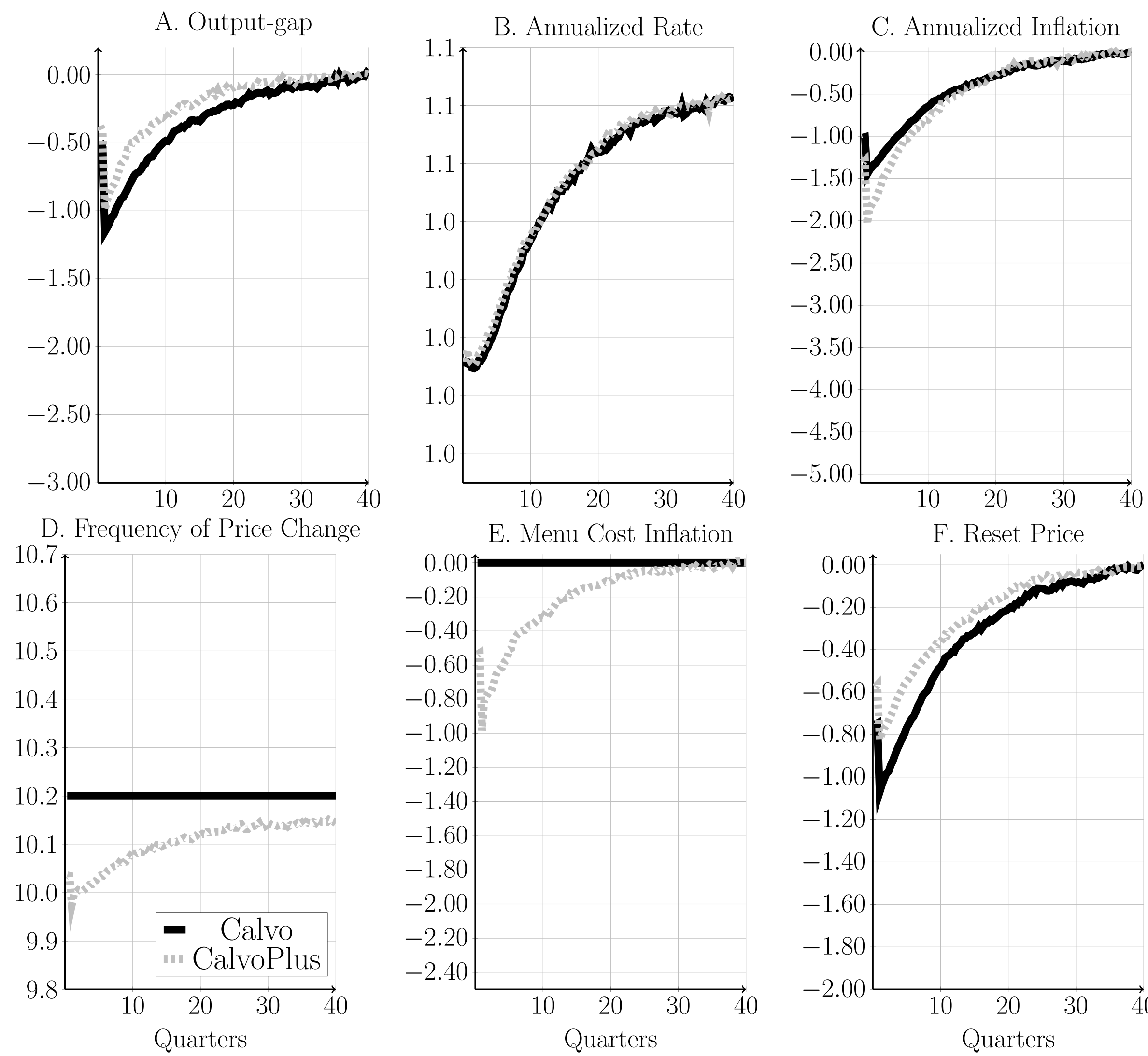
This figure describes the unconditional distribution of aggregate exogenous shocks and the distribution of aggregate shocks conditional of a zero interest rate in the CalvoPlus model. The upper row describes these distributions at 1% inflation target and the lower row describes these distributions at 3% inflation target.

Figure XVIII – Conditional and Unconditional Distribution of Aggregate Shocks in the Calvo model



This figure describes the unconditional distribution of aggregate exogenous shocks and the distribution of aggregate shocks conditional of a zero interest rate in the Calvo model. The upper row describes these distributions at 1% inflation target and the lower row describes these distributions at 3% inflation target.

Figure XIX – Non-Linear Impulse Response for Risk Premium Shock at Three Percent Percent Inflation Target



Panel A to F describe the non-linear median impulse response in the CalvoPlus model and in the Calvo model. Panel A, C, E and F are in percentage deviation from the terminal value. Interest rate and inflation are monthly rates.

- $\mathbb{S}td^{zlb}[\tilde{X}_t]] = \frac{\sum_t log(H_t)^2 I(R_t^*<1)}{I(R_t^*<1)} - \left(\frac{\sum_t log(H_t) I(R_t^*<1)}{I(R_t^*<1)}\right)^2$ with $H \in \{mc, \Delta, C, L, \Pi, P, \varphi, \Omega\}$.
- $\mathbb{S}td^{no\ zlb}[\tilde{X}_t]]) = \frac{\sum_t log(H_t)^2 I(R_t^*>1)}{I(R_t^*>1)} - \left(\frac{\sum_t log(H_t) I(R_t^*>1)}{I(R_t^*>1)}\right)^2$ with $H \in \{mc, \Delta, C, L, \Pi, P, \varphi, \Omega\}$.

P High-Inflation Price Statistics

This section of the appendix, describes the international and microeconomic evidence between micro-price statistics and inflation.

P.1 Internation Evidence

First, I construct a database for inflation and frequency of price change across different countries. Table X shows the statistics of each data base. It is important to remark that each database has different time periods, different frequency, different set of goods, different weight across goods. Importantly, since my objective is not to explain hyperinflation, I drop levels of inflation bigger than 30%—the result are insensitive to this exact number.

Table X – Frequency of price change and inflation across countries

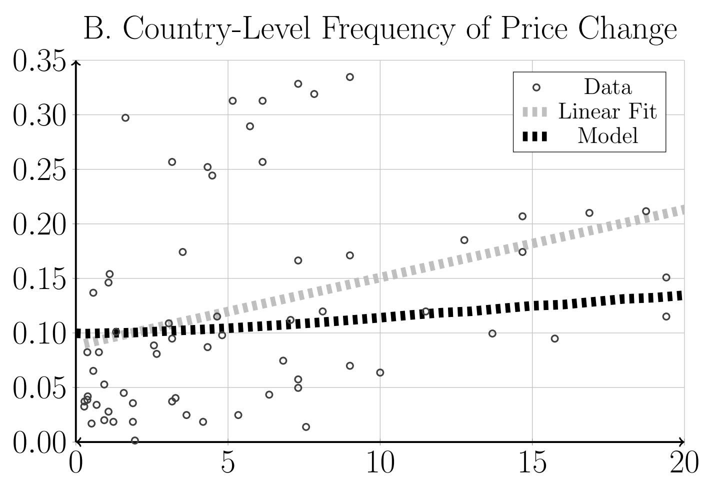
country	source	initial year	end year	mean inflation	mean frequency
Argentina	Alvarez et al (2015)	1991	1997	99202.1	42.6819
Brazil	Barros et al. (2009) Monthly	1996	2013	.423402	37.2333
Israel	Baharad and Eden (2004)	1991	1992	12.7513	26.2051
Mexico	Gagnon (2009)	1994	2002	14.3003	29.395
Norway	Wulfsberg(2016)	1975	2004	5.37058	22.1043
US	Nakamura at el. (2016)	1978	2014	3.69253	10.7967

Let Π_{ti} be the annualize inflation rate for time period t and country i, Ω_{ti} the monthly frequency of price change for country i and period t. I run the following regression to compute the sensitivity of inflation with respect to inflation:

$$\Omega_{ti} = \alpha_t + \beta_1 \Pi_{ti} + \beta_2 \Pi_{ti}^2 \tag{P.177}$$

Notice that there is a fixed effect, since there are different levels of frequencies for different countries. Additionally, this regression captures business cycle sensitivity. Next graph shows the relation between inflation and frequency in the data and in the model.

Figure XX – Micro-Price Statistics For Different Levels of Inflation Model-International Data



This figure describes international evidence on the pricing behavior of frequency of price change with respect to inflation.

Internationally evidence also reports a positive relation between the extensive margin component of inflation and average inflation. For the definition of the intensive-extensive component of inflation I follow Klenow and Kryvtsov (2008). In their paper, they decompose the total volatility of inflation between extensive and intensive margin using the following formula:

$$\mathbb{V}ar\left[\log(\Pi_t)\right] = \underbrace{\mathbb{V}ar\left[\Delta \log(p_t)\right]\mathbb{E}_t\left[\Omega_t\right]^2}_{\text{Intensive Marg. Component}} + \underbrace{\mathbb{V}ar\left[\Omega_t\right]\mathbb{E}_t\left[\Delta \log(p_{ti})\right]^2 + 2\mathbb{E}_t\left[\Omega_t\right]\mathbb{E}_t\left[\Delta \log(p_t)\right]\mathbb{C}ov(\Delta \log(p_t), \Omega_t)}_{\text{Intensive Marg. Component}} \tag{P.178}$$

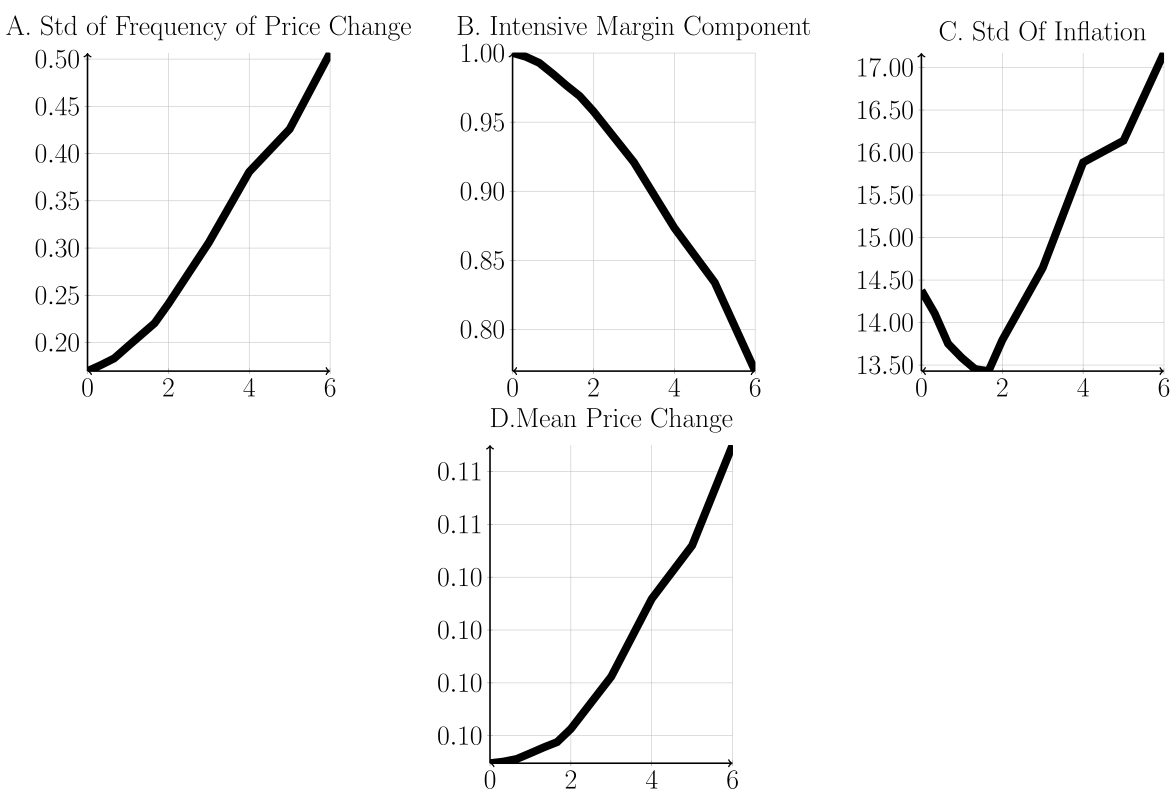
There are no so many computations of these statistics across countries. Table ZZ describes the computations I found in the literature.

Table XI – Inflation Volatility Decomposition

country	source	Average Inflation	Intensive Marginal	Extensive Margin
Mexico (1994-1999)	Gagnon (2009)	21	34.7	65.3
Mexico (1999-2001)	Gagnon (2009)	5	89.2	10.8
US (1988-2007)	Klenow and Kryvtsov (2008)	2.3	95	5
UK (1996-2015)	Carvalho and Kryvtsov (2015)	2	86	3

Wulfsberg (2016) also find a decreasing intensive component in high environment economies, but this methodology makes it incompatible with previous studies.

Figure XXI – Micro-Price Statistics For Different Levels of Inflation Model

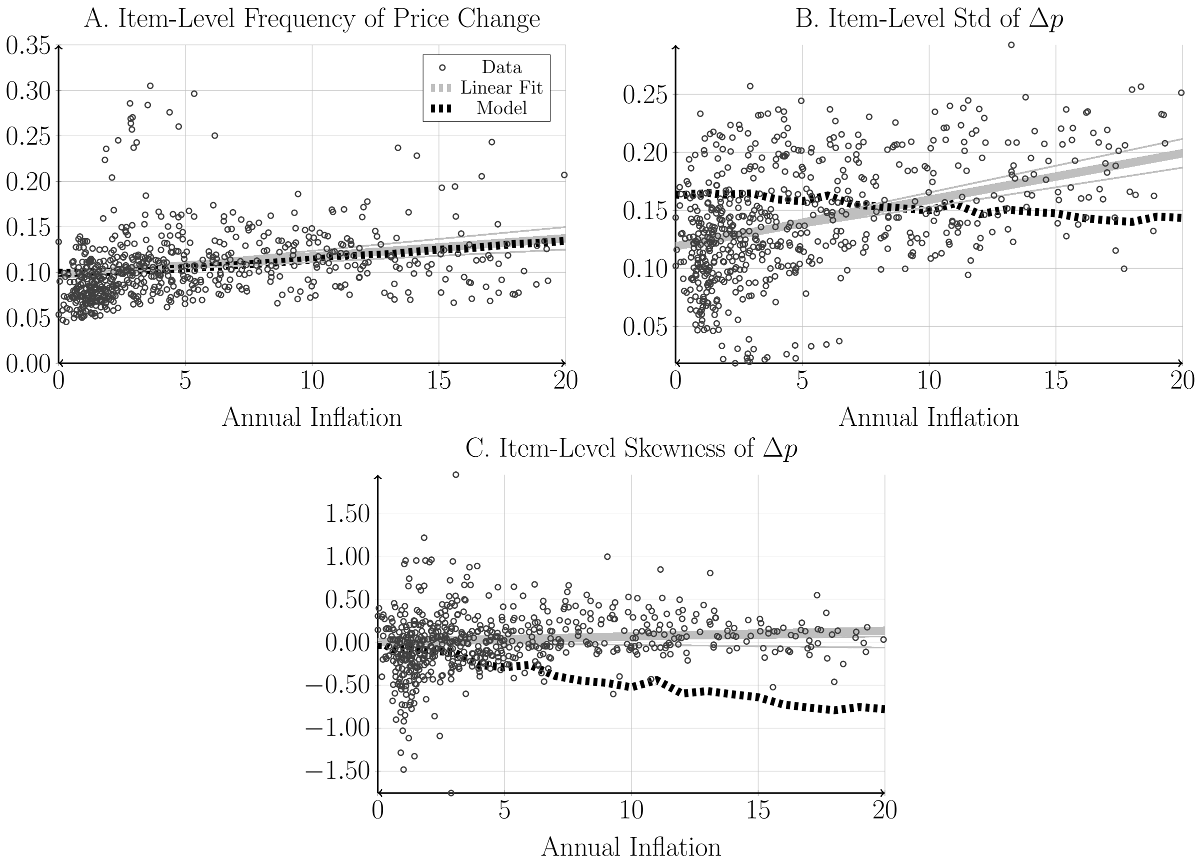


This figure describes the variance of the frequency of price changes, mean price dispersion intensive margin component of inflation and standard deviation of inflation in the model without zero lower bound. The standard deviation of frequency of price change and inflation are multiply by 100.

P.2 Product Level Evidence

To compute price statistics at product level, I apply the same statics as in the main text except the standardization. I compute the mean frequency of price change, mean, standard deviation and skewness of the size of price change. As with international price statistics, I drop products with average annualize inflation more than 25%. Table XXII describes the data-model comparison. In the case of the fit, I use the same regression than with international comparison. The result of the linear fix is robust to quadratic fit and robust regression.

Figure XXII – Micro-Price Statistics For Different Levels of Inflation Model-Data



This figure describes product evidence on pricing behavior on frequency of price change, standard deviation of price change and skewness of price change.