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Innovation, industry equilibrium, and discount rates

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Abstract

We develop a model to examine how discount rates affect the nature and composition of innovation within an industry. Challenging conventional wisdom, we show that higher discount rates do not discourage firm innovation when accounting for the industry equilibrium. Higher discount rates deter fresh entry—effectively acting as entry barriers—but encourage innovation through the intensive margin, which can lead to a higher industry innovation rate on net. Simultaneously, high discount rates foster explorative over exploitative innovation. The model rationalizes observed patterns of innovation cyclicality, and predicts that lower entry in downturns hedges innovating incumbents against higher discount rates.

Keywords: Vertical and horizontal innovation, creative destruction, time-varying discount rates, risk premia.

JEL Classification Numbers: G31; G12; O31

Non-technical summary

Since Schumpeter (1939), scholars have argued that innovation is key to understand the real economy. In recent years, the study of the determinants of corporate innovation has become particularly relevant, as firms' investment in research and development (henceforth, R&D) has increased dramatically. Despite this growing interest, the literature has so far neglected the role of discount rates in explaining firms' R&D. This is surprising, as existing studies show that discount rates are key to explain other corporate decisions, such as physical investment, initial public offerings, buyouts, mergers, or unemployment (i.e., Lamont, 2000; Pastor and Veronesi, 2005; Haddad, Loualiche, and Plosser, 2017; Hall, 2017). However, discount rates should be arguably more significant in explaining R&D, as it is a long term investment, has an extended gestation period, and bears an uncertain outcome.

This paper seeks to fill this gap and develops a model studying how discount rates affect corporate innovation in industry equilibrium. Corporate finance textbooks suggest that higher discount rates should penalize cash flows expected in the far future and, thus, should discourage investment, especially longer-term ones such as R&D. Yet, this line of reasoning neglects that a firm's R&D largely depends on the presence and decisions of competing firms, which are also affected by discount rates. Taking into account that firms do not innovate in isolation, our model thus shows that a higher market price of risk—the common component of firms' discount rates in the cross section—can lead to greater innovation rates by affecting the composition and nature of innovation within the industry—that is, whether it is performed by incumbents or entrants, and whether it is exploitative or explorative. Our results reveal that discount rate fluctuations help rationalize the cyclicality of R&D: We reconcile the Schumpeterian view that firm-level innovation should be countercyclical with the documented procyclicality of aggregate R&D. The model also shows that lower entry in downturns hedges innovating incumbents against higher discount rates. Importantly, our novel framework allows us to prove key results analytically while incorporating endogenous entry and heterogeneous firms.

Introduction

Since Schumpeter (1939), scholars have argued that innovation is key to understand the real economy. In recent years, the study of the determinants of corporate innovation has become particularly relevant, as firms' investment in research and development (henceforth, R&D) has increased dramatically.[1] Despite this growing interest, the literature has so far neglected the role of discount rates in explaining firms' R&D. This is surprising: Existing studies show that discount rates are key to explain other corporate decisions, such as physical investment, initial public offerings, buyouts, mergers, or hiring and layoff decisions (i.e., Lamont, 2000; Pastor and Veronesi, 2005; Malenko and Malenko, 2015; Haddad, Loualiche, and Plosser, 2017; Hall, 2017). Yet, discount rates should be arguably more significant in explaining R&D, as it is a long term investment, has an extended gestation period, and bears an uncertain outcome.

This paper seeks to fill this gap by developing a novel theoretical framework to study how discount rates affect corporate R&D, while stressing the importance of studying innovation in industry equilibrium. Corporate finance textbooks suggest that higher discount rates should penalize cash flows expected in the far future and, thus, should discourage investment, especially longer-term ones such as R&D. Yet, this line of reasoning neglects that a firm's R&D largely depends on the presence and decisions of competing firms, which are also affected by discount rates. As a starting point, we empirically document that higher discount rates need not reduce innovation. As the aggregate risk premium increases, US firms invest a *larger* fraction of their assets or sales on R&D, on average—i.e., discount rates appear to have a positive impact on the intensive innovation margin. Simultaneously, however, higher discount rates arguably discourage innovation in the extensive margin: A higher aggregate risk premium relates to significantly *fewer* firms. Figure 1 shows these joint patterns, and Table 1 in the Appendix provides additional supporting evidence.[2]

[1] See Doidge, Kahle, Karolyi, and Stulz (2018),Brown, Fazzari, and Petersen (2009) or De Ridder (2020).
[2] See Appendix A.1 for details on dataset construction and details on Figure 1 and Table 1.

Figure 1: Firm-level R&D, Number of Firms, and Discount Rates. The top panel plots the average (demeaned) ratio of R&D over total assets of public US firms in Compustat, net of firm-fixed effects, against the aggregate annualized risk premium, between 1982 and 2017. The bottom panel plots the average (demeaned) number of firms by industry-year, net of industry-fixed effects, for the same time period and set of industries, using data from the Business Dynamics Statistics (BDS) project, covering both public and private firms.

Taking into account that firms do not innovate in isolation, we thus propose a model that characterizes how discount rates affect the composition of innovation within an industry. Crucially, we show that a higher *market price of risk*—the common component of firms' discount rates in the cross section—significantly deters fresh entry and, simultaneously, encourages innovation by incumbents in the intensive margin. The mechanism rationalizes the empirical evidence in Figure 1. Furthermore, considering that the market price of risk is countercyclical, we show that discount rate fluctuations help rationalize the cyclicality of R&D: We reconcile the Schumpeterian view that firm-level innovation is countercyclical with the well-documented procyclicality of aggregate R&D. We also find that high discount rates foster explorative over exploitative innovation.

Our novel framework allows us to prove key results analytically while incorporating endogenous entry and heterogeneous firms. We consider an industry in which firms are subject to two sources of systematic risk: a diffusion risk directly affecting firms' cash flows, and a jump risk associated with changes in the state of the economy. The market price of risk associated with the diffusion risk is state-contingent. Consistent with the evidence (i.e., Braguinsky, Ohyama, Okazaki, and Syverson (2021)), firms in the industry may pursue two types of innovation: vertical (or explorative), which aims at major breakthroughs that improve the quality of technology, and horizontal (or exploitative), which aims at creating new products.[3] We further acknowledge that firms vary in their ability to innovate and produce by considering three types of firms: an initiator, exploiters, and entrants. The initiator is the leading firm in the industry. It represents the latest successful innovator to advance the technology frontier via a vertical breakthrough, starting a bundle of products building on such breakthrough. Exploiters are firms that, taking advantage of the latest vertical breakthrough, have successfully developed new products via horizontal breakthroughs, and solely focus on production. Lastly, entrants are startups on the sideline that invest in vertical and horizontal innovation, with the aim of becoming initiators (upon a vertical breakthrough) or exploiters (upon a horizontal breakthrough). Vertical breakthroughs cast the threat of creative destruction on the initiator and exploiters, then causing their exit. Conversely, horizontal breakthroughs cause a partial displacement by making some of the initiator's and exploiters' products obsolete, then eroding their revenues.

To disentangle the strengths at play, we develop our analysis in steps and start by consid-

[3] Additional studies on heterogeneous innovation include Manso, Balsmeier, and Fleming (2021), Akcigit and Kerr (2018), Hsieh, Klenow, and Shimizu (2021) and Arora, Belenzon, and Sheer (2021).

ering the case in which the market price of risk is constant. When abstracting from industry dynamics, we confirm the conventional wisdom that a higher market price of risk discourages a firm's investment in R&D. Yet, when allowing for endogenous industry dynamics, this result is overturned. Specifically, we prove formally that a higher market price of risk discourages entry by new firms and, yet, encourages innovation by incumbents, consistent with the evidence in Figure 1. Furthermore, compounding these offsetting effects, the model predicts that the market price of risk has a non-monotonic effect on the industry-level rate at which new technologies emerge. In particular, a higher market price of risk can spur the advent of new technologies if the higher R&D engagement by active firms (the intensive margin) more than offsets the decline in the mass of entrants (the extensive margin).

Notably, these predictions are robust to allowing the initiator not only to innovate "in house," but also to acquire entrants that attain breakthroughs (Phillips and Zhdanov, 2013; Cunningham, Ederer, and Ma, 2021). In this case too, we confirm our core prediction that the extensive innovation margin decreases with the market price of risk, whereas in-house innovation increases. Moreover, higher discount rates make acquisitions of innovating entrants less likely, as the extensive margin shrinks and target firms become scarcer.

Our model also reveals that different types of innovation—horizontal or vertical—exhibit different sensitivity to discount rates. While the rate of vertical innovation increases with the market price of risk due to the ensuing lower threat of creative destruction, the rate of horizontal innovation is non-monotonic. First, as the market price of risk increases, the lower rate of creative destruction spurs horizontal innovation. Simultaneously, however, exploiters face a greater threat of exit due to the initiator's greater innovation rate, which reduces the reward and the incentives to invest horizontally. Overall, this second strength dominates when the market price of risk is sufficiently high. Hence, a greater market price of risk stimulates the more explorative type of innovation.

We next allow the market price of risk to vary over time, which enables us to validate the predictions of our model empirically by capturing the cyclicality of R&D at both the firm and aggregate level. We assume that the economy switches over two alternative states, one with a low market price of risk (the good state or expansion) and the other with a high market price of risk (the bad state or recession), consistent with Lustig and Verdelhan (2012), among others. We show that active firms are more R&D-intensive when the market price of risk is higher (i.e., in bad states) but, at the same time, fewer firms are active. That is, active firms face lower

competition in innovation in bad states of the economy thanks to a lower threat of creative destruction and of product obsolescence which, in turn, encourage their investment in R&D. Whereas the aggregate contribution of entrants to innovation is higher in good states of the economy thanks to the greater mass of entrants, the firm-level R&D of active firms is yet higher in bad states. Our paper then reconciles the Schumpeterian view that firms should invest more in bad states of the economy with the evidence that R&D is procyclical at the aggregate level.[4]

Interestingly, the predictions of the model on R&D cyclicality are consistent with recent empirical studies. Babina, Bernstein, and Mezzanotti (2022) find that the intensive margin of innovation is resilient during downturns, whereas the extensive margin drops due to a substantial decline in patenting by entrepreneurs—aligned with our core prediction that incumbents benefit from lower entry by new firms. Their study also finds that innovation shifts towards more impactful patents during downturns—hinting at greater exploration when the market price of risk increases, consistent with our model and the evidence in Manso, Balsmeier, and Fleming (2021). Furthermore, the discount rate mechanism in our model strengthens when acknowledging that firms' ability to finance entry deteriorates during recessions—an aspect that we embed in an extension—aligned with the evidence in Brown, Fazzari, and Petersen (2009) and Howell, Lerner, Nanda, and Townsend (2020).

To further investigate the effect of fluctuations in the market price of risk on innovation, we compare our two-state economy with an identical economy in which the market price of risk is fixed at its two-state average. We find that the fluctuations in the market price of risk have the strongest impact on the extensive margin. Specifically, the mass of entrants is significantly *larger*, on average, when allowing for these fluctuations, and the rate of creative destruction is greater. Thus, fluctuations in the market price of risk induce a more prominent industry turnover that, in turn, spurs the advent of new technologies, consistent with the Schumpeterian view that creative destruction spurs innovation.

Lastly, the asset pricing implications of our model reveal that lower entry in downturns *hedges* innovating incumbents against the otherwise negative effect of higher discount rates on firm value. The model shows that the negative impact of higher discount rates on the intensive margin is more than offset by the positive effect of the associated lower rate of creative destruction, so that the expected value of technological improvements actually increases in periods of high discount rates. Moreover, while the literature suggests that competition in the

[4]E.g., Griliches (1984), Comin and Gertler (2006), Barlevy (2007), or Fabrizio and Tsolmon (2014).

product market makes incumbent firms *safer*,[5] we show instead that competition in innovation makes incumbent firms *riskier*. Thus, our model provides a mechanism to identify the nature of firm rivalry—either in product markets or in technology—through the impact of firm entry on risk premia, thus complementing the findings in Bloom, Schankerman, and Van Reenen (2013).

Related literature Our paper relates to the literature showing the significance of discount rates for various corporate decisions and aggregate dynamics (see the presidential address by Cochrane, 2011). In this strand, Pastor and Veronesi (2005) show that waves of initial public offerings are largely driven by declines in expected market returns. Malenko and Malenko (2015) and Haddad, Loualiche, and Plosser (2017) study the impact of discount rates on buyout activity. Opp, Parlour, and Walden (2014), Dou, Ji, and Wu (2021), and Chen, Dou, Guo, and Ji (2020) show that discount rate fluctuations affect competition in the product market—differently, our paper looks at firm's strategic interactions in the technological space, consistent with the evidence that these are two very different types of rivalries (see, e.g. Bloom, Schankerman, and Van Reenen, 2013). Taking a macroeconomic perspective, Hall (2017) shows that the time variation in discount rates is a strong determinant of unemployment dynamics. We contribute to this strand by showing that the level and fluctuations of the market price of risk have a first-order impact on R&D, challenging the conventional wisdom that larger discount rates discourage investment.

We also contribute to the corporate finance literature studying innovation. Previous models have considered the role of incentives schemes (Manso, 2011), firms' ownership structure (Ferreira, Manso, and Silva, 2014), takeovers (Phillips and Zhdanov, 2013), financing frictions and cash availability (Malamud and Zucchi, 2019; Lyandres and Palazzo, 2016), and debt financing (Geelen, Hajda, and Morellec, 2021). We look instead at the impact of discount rates in industry equilibrium. Our paper also relates to the empirical literature on competition in innovation. In this strand, Kogan, Papanikolaou, Seru, and Stoffman (2017) measure how a firm's innovation affects its rivals; Cunningham, Ederer, and Ma (2021) study the role of takeovers in preempting competition in innovation; Manso, Balsmeier, and Fleming (2021) show that firms engage in more explorative innovations during contractions; and Braguinsky et al. (2021) study how firms grow by innovating vertically and horizontally. The predictions of our model on the cyclicality of R&D rationalize the evidence in Howell et al. (2020), Manso, Balsmeier, and Fleming (2021),

[5]See, for instance, Bustamante and Donangelo (2017) and Babenko, Tserlukevich, and Boguth (2020), Corhay, Kung, and Schmid (2020b), and Loualiche (2021).

and Babina, Bernstein, and Mezzanotti (2022).

More generally, our paper relates to models studying firm decisions in industry equilibrium. In this strand, Miao (2005) uncovers a price feedback effect by which the availability of credit may discourage entry; Hackbarth and Miao (2012) elaborate on the link between mergers and industry dynamics; and Pindyck (2009) characterizes how uncertainty affects firms' entry incentives. In addition, Liu, Mian, and Sufi (2022) propose a model with a duopoly of innovating firms to jointly explain rising market concentration and falling productivity growth as the risk free rate falls to zero. While Liu, Mian, and Sufi (2022) do not allow for entry, our model does, which helps rationalize the observed cyclicality of R&D.[6] Relatedly, De Ridder (2020) and Corhay, Kung, and Schmid (2020a,b) stress the importance of firm entry in explaining aggregate trends.

Lastly, while a number of papers document that firms in less competitive product markets with higher markups exhibit higher expected returns (i.e., Bustamante and Donangelo, 2017; Corhay, Kung, and Schmid, 2020a; Loualiche, 2021), fewer papers have focused on competition in technology (innovation). As relevant exceptions, Bena and Garlappi (2020) consider a patent race model of two firms in which the expected return of one firm decreases with its own innovation output and increases with that of its rival, and Grotteria (2020) studies how lobbying relates to innovation and risk premia.

The paper is organized as follows. Section 1 presents the model. Section 2 analyzes the model implications when the market price of risk is constant, whereas Section 3 allows for time-variation in the market price of risk. Section 4 analyzes the asset pricing implications. Section 5 concludes. Technical developments are gathered in the Appendix.

1 The model

The economic environment Time is continuous, and the horizon is infinite. We consider a cluster of firms, or industry, which compete in innovation. Firms are subject to two sources of aggregate risk: a diffusion risk and a jump risk. These risks are both priced and affect the dynamics of the stochastic discount factor, denoted by ξ_t, which satisfies:

$$\frac{d\xi_t}{\xi_t} = -rdt - \eta(j_{t-})d\widetilde{B}_t + \sum_{j_t \neq j_{t-}} \left(e^{\theta(j_{t-},j_t)} - 1\right) d\widetilde{N}_t^{(j_{t-},j_t)}. \tag{1}$$

[6] Our model also differs from Liu, Mian, and Sufi (2022) as they impose no leapfrogging among firms, whereas we allow for it. We show that our results are robust to allowing the market leader to endogenously avoid leapfrogging by acquiring startups (see Section 2.3)

In this equation, r is the constant risk-free rate. $d\widetilde{B}_t$ is a standard Brownian motion representing the systematic source of diffusion risk, and $\eta(j_t)$ represents the associated market price of risk. $\tilde{N}_t^{(j_{t-},j_t)}$ is a compensated Poisson process with intensity $\tilde{\pi}_{j_{t-}}$, whereas $\theta(j_{t-}, j_t)$ represents the associated risk adjustment.

The jump risk represents switches in the state of the economy. The economy can be in two states $j = G, B$: a good (expansion) state G and a bad (recession) state B. The market price of the diffusion risk is state-contingent $\eta(j_t) \equiv \eta_j$, with $\eta_G < \eta_B$—that is, the market price of risk is countercyclical, as documented by Lustig and Verdelhan (2012) among others.[7] A switch in the state of the economy causes a jump in the stochastic discount factor, meaning that investors require a compensation for such risk. This compensation translates into a wedge between the transition intensity under the physical and risk neutral measure. Using the risk adjustment $\theta(j_{t-}, j_t) \equiv \theta_j$, the risk-neutral transition intensities satisfy $\pi_j = e^{\theta_j}\tilde{\pi}_j$ in each state. As in Bolton, Chen, and Wang (2013), we assume that $\theta_G = -\theta_B > 0$, which implies that the transition intensity from state G (respectively, B) to state B (G) is higher (smaller) under the risk-neutral probability measure than under the physical one. That is, risk averse agents expect the good (bad) state to be shorter (longer).

Innovation and firm types Consistent with the evidence, we acknowledge that firms may pursue two types of innovation: Vertical (or explorative) or horizontal (or exploitative).[8] Specifically, vertical innovation represents major breakthroughs in the quality of technology, denoted by q_t. Conversely, horizontal innovation builds on the latest vertical breakthrough and aims at introducing new products. As in Howitt (1999), horizontal innovation applied to a given quality level q_t eventually runs into diminishing returns to scale. In the following, we will exploit the concept of "technological cluster," representing the collection of products that stem from a given increase in the quality of technology.

We assume that the industry features three types of firms: an initiator, exploiters, and entrants. The initiator, denoted by U_j in each state j, represents the latest vertical innovator starting a new technological cluster. It produces and sells products that build on the latest technological frontier and, meanwhile, continues to invest in innovation. Exploiters, denoted

[7] As shown by Campbell and Cochrane (1999), the countercyclicality of the market price of risk can be driven by, e.g., time-varying risk aversion.

[8] Consistent with Arora, Belenzon, and Sheer (2021), vertical innovation is more related to "research" whereas horizontal innovation to "development." Hsieh, Klenow, and Shimizu (2021) show that both are key to understand the patterns of growth.

by X_j, are firms that have successfully developed new products via horizontal breakthroughs, and solely focus on production (i.e., they do not invest in innovation). Entrants, denoted by the value function W_j, are startups on the sideline. They invest in vertical and horizontal innovation and have the potential to become the new initiator (if they attain a vertical breakthrough) or an exploiter (if they attain a horizontal breakthrough). We next describe these firm types in detail.

Initiator The latest vertical innovator improving the industry's quality level q_t becomes the initiator of a new technological cluster and drives the existing producers (that is, the previous initiator and exploiters) out of the market. Using this novel technology, the initiator manufactures a mass M_t of products. In each product line i, the firm faces the following demand function:[9]

$$p_{it} = \Gamma_j \left(\frac{Y_{it}}{q_t} \right)^{-\beta}, \tag{2}$$

where p_{it} represents the selling price associated to product i, Y_{it} represents quantity, and $\beta \in (0, 1)$ is the inverse of the price elasticity of demand.[10] Γ_j represents a demand-shift parameter, which varies with the state of the economy j. We assume that the cost of production is normalized to one in all product lines. Following previous literature, we assume that all product lines exhibit the same demand function, and each product line is a monopoly until an entrant attains a breakthrough, as we explain below.[11]

The initiator earns revenues from producing the M_t goods and, at the same time, continues to invest in innovation. We denote by z_t the initiator's innovation intensity at time t. We follow previous literature in capturing the key features of innovation: It is costly and has an uncertain outcome. That is, if the firm bears the following flow cost

$$\Phi(z, q, M) = \zeta \frac{z_t^2}{2} q_t M_t, \quad \zeta > 0, \tag{3}$$

it attains a breakthrough at Poisson rate ϕz_t, where ϕ is a positive constant. This specification

[9] Our main results continue to hold when we assume that production decisions are not optimized—that is, the firm's expected revenues are exogenous and constant. That is, our results are not driven by the particular functional form of the firm's demand function.

[10] As we express p and Y as a function of time (as captured by the subscript t), we omit their dependence to the state of the economy (subscript j).

[11] The assumption that each product line is a monopoly is standard in this strand, see Klette and Kortum (2004), Aghion and Howitt (1992), Howitt (1999), Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018), among many others. The assumption is consistent with firms earning monopolistic rents from patented innovation. As all product lines face the same demand function, we drop the subscript i in the following.

implies that a breakthrough is more likely if the firm spends more on innovation. It also captures the idea that innovation is more costly if quality is greater or if the mass of current product lines is larger.[12] We assume that when the initiator attains a breakthrough, quality jumps by a factor $\lambda > 1$ and the mass of product lines jumps by a factor $\varphi > 1$. That is, because the initiator has specific "working" knowledge of the particular industry, an innovation increasing the quality of technology results in the creation of new products, in the spirit of Nelson (1959) and Akcigit, Hanley, and Serrano-Velarde (2021).

Absent breakthroughs by other firms, the cash flows of the initiator satisfy:

$$\begin{aligned} dC_t &= \left[Y_t \left(p_t - 1\right) M_t - \Phi(z, q, M)\right] dt + \sigma Y_t M_t d\widetilde{B}_t^U \\ &= \left[Y_t \left(p_t - 1\right) M_t - \Phi(z, q, M)\right] dt + \sigma Y_t M_t \left[\rho d\widetilde{B}_t + \sqrt{1-\rho^2} d\widetilde{B}_t^{U\perp}\right]. \end{aligned} \tag{4}$$

The first term represents the initiator's profits from production in the M_t product lines net of R&D expenditures. Throughout our analysis, we focus on cases in which this term is positive, so to avoid the degenerate case in which the initiator always makes losses in expectation. The second term represents the volatility of the initiator's cash flows, which increases with the firm's production rate. The parameter σ is a positive constant, and $\widetilde{B}^U$ is a standard Brownian motion under the physical probability measure. The Brownian motion $\widetilde{B}^U$ is correlated with the aggregate shock $\widetilde{B}$ by a factor $\rho \geq 0$. That is, $\widetilde{B}^U$ can be decomposed into the orthogonal components $\widetilde{B}_t$ and $\widetilde{B}_t^{U\perp}$ through ρ.

Consistent with Argente, Lee, and Moreira (2021), the initiator loses some of its product lines if an entrant attains a horizontal breakthrough. In this case, new products are launched, making the initiator's products obsolete. Namely, if an entrant attains a horizontal breakthrough creating a mass ωM_{t-} of new products, the initiator's product lines drop from M_{t-} to $M_t = M_{t-}(1 - \omega\delta)$, and so do cash flows.[13] The parameter $\delta \in (0, 1]$ represents the degree of overlap between new and existing products. A greater overlap means that more of the initiator's products become obsolete due to an entrant's breakthrough.

In addition to partial displacement due to horizontal breakthroughs, the initiator is hit by creative destruction if an entrant attains a vertical breakthrough. When creative destruction

[12] Scalability of the flow cost of innovation in quality or product lines is consistent with previous models of endogenous growth, see Aghion, Akcigit, and Howitt (2014) for a survey as well as Akcigit, Hanley, and Serrano-Velarde (2021) or Acemoglu et al. (2018) for recent contributions.

[13] As shown by Equation (4), the cash flows of the initiator scale up with its mass of product lines M_t.

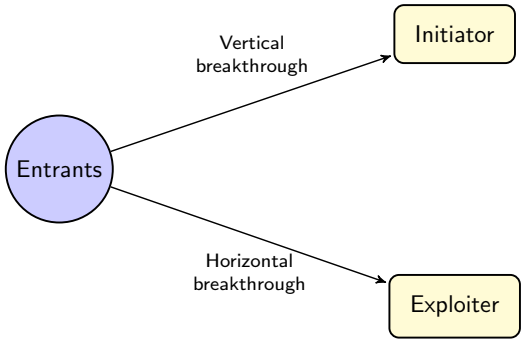

Diagram 1: Continuation value of entrant conditional on breakthrough type.

hits, the successful entrant takes over the initiator's market position, and the initiator liquidates its assets and exits. We assume that liquidation is costly, as the initiator recovers just a fraction $\alpha \in [0, 1)$ of its value.

Entrants There is a continuum of entrants on the sideline, whose endogenous mass is denoted by μ. Entrants only invest in innovation and can be interpreted as startups. Because entrants do not have ongoing production—that is, differently from the initiator, they do not have working knowledge of specific products—they do not benefit from the synergy between vertical and horizontal R&D as the initiator does. Thus, entrants need to spend on vertical or horizontal innovation separately.

We denote an entrant's innovation rate targeting vertical breakthroughs by v_t at any time t. Similar to the initiator, v_t governs the Poisson rate of vertical breakthroughs—given by $\phi_v v_t$ with ϕ_v being a positive constant—and entails the flow cost:

$$\Phi_v(v, q, M) = \zeta_v \frac{v_t^2}{2} q_t M_t, \quad \zeta_v > 0. \tag{5}$$

When the entrant attains a vertical breakthrough, the industry's technological quality jumps by a factor $\Lambda > 1$, a new technological cluster is created, and the entrant becomes the new initiator, as shown in the upper branch in Diagram 1.

In addition, we denote by h_t an entrant's innovation rate targeting horizontal breakthroughs.

When spending the amount

$$\Phi_h(h, q, M) = \zeta_h \frac{h_t^2}{2} q_t M_t, \quad \zeta_h > 0, \tag{6}$$

an entrant attains a horizontal breakthrough at a Poisson rate $\phi_h h$, with $\phi_h > 0$. The greater h_t is, the more likely the entrants will attain a breakthrough and create a mass of new products $M_{Xt} = \omega M_{t-}$, where $\omega \in [0, 1]$ and M_{t-} represents the mass of the initiator's products before the breakthrough. As M_t decreases as more products are introduced within a technological cluster, horizontal innovation run into diminishing returns to scale.[14] Once an entrant attains a horizontal breakthrough, it becomes an exploiter thereafter, as illustrated in the lower branch of Diagram 1.

Entrants are exposed to random shocks—for instance, random outflows or windfalls in the development of new ideas or products. Specifically, entrants' cash flows satisfy:

$$dC_t^W = \left[-\frac{1}{2} \left(\zeta_v v_t^2 + \zeta_h h_t^2 \right) dt + \sigma_W d\widetilde{B}_t^W \right] M_t q_t \tag{7}$$

where $\sigma_W > 0$ and $\widetilde{B}_t^W$ is a standard Brownian motion under the physical measure correlated with the aggregate shock $\widetilde{B}_t$ by a factor $\rho_W \geq 0$.[15] Equation (7) implies that entrants have negative cash flows in expectation, consistent with the evidence that startups typically lack steady revenues. As entrants aim at improving the quality and expand the products launched by the initiator, their innovation costs and volatility depend on q_t and M_t.

At the outset, entrants face an entry cost $K_t = \kappa q_t M_t$ to start investing in innovation, which can then be interpreted as the cost of installing the firm's technological capital. The magnitude of the cost K_t varies over time due to technological improvements in q_t or due to the expansion or contraction of M_t. As a result, if the initiator or other entrants attain a breakthrough, an entrant needs to adjust its technological capital in proportion to the ensuing change in q_t and/or M_t, consistent with Luttmer (2007).

Exploiters Entrants who successfully attain a horizontal breakthrough—then creating a mass of new products M_{Xt}—become the monopolistic producers in such new product lines. These

[14] Notably, because entrants aim to improve on the initiator's technology and products, their innovation cost is a function of current quality q_t and of the initiator's mass of product lines M_t.

[15] As for the initiator, we can decompose the Brownian motion $\widetilde{B}_t^W$ into the systematic source of risk and an orthogonal component, representing purely idiosyncratic risk.

firms, which we refer to as exploiters, give up on innovation and maximize their value by choosing the production quantity Y_{Xt} in their product lines. As the initiator, exploiters face the demand function (2) in each product line. An exploiter's cash flows are given by:

$$dC_t^X = Y_{Xt}\left(p_{Xt} - 1\right) M_{Xt} dt + \sigma_X Y_{Xt} M_{Xt} d\widetilde{B}_t^X, \tag{8}$$

where σ_X is a positive constant, and $\widetilde{B}_t^X$ is a standard Brownian motion under the physical probability measure that is correlated with the aggregate shock $\widetilde{B}_t$ by a factor $\rho_X \geq 0$.[16] Because the initiator and the exploiters both produce goods in the same industry, we assume that their exposure to aggregate risk is the same, so $\rho_X = \rho$. As for the initiator, an exploiter's cash flow volatility increases with its production rate.

As the initiator, exploiters lose a fraction $\omega\delta$ of their product lines when entrants attain horizontal breakthroughs. In addition, exploiters face the threat of exit if a vertical breakthrough improves the current quality q_t. When this happens, exploiters liquidate and recover just a fraction $\alpha_X \in [0, 1)$ of their value. Notably, the exploiters are subject to the threat of exit when either the initiator or the entrants attain a vertical breakthrough.

Industry equilibrium We consider an industry equilibrium in which: (1) the initiator maximizes its value by choosing its optimal production and innovation rate; (2) exploiters maximize their value by choosing their optimal production rate; (3) entrants maximize their value by choosing their optimal vertical and horizontal innovation rates; (4) the mass of active entrants makes the free-entry condition binding at any time.

In equilibrium, the rate of creative destruction Ψ_{vt} is derived endogenously as the rate at which entrants, on aggregate, attain a vertical breakthrough starting a new technological cluster. In turn, the equilibrium rate of horizontal displacement Ψ_{ht} is the rate at which entrants, on aggregate, attain horizontal breakthroughs, then creating new products.

2 Constant market price of risk

To disentangle the forces at play, we start by considering the case in which there is only one state of the economy, in which the market price of risk is constant and denoted by η.[17]

[16] As for the other firms, the Brownian motion $\widetilde{B}_t^X$ can be decomposed into the orthogonal components $\widetilde{B}_t$ and $\widetilde{B}_t^{X\perp}$ through ρ_X, where $\widetilde{B}_t^{X\perp}$ is independent to the aggregate (priced) risk $\widetilde{B}_t$.

[17] As there is just one state, the demand shift parameter $\Gamma_j = \Gamma$ is constant too.

2.1 Model solution

By Girsanov theorem, we derive the dynamics of cash flow under the risk neutral measure. Using standard arguments, the value of the initiator satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} rU(q,M) = \max_{z,Y} \; & MY(p-1-\sigma\eta\rho) - \zeta\frac{z^2}{2}qM + \phi z\left[U(\lambda q,\varphi M) - U(q,M)\right] \\ & + \Psi_v\left[\alpha U(q,M) - U(q,M)\right] + \Psi_h\left[U(q,M(1-\omega\delta)) - U(q,M)\right]. \end{aligned} \tag{9}$$

The left-hand side is the return required by risk-neutral investors. The right-hand side is the expected change in firm value on an infinitesimal time interval. Namely, the first two terms are the risk-adjusted expected cash flows net of R&D. The third term is the expected change in firm value due to a breakthrough by the initiator, which triggers an increase in quality and an expansion in the mass of products. The fourth term represents the effect of creative destruction triggered by entrants' vertical innovations (occurring at rate Ψ_v), in which case the initiator exits and recovers just a fraction α of its value. The last term is the effect of obsolescence triggered by entrants' horizontal innovations (occurring at rate Ψ_h), which erodes a fraction $\omega\delta$ of the initiator's product lines.

We conjecture that the value of the initiator scales with quality q_t and with the mass of product lines M_t, $U(q_t, M_t) = q_t M_t u$, where u represents the initiator's scaled value. Also, we define by $y \equiv Y_t/q_t$ the production quantity in each product line scaled by quality. Substituting these definitions into equation (9) and differentiating the resulting equation with respect to y gives the optimal production quantity and the associated selling price:

$$y(\eta) = \left(\frac{\Gamma(1-\beta)}{1+\sigma\eta\rho}\right)^{\frac{1}{\beta}} \quad \Rightarrow \quad p(\eta) = \Gamma y^{-\beta} = \frac{1+\sigma\eta\rho}{1-\beta}. \tag{10}$$

As illustrated by equation (4), the initiator's exposure to aggregate risk is endogenous as so is its production quantity. Equation (10) shows that if the market price of risk η is greater, the firm reduces its production quantity and increases the selling price. That is, the firm effectively reduces its exposure to aggregate risk and, by increasing the selling price, it passes the higher price of risk on to the consumers. By calculations, $\Upsilon(\eta) \equiv \beta\left(\frac{1-\beta}{1+\sigma\eta\rho}\right)^{\frac{1}{\beta}-1}\Gamma^{\frac{1}{\beta}}$ represents the initiator's risk-adjusted profits from production.

Furthermore, differentiating the HJB equation with respect to z gives the optimal innovation

rate:

$$z(\eta) = \frac{\phi}{\zeta}(\lambda\varphi - 1)\, u(\eta). \tag{11}$$

This expression suggests that the higher the value of the initiator, the greater its innovation rate, as the surplus from attaining a technological breakthrough widens. Moreover, the optimal innovation rate increases if R&D expenditures are more likely to translate into technological breakthroughs (higher ϕ), if the returns to innovation are greater (larger λ or φ), or if innovation is less costly (ζ is smaller).

Consider now the dynamics of the exploiters. Recall that exploiters are entrants that have attained a horizontal breakthrough creating a mass of new product lines $M_{Xt} = \omega M_{t-}$. Exploiter value satisfies:

$$\begin{aligned} rX(q, M_X) = \max_{Y_X} \quad & M_X Y_X (p_X - 1 - \eta\rho\sigma_X) + (\Psi_v + \phi z)(\alpha_X X(q, M_X) - X(q, M_X)) \\ & + \Psi_h \left[X(q, (1-\omega\delta)M_X) - X(q, M_X)\right]. \end{aligned} \tag{12}$$

As for equation (9), the right-hand side is the expected change in exploiter value over an infinitesimal time interval. The first-term represents the exploiter's risk-adjusted expected profits. The second term represents the effect of vertical innovations by entrants (occurring at rate Ψ_v) or by the initiator (occurring at rate ϕz), which cause the exit of the incumbent exploiters. The third term represents the effect of horizontal innovations by entrants (occurring at rate Ψ_h), which cause the exploiters to lose a fraction of their product lines. Exploiters maximize their value by choosing their optimal production quantity Y_X.

We conjecture that the exploiter value function satisfies $X(q_t, M_{Xt}) = q_t M_{Xt} \tilde{x}$, where $\tilde{x}$ represents the exploiter value scaled by the industry's quality level q_t and by the mass of its active product lines M_{Xt}. We define $y_X \equiv Y_{Xt}/q_t$ as an exploiter's production quantity per active product line scaled by quality. When an exploiter starts production, its mass of product lines can be expressed as a function of the product lines of the initiator: $M_{Xt} = \frac{M_t}{1-\omega\delta}\omega$.[18] Thus, we can express the exploiter value as a function of the active product lines of the initiator M_t as follows:

$$X(q_t, M_{Xt}) = q_t M_{Xt} \tilde{x} = q_t M_t \tilde{x} \frac{\omega}{1-\omega\delta} = q_t M_t x. \tag{13}$$

[18] Recall that right after a horizontal breakthrough leading to the emergence of a new exploiter, the initiator's product lines are $M_t = M_{t-}(1-\omega\delta)$.

To make the scaled value of the exploiter comparable to the other scaled quantities, we define $x = \tilde{x}\frac{\omega}{1-\omega\delta}$. Maximizing the ensuing scaled HJB equation with respect to y_X gives

$$y_X(\eta) = \left(\frac{\Gamma(1-\beta)}{1+\eta\rho\sigma_X}\right)^{\frac{1}{\beta}} \tag{14}$$

and the associated selling price is $p_X = \frac{1+\sigma_X\eta\rho}{1-\beta}$. Notably, the initiator and the exploiters choose a different production quantity due to the difference in their cash flow volatilities (see equation (10)), which in turn results in a different exposure to systematic risk.

Next, we study the dynamics of entrants, whose value $W(q, M)$ is a function of the current quality level and the product lines they try to improve on. Entrant value satisfies:

$$\begin{aligned} rW(q,M) = \max_{v,h} \quad & -qM\left(\eta\rho_W\sigma_W + \frac{\zeta_v}{2}v^2 + \frac{\zeta_h}{2}h^2\right) + \phi_v v\left[U(\Lambda q, M) - W(q,M)\right] \\ & + \phi_h h\left[X(q,\omega M) - W(q,M)\right] + \phi z\left[W(\lambda q, \varphi M) - W(q,M) - K(\lambda\varphi - 1)\right] \\ & + \Psi_v^-\left[W(\Lambda q, M) - W(q,M) - K(\Lambda - 1)\right] + \Psi_h^-\left[W(q, M(1-\omega\delta)) - W(q,M) + K\omega\delta\right]. \end{aligned} \tag{15}$$

The first term on the right-hand side represents an entrant's risk-adjusted expected outflow on any time interval. The second term represents the effect of a vertical breakthrough by the entrant occurring at rate $\phi_v v$, in which case it becomes the new industry initiator. The third term represents the effect of a horizontal breakthrough by the entrant occurring at rate $\phi_h h$, in which case it becomes an exploiter. The fourth, fifth, and sixth terms represent the effect of breakthroughs by the current initiator (occurring at rate ϕz), vertical breakthroughs by other entrants (occurring at rate Ψ_v^-), or horizontal breakthroughs by other entrants (occurring at rate Ψ_h^-), respectively. The fourth and fifth terms imply that, whenever the initiator or other entrants attain vertical breakthroughs, the entrant needs to catch up with the new technology, consistent with Luttmer (2007). Catching up requires an upgrade cost proportional to the size of the breakthrough—depending on whether the breakthrough is attained by the initiator or an entrant, it is, respectively, $K(\lambda\varphi - 1)$ or $K(\Lambda - 1)$. Conversely, as in Howitt (1999), horizontal breakthroughs erode the value of the initiator and of the exploiters, as their product lines are competed away. It follows that the entrants' perspective earnings from innovation fall, and the entrant responds by adjusting its capital downwards by $K\omega\delta$, as illustrated by the last term in equation (15).

As for the other firms in the model, we conjecture that the entrant value scales with $M_t q_t$,

that is, $W(q_t, M_t) = wq_tM_t$ where we denote by w the scaled value of a perspective entrant. Differentiating the resulting scaled HJB equation with respect to v gives the optimal vertical innovation rate of entrants:

$$v(\eta) = \frac{\phi_v}{\zeta_v}(\Lambda u(\eta) - w). \tag{16}$$

This expression suggests that the entrants' engagement in vertical innovation increase if the value of the initiator is greater—in which case the "reward" upon attaining a vertical breakthrough is more attractive. In turn, differentiating the scaled HJB equation with respect to h gives the optimal horizontal innovation rate:

$$h(\eta) = \frac{\phi_h}{\zeta_h}(\omega x(\eta) - w). \tag{17}$$

The engagement in horizontal innovation then increases as exploiters are more valuable—in which case, the "reward" upon a horizontal breakthrough is greater.

Aggregating the rate of vertical innovation across active entrants μ gives the rate of creative destruction:

$$\Psi_v(\eta) = \mu(\eta)\,\phi_v v(\eta). \tag{18}$$

In turn, the rate of horizontal displacement obtains by aggregating the rate of horizontal innovation across the mass of active entrants:

$$\Psi_h(\eta) = \mu(\eta)\,\phi_h h(\eta). \tag{19}$$

In these expressions, $\mu(\eta)$ is endogenously determined so that the free-entry condition $w = \kappa$ holds. Notably, Ψ_{vt} and Ψ_{ht} affect and are affected by the initiator's and the exploiters' value. Lastly, we pin down the aggregate rate at which new technological clusters endogenously arise, denoted by $\mathcal{I}(z, \Psi_v)$, which satisfies:

$$\mathcal{I}(z, \Psi_v) = \phi z(\eta) + \Psi_v(\eta) = \underbrace{\frac{\phi^2}{\zeta}u(\eta)(\lambda\varphi - 1)}_{\text{Initiator}} + \underbrace{\mu(\eta)\frac{\phi_v^2}{\zeta_v}[\Lambda u(\eta) - \kappa]}_{\text{Entrants}} \tag{20}$$

The first term represents the contribution of the initiator to the aggregate vertical innovation rate, whereas the second term is the contribution of entrants.

2.2 Model analysis

We analyze the model implications in steps. We start by considering simpler cases for which we obtain analytical results. First, we analyze firms in isolation, instead of studying them in the industry equilibrium. Second, we allow for endogenous industry dynamics in two corner cases: an industry in which entrants engage in vertical innovation only, and an industry in which entrants invest in horizontal innovation only.

2.2.1 Exogenous industry dynamics

Suppose that the rate of creative destruction Ψ_v and the rate of horizontal displacement Ψ_h are exogenous and constant. In this case, the value of the initiator continues to satisfy equation (9), but Ψ_v and Ψ_h are insensitive to the market price of risk η. Under these assumptions, the value of the initiator is given by:

$$u(\eta) = \frac{r + \Psi_v(1-\alpha) + \Psi_h\omega\delta - \sqrt{(r + \Psi_v(1-\alpha) + \Psi_h\omega\delta)^2 - 2\Upsilon(\eta)\frac{\phi^2}{\zeta}(\lambda\varphi - 1)^2}}{\phi^2(\lambda\varphi - 1)^2\zeta^{-1}}. \tag{21}$$

The next proposition follows (see Appendix A.2.1 for a proof).

Proposition 1 *For exogenous Ψ_v and Ψ_h, the initiator's innovation rate satisfies:*

$$z(\eta) = \frac{r + \Psi_v(1-\alpha) + \Psi_h\omega\delta - \sqrt{(r + \Psi_v(1-\alpha) + \Psi_h\omega\delta)^2 - 2\Upsilon(\eta)\frac{\phi^2}{\zeta}(\lambda\varphi - 1)^2}}{\phi(\lambda\varphi - 1)}, \tag{22}$$

which is a decreasing function of the market price of risk η.

By abstracting from endogenous industry dynamics—then neglecting that Ψ_v and Ψ_h are themselves functions of η in equilibrium—Proposition 1 shows that a greater market price of risk leads to a lower innovation rate. By decreasing the expected profits from production—and, thus, the expected surplus from innovation—a greater η decreases the initiator's optimal investment in innovation. This result is in line with the received wisdom that a greater market price of risk depresses long-term investment such as R&D.

Consider next the value of exploiters when Ψ_v and Ψ_h are exogenous, which is given by:

$$x(\eta) = \frac{1}{[r + (\phi z + \Psi_v)(1-\alpha_X) + \Psi_h\omega\delta]}\frac{\beta\omega}{1-\omega}\left(\frac{1-\beta}{1+\sigma_X\rho\eta}\right)^{\frac{1}{\beta}-1}\Gamma^{\frac{1}{\beta}}. \tag{23}$$

This equation illustrates that, when Ψ_v and Ψ_h are exogenous, the value of the exploiter also decreases with η. In the following, we will show that when Ψ_v, and Ψ_h are endogenous, the net impact of η on x is more nuanced.

2.2.2 Endogenous industry dynamics in corner cases

We now focus on two corner cases featuring endogenous industry dynamics.

Entrants only engage in vertical innovation If entrants invest in vertical innovation only, the industry features two types of firms: the initiator and the entrants.[19] In this case, we can solve for the value of the initiator in closed form, which satisfies:

$$u(\eta) = \frac{1}{\Lambda}\left(\kappa + \frac{\sqrt{2\zeta_v(r\kappa + \eta\rho_W\sigma_W)}}{\phi_v}\right). \tag{24}$$

Notably, the value of the initiator is an increasing function of the entrants' exposure to aggregate risk, $\eta\rho_W\sigma_W$. Recall that the industry equilibrium requires that entrant value w equals the entry cost κ due to the free-entry condition. To offset a greater discounting due to a larger η—which should push the entrant value down—the surplus from innovation needs to increase through an increase in the value of the initiator u, and the mass of active entrants adjusts accordingly—then pushing entrant value up. The next proposition illustrates the sensitivity of the endogenous equilibrium quantities to η (see Appendix A.2.2).

Proposition 2 *When entrants only invest in vertical innovation, the innovation rate of the initiator satisfies:*

$$z(\eta) = \frac{\phi(\lambda - 1)}{\zeta\Lambda}\left[\kappa + \sqrt{\frac{2\zeta_v(r\kappa + \eta\rho_W\sigma_W)}{\phi_v^2}}\right] \tag{25}$$

and the vertical innovation rate of active entrants satisfies:

$$v(\eta) = \sqrt{\frac{2(r\kappa + \eta\rho_W\sigma_W)}{\zeta_v}}. \tag{26}$$

Both $z(\eta)$ and $v(\eta)$ increase with η. At the same time, the mass of active entrants $\mu(\eta)$ as well as the rate of creative destruction $\Psi_v(\eta)$ decrease with η.

[19] In this case, there are no exploiters as entrants do not pursue horizontal innovation. The initiator is subject to creative destruction, but there is no horizontal displacement.

In contrast with Proposition 1 (in which the initiator is considered in isolation), Proposition 2 shows that the optimal innovation rate of the initiator $z(\eta)$ and of active entrants $v(\eta)$ *increase* with η when accounting for the industry equilibrium. Proposition 2 also shows that the mass of entrants μ decreases with η, meaning that the market price of risk effectively acts as an entry barrier. Hence, a higher η bears two offsetting effects on the entrants' contribution to innovation, captured by the rate of creative destruction Ψ_v. First, active entrants invest more in innovation, as v increases with η. Second, the mass of active entrants shrinks. Proposition 1 illustrates that this second strength dominates and, thus, Ψ_v decreases with η. As η rises, the initiator is less threatened by exit and, thus, is more valuable and has greater incentives to invest in R&D.

Entrants only engage in horizontal innovation We next consider the case in which entrants only pursue horizontal innovation. As in the full model, there are three types of firms in the economy: initiator, entrants, and exploiters. Differently, the initiator is only subject to the threat of horizontal displacement. Entrants attaining a horizontal breakthrough become exploiters, whose value satisfies:

$$x(\eta) = \frac{1}{\omega}\left(\kappa + \frac{\sqrt{2\zeta_h(r\kappa + \eta\rho_W\sigma_W)}}{\phi_h}\right). \tag{27}$$

Differently from equation (23) in which the exploiter is considered in isolation, equation (27) shows that the exploiter value increases with η in the industry equilibrium. The industry equilibrium requires that entrant value equals the entry cost for the free-entry condition to hold. Thus, to offset the value-decreasing effect of a greater η, the surplus from horizontal innovation needs to increase through an increase in the exploiter value and an adjustment in the mass of entrants. We prove the following result (see Appendix A.2.3).

Proposition 3 *When entrants invest in horizontal innovation only, their investment in innovation satisfies*

$$h(\eta) = \sqrt{\frac{2(r\kappa + \eta\rho_W\sigma_W)}{\zeta_h}}, \tag{28}$$

which is an increasing function of η.

The incentive to invest in horizontal innovation stems from the expected value of becoming an exploiter. If the exploiter value increases with η, horizontal innovation increases too. The resulting monotonicity of h with respect to η shown in Proposition 3 is noteworthy because,

as discussed in Section 2.2.3, this result does not hold in the full model with both vertical and horizontal innovation. That is, the analysis in these corner cases helps us pin down the strengths in the full model, which we analyze next.

2.2.3 The full model

When entrants invest in both horizontal and vertical innovation, the rate of creative destruction and of horizontal displacement are jointly solved endogenously. While the richness of this case prevents us from obtaining closed-form solutions, we investigate it numerically.

Baseline parameterization Table 2 reports our baseline parameterization. We set the risk-free rate to 1%. Following previous contributions, we normalize $\phi = \phi_v = \phi_h$ to one.[20] We assume that the entrants' vertical R&D cost parameter ζ_v is ten times larger than the initiator's cost ζ, which is in the ballpark of Akcigit and Kerr (2018). We also assume that ζ_h is smaller than ζ_v to acknowledge that horizontal innovation is less costly than vertical innovation. The size of quality jumps $\lambda = 1.055$ and $\Lambda = 1.12$ are also in line with Akcigit and Kerr (2018). We set $\varphi = 1.14$, which is consistent with the estimates of Argente, Lee, and Moreira (2021) about the contribution of new products to sales growth. The inequality $\varphi > \lambda$ implies that the breakthroughs by the initiator is more exploitative than explorative, consistent with Gao, Hsu, and Li (2018) among others. We set δ to 0.2, which captures the overlap between existing and new innovations reported by the OECD (2015).[21] Furthermore, we set ω to 0.25, which implies that horizontal innovations lead to a 5% drop in the initiator's output, aligned with the estimates of Kogan et al. (2017).

We set $\beta = 0.13$, so that markups are consistent with the estimates by Hall (2018). Moreover, we normalize $\Gamma = 1$. We calibrate σ so that the initiator's cash flow volatility is about 11% (as in Malamud and Zucchi, 2019). Moreover, we assume that $\sigma_X < \sigma$ to acknowledge that, differently from initiators, exploiters do not have an active R&D program and, thus, their cash flow volatility is smaller.[22] In turn, we assume that the entrants' volatility is greater and equal to 20%—consistently, Begenau and Palazzo (2021) show that entrants exhibited greater volatility and R&D expenditures over time. We acknowledge that entrants are comparatively

[20] See, for instance, Akcigit and Kerr (2018) or Akcigit, Hanley, and Serrano-Velarde (2021).

[21] The degree of overlap is captured by the backward citation index, see OECD (2015). The report shows that, depending on the sector, the index ranges from slightly below 0.1 to slightly above 0.3.

[22] Recall that volatility for these firms is given by σX and $\sigma_X X_s$.

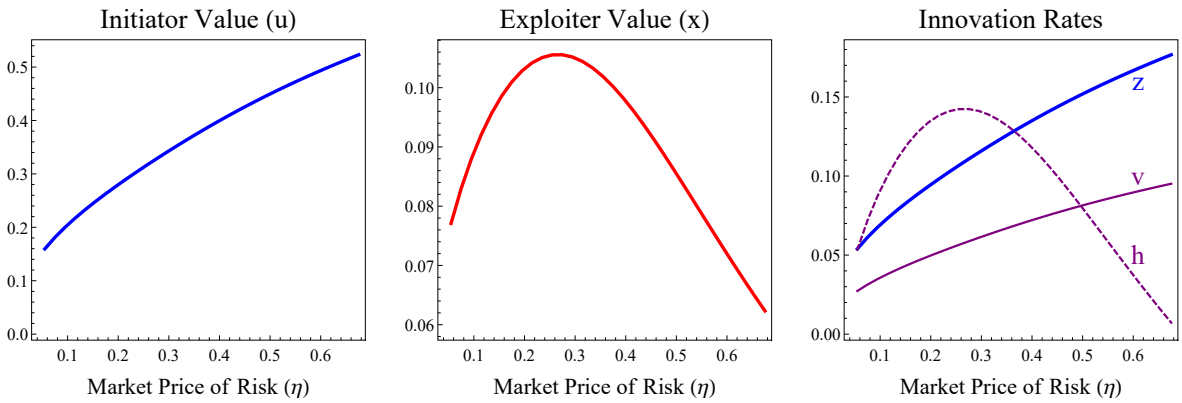

Figure 2: FIRM VALUES AND OPTIMAL INNOVATION RATES. The figure shows the initiator (left panel) and the exploiter values (middle panel), as well as the initiator's and the entrants' optimal innovation rates (right panel) as a function of η.

more exposed to idiosyncratic risk than actively-producing firms (initiator and exploiter)—consistently, we assume that $\rho = 0.55$ and $\rho_W = 0.2$. We set the exploiter's recovery rate in liquidation to 0.85, consistent with Korteweg (2010). By setting a lower recovery rate for the initiator—which, differently from the exploiters, invests in R&D—we recognize that R&D entails asset intangibility, which leads to a greater value loss in liquidation. We set the magnitude of the entry cost to $\kappa_E = 0.015$, which gives a rate of creative destruction consistent with Acemoglu et al. (2018).

The equilibrium impact of the market price of risk on innovation Consistent with the analytical results in Proposition 2, Figure 2 shows that z and v increase with η. That is, when considering the industry equilibrium—and, thus, recognizing that a firm's incentives to invest in R&D depend on other firms' decisions—η has a positive effect on active firms' innovation rate aimed at starting new technological clusters. This result overturns the conventional wisdom that discount rates frustrate long-term investment such as R&D.

Figure 3 also indicates that the mass of active entrants decreases with η. That is, our model suggests that a higher η effectively acts as an entry barrier. Confirming the result in Proposition 2, Figure 3 shows that the declining pattern of μ more than offsets the increasing pattern of v in η—as a result, the rate of creative destruction Ψ_v decreases with η. A lower Ψ_v implies less competitive pressure on active firms, which spurs the more explorative R&D investment aimed at starting new technological clusters.

Focusing on horizontal innovation, Figure 2 also shows that h is hump-shaped in η. As illus-

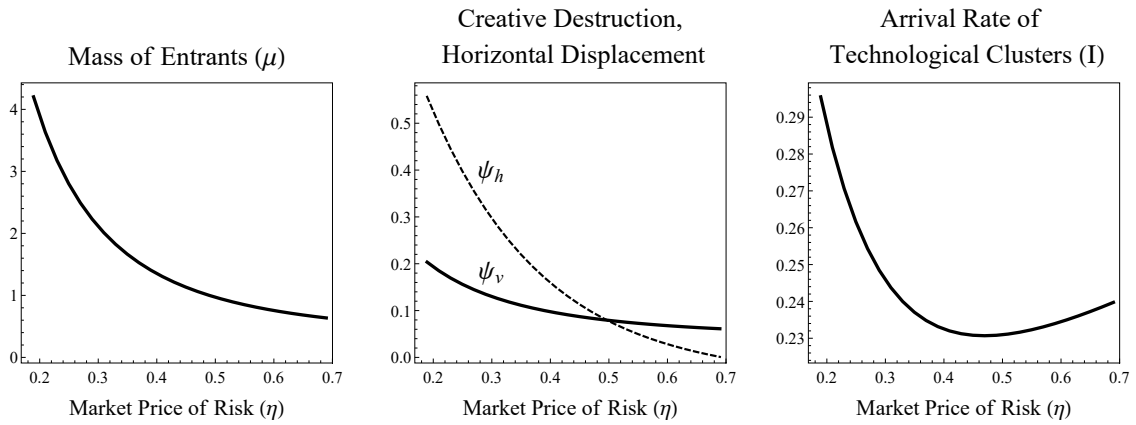

Figure 3: INDUSTRY EQUILIBRIUM AND THE MARKET PRICE OF RISK. The figure shows the mass of active entrants (μ), the rate of creative destruction (Ψ_v), the rate of horizontal displacement (Ψ_h), and the endogenous arrival rate of new technological clusters ($\mathcal{I}$) as a function of η.

trated by equation (17), investment in horizontal innovation is directly linked to the prospect of becoming an exploiter. Consistently, Figure 2 shows that the sensitivity of h to η is largely driven by the non-monotonic impact of η on the exploiter value, x.[23] This result is in contrast with Proposition 3, showing that h increases with η if entrants only invest in horizontal innovation. In fact, the interaction between competition in the vertical and horizontal dimensions triggers nontrivial dynamics. At lower levels of η, h increases with η as the rate of creative destruction concurrently declines, and so does the associated liquidation risk of the exploiter—that is, the lower threat of creative destruction spurs horizontal innovation. However, as η increases further, the higher innovation rate of the initiator increases the exploiters' threat of exit, leads to a decline in the exploiter value x, and reduces the entrants' incentives to invest in horizontal innovation.

These results illustrate that a higher market price of risk stimulates the more explorative type of innovation, i.e., vertical innovation. This can be seen at both the firm and industry level. At firm level, the right panel of Figure 2 illustrates that entrants shift resources from h to v when η is sufficiently large. At the industry level, Figure 3 shows that the rate of creative destruction Ψ_v is greater than the rate of horizontal displacement Ψ_h when η is sufficiently high, meaning that entrants invest more in vertical innovation, on aggregate.

A question then arises as to what is the net impact of the market price of risk on the rate $\mathcal{I}$ at which new technological clusters arise. As illustrated by equation (20), this quantity is the sum

[23] Conversely, the figure shows that the initiator value increases with η, consistent with Proposition 2.

of the contribution of the initiator (ϕz) and of the entrants (Ψ_v). Because z increases whereas Ψ_v decreases with η, the sensitivity of $\mathcal{I}$ to η is ambiguous. Figure 3 shows that, under our baseline parameterization, $\mathcal{I}$ is U-shaped in η. That is, perhaps surprisingly, our model shows that an increase in the market price of risk can stimulate the advent of new technological clusters. This prediction contrasts with the textbook intuition that discount rates frustrate innovation. Our model suggests that the market price of risk importantly affects the composition of innovation within an industry—hence, a higher market price of risk needs not lead to a reduction in the industry innovation rate.

The interaction between vertical and horizontal innovation As illustrated, considering both vertical and horizontal R&D is key to understand the equilibrium dynamics of an innovative industry. To further investigate the interaction between vertical and horizontal innovation, Table 3 exhibits the model's endogenous quantities in the cases in which entrants invest in either horizontal or vertical innovation only (as analyzed in Section 2.2.2) and in the full case, for different values of ω. Because horizontal innovation is less profitable when ω is smaller, the case with horizontal innovation only exists *if* ω is sufficiently large.

Introducing horizontal innovation—i.e., moving from the case with vertical innovation only to the full case—has an ambiguous effect on the mass of entrants μ. If horizontal innovation is sufficiently appealing (ω is larger), μ should increase. At the same time, however, horizontal innovation frustrates vertical innovation by making the initiator and exploiters more exposed to product obsolescence—a strength that reduces the mass of entrants μ. Table 3 suggests that, under our baseline parameterization, the second strength dominates and, thus, μ decreases when introducing horizontal innovation.[24]

These opposing strengths imply a tension regarding the effects of horizontal innovation (captured by ω) on entry. Indeed, Figure 4 illustrates that the mass of active entrants μ is U-shaped in ω. The reason is that the surplus from horizontal innovation increases with ω, but the surplus from vertical innovation declines. Thus, the rate of horizontal displacement Ψ_h sharply increases whereas the rate of creative destruction Ψ_v decreases with ω, which causes the arrival rate of technological clusters $\mathcal{I}$ to be decreasing in ω too.

Table 3 also shows that horizontal innovation decreases the initiator's R&D rate (z). The

[24] In unreported results, we find that the first strength dominates if ω is unrealistically high, in which case the incentives to invest in horizontal innovation are disproportionately greater than those to invest in vertical innovation.

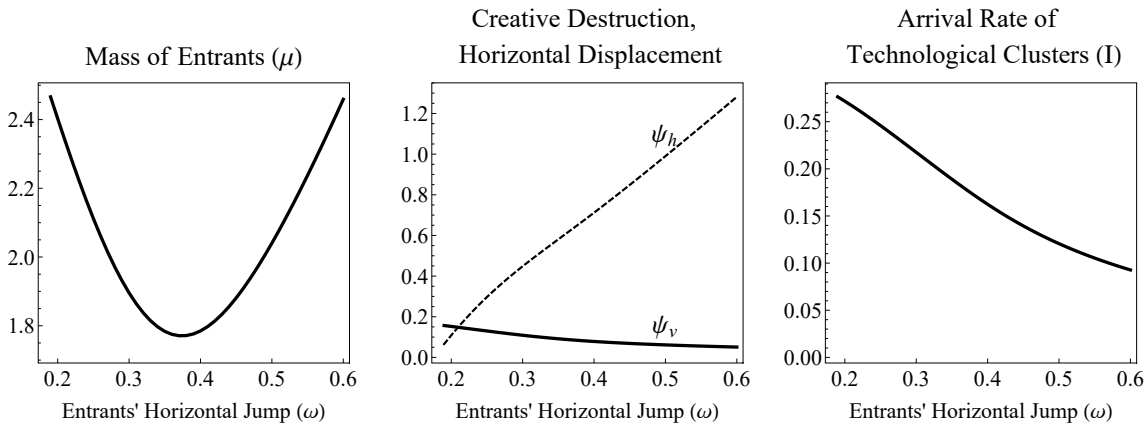

Figure 4: HORIZONTAL INNOVATION AND INDUSTRY EQUILIBRIUM. The figure shows the mass of active entrants (μ), the rate of creative destruction (Ψ_v) and of horizontal displacement (Ψ_h), and the arrival rate of technological clusters ($\mathcal{I}$) as a function of ω.

drop is wider if ω is larger, in which case horizontal breakthroughs trigger a sharper drop in the initiator's product lines. Similarly, the entrants' rate of vertical R&D v drops notably if ω is larger. Folding in the effects on μ and v, Table 3 shows that the rate of creative destruction decreases when introducing horizontal innovation—i.e., Ψ_v is lower in the full case than in the case with vertical innovation only. This result, together with the aforementioned impact on z, implies that horizontal innovation frustrates vertical innovation. Moreover, a greater emphasis on horizontal innovation leads to a lower industry turnover– on average, the initiator is expected to remain the technology leader for longer.

Consider now the effect of introducing vertical innovation, i.e., moving from the case with horizontal innovation only to the full case. Table 3 shows that vertical innovation has a positive impact on the rate of horizontal displacement. Consistently, Braguinsky et al. (2021) show that vertical R&D has notable spillovers to horizontal R&D. In fact, the upside associated with vertical innovation—i.e., the prospect of becoming the next initiator—spurs an increase in the mass of entrants that, in turn, boosts the aggregate rate of horizontal displacement too. Yet, the rate of horizontal innovation h sharply drops—i.e., because vertical innovation promises a greater upside potential (the perspective of becoming the new initiator), active entrants in the full case shift from horizontal to vertical innovation.

Our model also shows that the magnitude of the entry cost largely impacts the type of innovation pursued by firms in the industry. Namely, the left panel of Figure 5 shows that a greater entry cost κ fosters vertical innovation but deters horizontal innovation. As shown in the

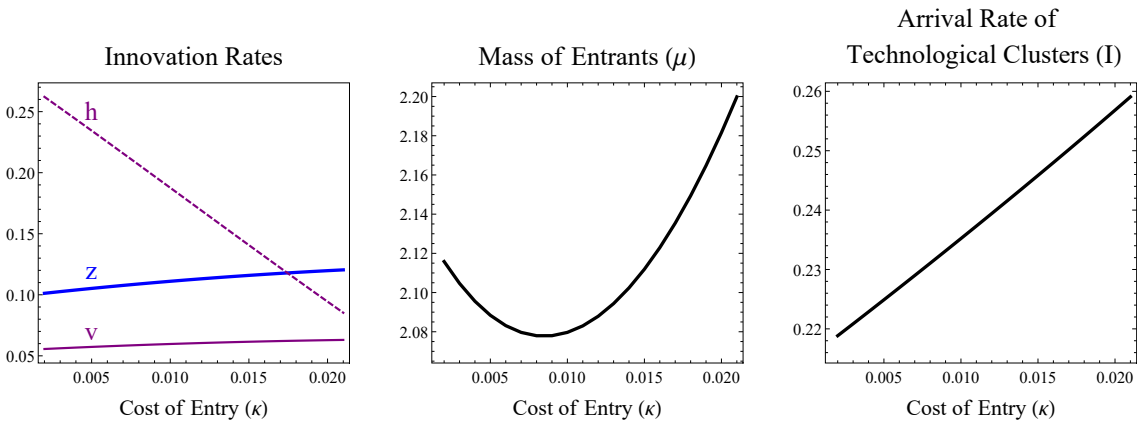

Figure 5: ENTRY COST AND INDUSTRY EQUILIBRIUM. The figure shows the initiator's and the entrants' innovation rates (z, v, and h), the mass of active entrants (μ), and the arrival rate of technological clusters ($\mathcal{I}$) as a function of κ.

middle panel, this translates into the mass of entrants being U-shaped with κ. In fact, for low levels of κ, a greater cost of entry curbs entrants' incentives to invest in horizontal innovation; by contrast, when κ is sufficiently large, the negative effect of κ on horizontal innovation is shadowed by their increased incentive to innovate vertically. This positive effect, together with the increasing pattern of z in κ, jointly explain why the arrival rate of technological clusters increases with the cost of entry.

2.3 Allowing the initiator to take over entrants

In addition to innovating "in house," initiators often take over startups (e.g., Phillips and Zhdanov, 2013; Cunningham, Ederer, and Ma, 2021). In this section, we assess if our core predictions are confirmed when allowing the initiator to acquire entrants. For simplicity, we focus on the case with vertical innovation only.

We assume that the initiator bears a search cost $\zeta_s sqM$ for finding entrants that attain a breakthrough, where s denotes the search intensity and $\zeta_s > 0$ is a cost coefficient. The initiator acquires an entrant with (endogenous) probability $s/(1+s)$—i.e., the probability increases with the search intensity—at the endogenous cost A_t. With probability $1/(1+s)$, the initiator fails to take over the successful entrant, and the entrant becomes the new initiator. Furthermore, the entrant pays a setup cost G_t upon taking over the initiators' market position, and bears no cost if the entrant is acquired instead.

Under these assumptions, the value of the initiator satisfies the following HJB equation:

$$rU(q,M) = \max_{z,Y,s} \; MY(p-1-\sigma\eta\rho) - \frac{\zeta^2}{2}zqM - \zeta_s sqM + \phi z\left[U(\lambda q, \varphi M) - U(q,M)\right] + \Psi_v\left[\frac{1}{1+s}\left(\alpha U(q,M) - U(q,M)\right) + \frac{s}{1+s}\left(U(\Lambda q, M) - U(q,M) - A\right)\right], \tag{29}$$

where last term in this equation implies that, when an entrant attains a breakthrough, the initiator acquires it with probability $s/(1+s)$ at the cost A_t. Conversely, with probability $1/(1+s)$, the initiator does not acquire the successful entrant and exits. To solve the initiator's problem, we resort to scaled quantities (see Appendix A.2.4), and we define $a = A_t/(q_t M_t)$ and $g = G_t/(q_t M_t)$ as the scaled (time-invariant) acquisition cost and setup cost, respectively. Differentiating with respect to s yields the optimal search intensity:

$$s = \sqrt{\frac{\Psi_v\left[u(\Lambda-\alpha) - \Lambda a\right]}{\zeta_s}} - 1, \tag{30}$$

meaning that the initiator has greater incentives to search for a target, either if the rate of creative destruction rises, or if the cost of acquiring the target decreases.

In turn, maximizing the entrants' HJB equation with respect to v gives the expression for the optimal innovation rate of the entrant (see Appendix A.2.4):

$$v = \frac{\phi_E}{\zeta_E}\left[\frac{\Lambda(u-g+as)}{1+s} - w\right]. \tag{31}$$

Differently from the expression of v in Proposition 2, the entrant's optimal innovation rate is now a function of the intensity s with which the initiator searches for targets, and of the payoff a that the entrant receives upon being acquired.

The initiator and the successful entrant negotiate over the terms of the deal. We consider a Nash bargaining solution where $b \in (0,1)$ corresponds to the bargaining power of the initiator. Solving for this bargaining game gives the equilibrium acquisition cost:

$$a = u - \frac{u}{\Lambda}\alpha(1-b) - bg, \tag{32}$$

which is increasing in the initiator value and in the return to entrant's innovation Λ. Moreover, it decreases with the setup cost g.

Figure 6 summarizes the main takeaways of this extension. The left and middle panels show

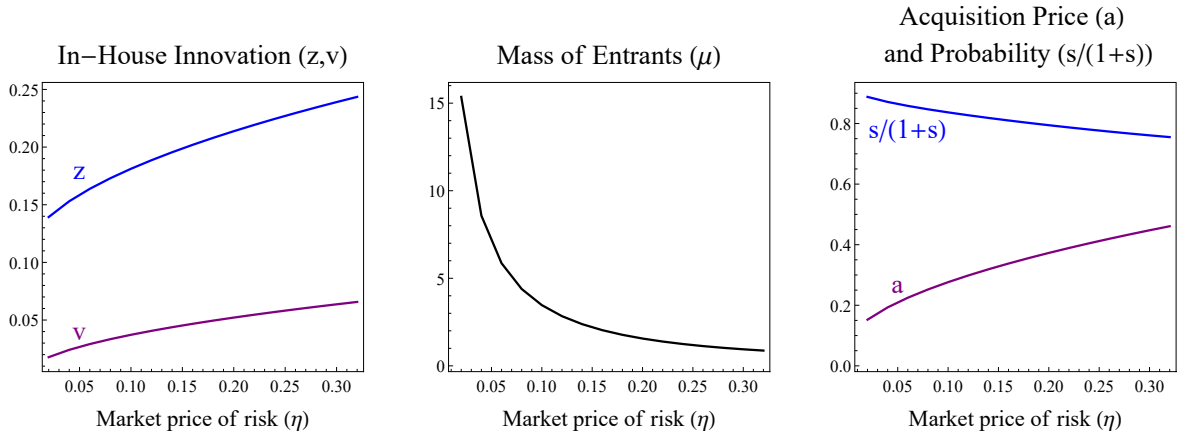

Figure 6: ALLOWING FOR TAKEOVERS. The figure shows the initiator and the entrants' in-house optimal innovation rates (z and v), the mass of entrants (μ), and the acquisition price and probability (a and $s/(1+s)$) as a function of η.

that the core predictions of Proposition 2 are confirmed: The optimal innovation rates z and v increase with η, whereas the mass of entrants decreases. Additionally, the right panel shows that acquisitions become more likely as η decreases, in which case the mass of entrants (thus, potential targets) increases. This result is consistent with the evidence in Haddad, Loualiche, and Plosser (2017) that merger activity increases with lower discount rates. In addition, we find that the endogenous acquisition cost is increasing with η: As targets become relatively more scarce, the initiator pays a higher premium in equilibrium.

3 Time-varying market price of risk

We next assume that the market price of risk varies with the state of the economy, being η_G in the good state and $\eta_B > \eta_G$ in the bad state.[25] Before analyzing the full model with both vertical and horizontal innovation, it is worth considering again the corner cases analogous to those in Section 2.2.2. We show the following results (see Appendix A.3.2).

Proposition 4 *Assume that $\eta_B > \eta_G$. If entrants invest in vertical innovation only, the initiator's and active entrants' innovation rates respectively satisfy*

$$z_j(\eta_j, \eta_{j-}) = \frac{\phi}{\zeta}(\lambda\varphi - 1)\, u_j(\eta_j, \eta_{j-}),$$
$$v_j(\eta_j, \eta_{j-}) = \frac{\phi_v}{\zeta_v}\left[\Lambda u_j(\eta_j, \eta_{j-}) - \kappa\right],$$

[25] Derivations of the firm's optimal choices and of the industry equilibrium are in Appendix A.3.

and are countercyclical, $z_B > z_G$ and $v_B > v_G$. Conversely, the rate of creative destruction Ψ_{vj} and the mass of entrants μ_j are procyclical, $\Psi_{vG} > \Psi_{vB}$ and $\mu_G > \mu_B$. The extensive innovation margin (μ_j) is more sensitive to variations in the market price of risk than the intensive margin (v_j). If, instead, entrants only invest in horizontal innovation, their optimal innovation rate satisfies

$$h_j(\eta_j, \eta_{j-}) = \frac{\phi_h}{\zeta_h} \left[\omega x_j(\eta_j, \eta_{j-}) - \kappa\right]. \tag{33}$$

and is countercyclical, $h_B > h_G$.

Proposition 4 illustrates how time variation in the market price of risk affects the industry equilibrium. It shows that the innovation rate of active firms is countercyclical—that is, it is greater when the market price of risk is larger. This holds in both corner cases with either vertical or horizontal innovation only. Proposition 4 also shows that the greater market price of risk in state B bears a negative impact on the *extensive* margin—i.e., the mass of active entrants declines. Moreover, the variation in the extensive margin is greater than the variation in the intensive margin. This implies that the procyclicality of μ then extends to the rate of creative destruction Ψ_v, which is procyclical too. In sum, in the good (bad) state, the mass of active entrants is larger (smaller), creative destruction is higher (lower), and incumbent firms reduce (increase) their R&D investment.

Innovation cyclicality and the market price of risk We next analyze the model featuring both vertical and horizontal innovation. On top of the parameters in Table 2, we assume that $\tilde{\pi}_G = 0.1$ and $\tilde{\pi}_B = 0.4$ under the physical measure, meaning that the good and the bad states are expected to last 10 and 2.5 years, respectively. Moreover, we set $\theta_G = -\theta_B = 0.08$, which implies that risk averse investors expect the good state to be shorter and the bad state to be longer than under the physical measure. Throughout the analysis, we consider two cases. First, we only allow η_j to vary across different states and set $\Gamma_G = \Gamma_B = 1$ (see equation (2)). Second, to acknowledge that variations in demand may impact R&D (e.g., Caballero and Hammour, 1994), we allow the demand function to vary. We keep $\Gamma_B = 1$ and set $\Gamma_G = 1.02$, so that the profit wedge between the good and the bad state is about 30%.[26]

Table 4 compares the endogenous quantities in the G and B states. In the top panel, we only vary η_j across states, whereas we also let Γ_j vary in the middle panel. Consistent with

[26]Such variation in profits across states is consistent with the change in total (detrended) earnings before interest, depreciation and amortization of R&D-active firms in Compustat between peaks and troughs.

Proposition 4, investment in innovation by active firms is countercyclical—i.e., higher in state B, in which the market price of risk is larger. This result aligns with the Schumpeterian view that firms should invest more in innovation in recessions than in expansion, as the opportunity cost of foregone revenues is smaller. In our model, this is the case both when abstracting (top panel) and when accounting for time-varying demand (middle panel). Hence, fluctuations in the market price of risk can alone generate countercyclical innovation rates.

Table 4 also shows that the mass of entrants is procyclical. That is, the higher market price of risk in the B state has the most detrimental effect on the extensive margin, by reducing the mass of entrants. At the industry level, the table also shows that the procyclicality of μ more than offsets the countercyclicality of firm-level innovation in both the vertical and horizontal dimension, so that Ψ_v and Ψ_h are both procyclical. Hence, active firms face greater competition in innovation—through greater creative destruction and horizontal displacement—when the market price of risk is lower in the good state. The aggregate innovation rate at which new technological clusters arise, $\mathcal{I}$, is also procyclical.

Our analysis then illustrates that variations in discount rates are an important driver of innovation cyclicality within an industry. Namely, when the market price of risk is high (in bad states of the economy), the mass of entrants should shrink, and active firms (initiator and entrants) should invest more in innovation. In contrast, when the market price of risk is low (in good states of the economy), we should see a considerable increase in the mass of entrants, which in turn should lead to a reduction in the innovation rate of active firms.

These results provide novel theoretical grounds to the evidence on R&D cyclicality, by posing the accent on the effect of discount rate fluctuations. Our paper can rationalize the observed procyclicality of innovation rates at the aggregate level—a pattern that has been consistently reported starting from Griliches (1984)—with the Schumpeterian view that firms should invest more in recessions than expansions. We predict that the procyclicality of aggregate R&D comes from the extensive margin, in line with the evidence in Brown, Fazzari, and Petersen (2009), Babina, Bernstein, and Mezzanotti (2022), and Howell et al. (2020).[27] Furthermore, Babina, Bernstein, and Mezzanotti (2022) find that, in downturns, the intensive margin of innovation is resilient whereas the extensive margin drops due to a substantial decline in patenting by entrepreneurs—consistent with our prediction that incumbents benefit from lower competition

[27] As we elaborate later in this section, the evidence in Brown, Fazzari, and Petersen (2009) and Howell et al. (2020) further emphasize the role of financing frictions in explaining innovation cyclicality.

in downturns.

Table 4 also shows that entrants allocate relatively more resources to explorative innovation during recessions: The ratio of v over total investment in innovation $v+h$ is strictly higher in the B state under all specifications. The result echoes the prediction in our single state model that higher discount rates stimulate vertical innovation (see Figure 3). The result is then consistent with recent firm-level evidence: Manso, Balsmeier, and Fleming (2021) find that exploration strategies are more prevalent in recessions, whereas Babina, Bernstein, and Mezzanotti (2022) report that innovation shifts towards more impactful patents in downturns.

The impact of fluctuations in the market price of risk We next investigate the impact of *fluctuations* in the market price of risk vis-à-vis an environment in which η is fixed. To this end, the last two columns of Table 4 report the model endogenous quantities in the one-state model (as analyzed in Section 2) and their averages in the two-state case. To make the one- and the two-state cases comparable, we assume that the time-invariant market price of risk in the one-state is equal to the average in the two-state model.[28]

Table 4 shows that fluctuations in the market price of risk affect the firm-level innovation rate only slightly under our baseline parameterization, with the entrants' horizontal innovation rate being only modestly smaller in the two-state model, on average. In turn, fluctuations in the market price of risk have a considerable impact on the mass of entrants, consistent with Proposition 4. Table 4 shows that the average mass of entrants in the two-state model is greater than its counterpart in the one-state. Moreover, the greater mass of entrants implies that the rate of creative destruction is greater, on average, in the two-states vis-à-vis the one-state model. As a result, the rate of arrival of new technological clusters $\mathcal{I}$ is greater in the two-state case, on average. In other words, our model shows that fluctuations in the market price of risk induce a greater industry turnover, which fosters the emergence of new technological clusters. These patterns are confirmed in the middle panel, in which we also account for demand-shifts over the business cycle. Thus, our paper supports the view that fluctuations are not detrimental to the industry equilibrium.[29]

Finally, Figure 7 investigates the sensitivity of equilibrium quantities to η_G and η_B. It shows

[28] We assume $\bar{\eta} = \frac{\eta_G \pi_B + \eta_B \pi_G}{\pi_B + \pi_G}$ in the one-state. In the middle panel, $\bar{\Gamma} = \frac{\Gamma_G \pi_B + \Gamma_B \pi_G}{\pi_B + \pi_G}$ in the one-state.

[29] Manso, Balsmeier, and Fleming (2021) show that macroeconomic fluctuations stir a more balanced mix between explorative and exploitative innovation. In line with Manso, Balsmeier, and Fleming (2021), in unreported results we find that h can be procyclical when allowing for time variation in Γ.

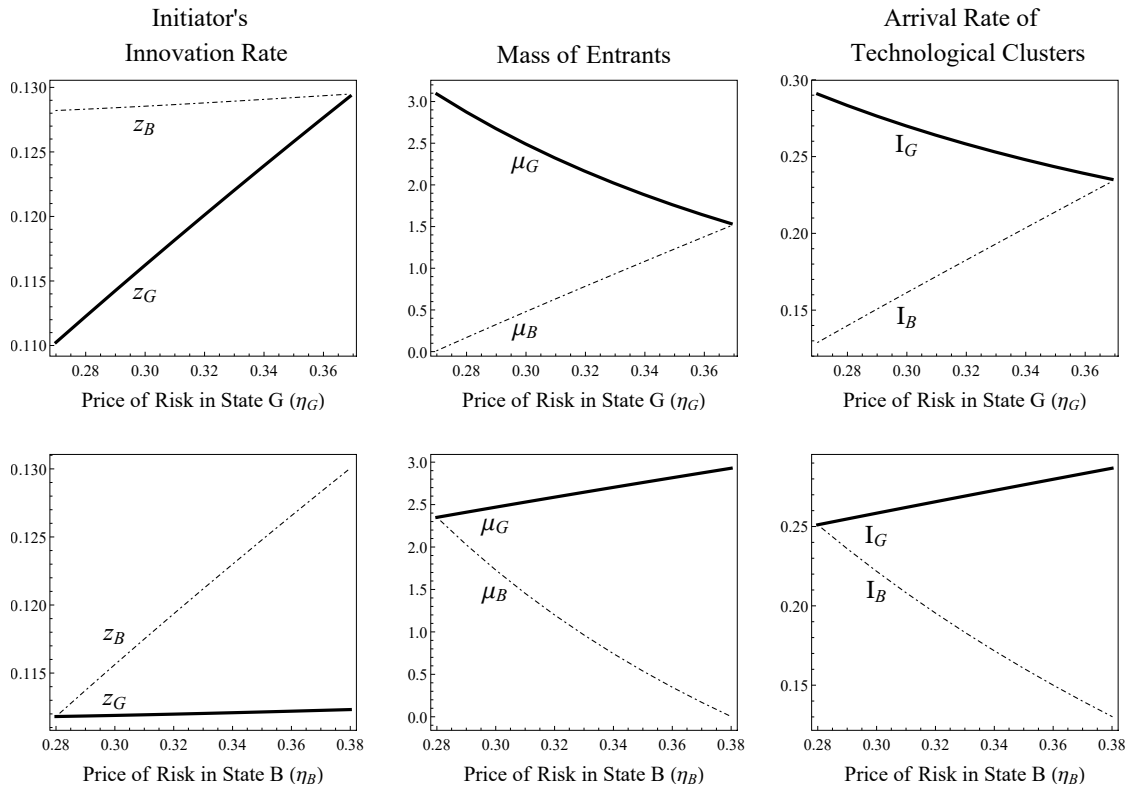

Figure 7: TIME-VARYING EXTENSIVE AND INTENSIVE MARGINS. The figure shows how the initiator's innovation rate (z_j), the mass of entrants (μ_j), and the arrival rate of technological clusters ($\mathcal{I}_j$) vary with the market price of risk in the contemporaneous (η_j) and non-contemporaneous (η_{-j}) state. The top (respectively, bottom) panel varies η_G (η_B).

that z increases with the magnitude of the market price of risk in the contemporaneous state j—consistent with the result in the one-state model—whereas it is quite insensitive to the market price of risk in the non-contemporaneous state $j-$. By contrast, the mass of entrants μ and the arrival rate of new technological clusters $\mathcal{I}$ are notably sensitive to the market price of risk in both j and j^-—they decrease with the market price of risk in the contemporaneous state j but increase with the market price of risk in the non-contemporaneous state j^-. Specifically, an increase in the market price of risk shifts entrants from the contemporaneous to the non-contemporaneous state, then also affecting the arrival rate of new technologies. This effect then sheds light on the importance of the market price of risk in transferring resources across states of the economy.

3.1 Time-varying ability to finance entry

As shown by Brown, Fazzari, and Petersen (2009) and Howell et al. (2020), entrants are largely exposed to shifts in the supply of finance over the business cycle.[30] We thus investigate whether our results continue to hold when allowing for time-variation in the entrants' ability to finance entry. We follow Malamud and Zucchi (2019) and allow the entry cost to include a financial component. We denote this financing cost by κ_{Fj}, $j = G, B$, so that the cost of entry is then $\kappa + \kappa_{Fj}$. Consistent with Brown, Fazzari, and Petersen (2009), we assume that $\kappa_{FG} < \kappa_{FB}$, i.e., the financing cost rises in the bad state of the economy.

The bottom panel of Table 4 shows that our results continue to hold and are amplified in magnitude in this extension.[31] Compared to our baseline in which only η is time-varying, allowing for variation in the cost of financing leads to a greater gap in the mass of entrants between the two states. That is, the greater financing cost in state B makes the mass of entrants even more procyclical, which bolsters the procyclicality in the rate of creative destruction and the rate of horizontal displacement. As a result, the initiator increases its investment in state B compared to our baseline case, strengthening the countercyclicality in the initiator's innovation rate.[32] In line with Howell et al. (2020), our paper thus shows that the aggregate contribution of entrants to innovation is higher in good states of the economy—nonetheless, the firm-level investment of active startups is higher in bad states. Fluctuations in the market price of risk continue to spur greater creative destruction and foster the arrival of new technological clusters.

4 Asset pricing implications

We conclude by analyzing the model implications for valuations and firm-level risk premia. A heuristic derivation of risk premia involves a comparison of the HJB equations under the physical and risk-neutral measures, as in Bolton, Chen, and Wang (2013). In each j, the risk premium

[30] Conversely, they do not find a significant imprint of these shifts on mature, incumbent firms.

[31] We continue to use the same parameters as in our baseline calibration and, additionally, we normalize κ_{FG} to zero and assume that κ_{FB} increases the cost of entry by 4%.

[32] These results are consistent with Malamud and Zucchi (2019), who show that financing frictions slow down creative destruction, reducing the entrants' contribution to growth but increasing that of incumbents.

of the initiator ($\mathcal{R}_{U,j}$) and of the exploiter ($\mathcal{R}_{X,j}$) respectively satisfy:[33]

$$\mathcal{R}_{U,j} \equiv \rho\sigma\eta_j \frac{y_j}{u_j} + \tilde{\pi}_j \left(\mathrm{e}^{\theta_j} - 1\right) \frac{u_j - u_{-j}}{u_j}, \tag{34}$$

$$\mathcal{R}_{X,j} \equiv \rho\sigma_X\eta_j \frac{\omega}{1-\delta\omega} \frac{y_{X,j}}{x_j} + \tilde{\pi}_j \left(\mathrm{e}^{\theta_j} - 1\right) \frac{x_j - x_{-j}}{x_j}. \tag{35}$$

In these expressions, the first term is the risk premium associated with the diffusion risk, and the second term is the premium associated with changes in the state of the economy.[34]

Notably, the first insight that stems from our analysis is that the resolution of the idiosyncratic uncertainty associated with innovation breakthroughs within the industry affect firms' exposure to systematic risk (i.e., Berk, Green, and Naik, 2004; Kumar and Li, 2016). In what follows, we develop two additional implications about the impact of competition in innovation on asset prices.

The extensive innovation margin as a hedge As a core prediction, our model shows that lower entry during downturns hedges innovating incumbents against higher discount rates. We begin by uncovering the mechanism behind this result in the one-state case with vertical innovation only, for which we obtain analytical results.

Corollary 5 *When entrants only invest in vertical innovation and η is not time-varying, the value of the initiator $u(\eta)$ increases with η, and its risk loading $\rho\sigma\frac{y(\eta)}{u(\eta)}$ decreases with η.*

Corollary 5 follows from Proposition 2. For higher levels of η, the initiator (that is, the innovating incumbent) faces a lower threat of exit as the rate of creative destruction decreases. Thus, the value of the initiator $u(\eta)$ increases, meaning that the lower rate of creative destruction hedges the initiator against a higher market price of risk. Consistently, the risk loading of the initiator, $\rho\sigma\frac{y(\eta)}{u(\eta)}$, falls as the market price of risk increases.

The left panel of Figure 2 confirms that the predictions in Corollary 5 also hold when allowing for both vertical and horizontal innovation. Furthermore, the middle panel of Figure 2 confirms that, for low values of η, the lower threat of entry also hedges exploiters (that is, the non-innovating producers) against reductions in value due to higher discount rates. The value of exploiters is, however, non-monotonic in η (see Section 2.2.3). While a greater η reduces the rate

[33] We do not elaborate on the corresponding risk premium of the entrant as it is likely unobservable by the econometrician (entrants should be interpreted as startups).

[34] The risk premium in the one-state case can be thus obtained by setting $\tilde{\pi}_j = 0$ in equation (34).

of creative destruction, it also incentivizes the initiator to innovate more which, then, increases the exit threat and reduces value for exploiters.

The next corollary focuses instead on the case with time-varying market price of risk.

Corollary 6 *When entrants only invest in vertical innovation and the market price of risk is time-varying, the initiators' risk premium associated to changes in the state of the economy,* $\tilde{\pi}_j \left(e^{\theta_j} - 1\right) \frac{u_j - u_{-j}}{u_j}$*, is strictly negative.*

When allowing the market price of risk to vary over time, the extensive innovation margin continues to act as a hedge, so that the value of the initiator is countercyclical. Crucially, such countercyclicality implies that the risk premium associated with jump risk (i.e., the second term in equation (34)) is unconditionally negative.[35] The same result in Corollary 6 applies to the full model with both vertical and horizontal innovation, given that the initiator's innovation rate (and the initiator's value) is countercyclical (see Table 4). Hence, in the two-state model, the extensive innovation margin acts as a hedge against cyclical fluctuations in discount rates.

Competition in the intensive margin and risk premia We now investigate how the interactions among active firms in the industry affect risk premia. We start by studying how incumbents' risk premia are affected by entrants' innovation rates in the corner cases, for which we obtain analytical proofs (see Appendix A.4.1).

Proposition 7 *If entrants only engage in vertical innovation, the initiator's risk premium,* $\mathcal{R}_{U,j}$*, is increasing in both the frequency and size of entrants' innovations (*ϕ_v *and* Λ*). Similarly, if entrants only engage in horizontal innovation, the exploiter's risk premium,* $\mathcal{R}_{X,j}$*, is increasing in the frequency and size of entrants' innovations (*ϕ_h *and* ω*) and in the degree of overlap between the ensuing new products and those of the initiator (*δ*).*

Proposition 7 shows that a higher likelihood or size of entrants' breakthroughs—in either the vertical or horizontal dimension—results in higher risk premia for the initiator and exploiters, respectively. All else equal, if breakthroughs are more likely or more profitable, entrants invest more in R&D, which increases the threat of exit or horizontal displacement for incumbent firms. Hence, greater innovation rates by entrants make both the initiator and the exploiters riskier.

[35] Corollary 6 follows directly from Proposition 4 and the assumption $\theta_G = -\theta_B > 0$, capturing that risk-averse agents expect recessions (respectively, expansions) to be longer (shorter), as in Bolton, Chen, and Wang (2013).

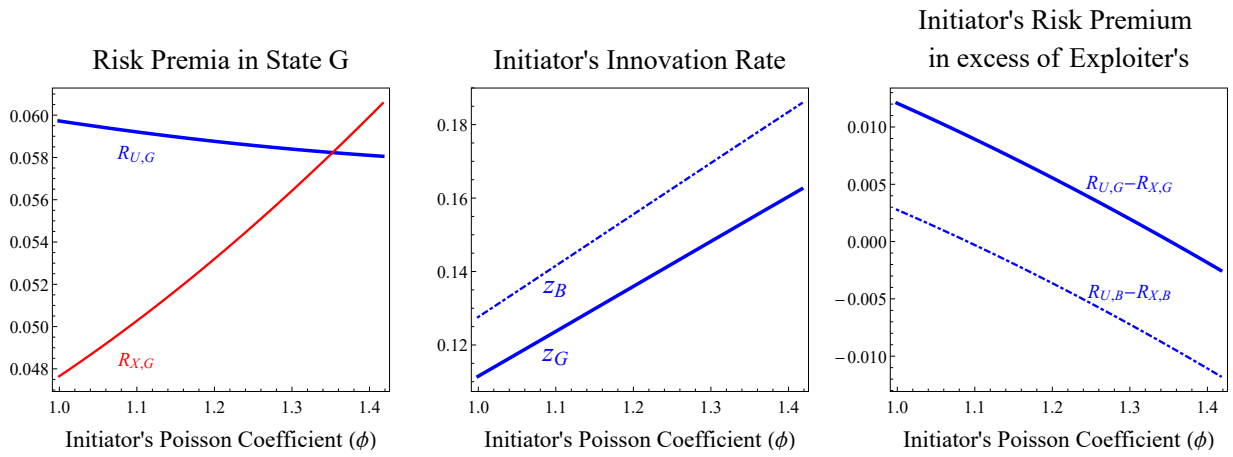

Figure 8: Risk premia and the initiator's innovation intensity. The figure shows the risk premia of the initiator and of the exploiters in state G, the initiator's optimal innovation rate, and the risk premium of the initiator in excess of the exploiters' as a function of the initiator's Poisson intensity ϕ.

Under our baseline calibration, this prediction extends to the full model with both vertical and horizontal innovation.

The full model also reveals that the risk premium of the initiator decreases with its own innovation—indicating that a firm's own innovation acts as insurance. The result is illustrated in the left panel in Figure 8. A higher breakthrough intensity for the initiator (captured by an increase in ϕ) boosts its incentives to invest in innovation (leading to a higher value of z_j), which in turn reduces its risk premium. Simultaneously, the middle panel in Figure 8 shows that an increase in ϕ leads to a higher risk premium for the exploiter, as it increases its threat of exit due to an initiator's breakthrough.

In sum, either when we look at interactions between entrants and incumbents or between the initiator and exploiters, the model reveals that innovation by competitors increases a firm's risk premium, whereas a firm's own innovation acts as insurance. These results are consistent with the patent race model without entry by Bena and Garlappi (2020). Bena and Garlappi (2020), however, consider a setting in which all firms innovate and further predict that market leaders are always less risky. By contrast, we consider an industry with both innovating and non-innovating firms and show that the initiator needs not be safer than exploiters, as illustrated in the right panel of Figure 8. Relatedly, Bena, Fisher, Knesl, and Vahl (2022) document that non-innovating firms earn significantly lower returns.

5 Concluding remarks

The study of corporate innovation is key to understand the real economy. Our paper highlights that discount rates are an important determinant of R&D decisions. In contrast with the conventional wisdom that higher discount rates deter long-term investment, we show that higher discount rates can encourage innovation in the intensive margin, and spur the emergence of new technologies stemming from explorative innovation. That is, discount rates affect both the nature and composition of R&D within an industry.

Our results further highlight that discount rate fluctuations help rationalize the documented cyclicality of R&D. Importantly, the model shows that the extensive innovation margin is countercyclical, whereas the intensive margin is procyclical. This result reconciles the Schumpeterian view that firms should innovate more intensively in recessions with the observed procyclicality of aggregate R&D investment in the data. The model also uncovers novel asset pricing implications. In particular, we show that the lower threat of entry by new firms in downturns hedges innovating incumbents against higher discount rates.

Overall, our findings shed light on the importance of studying firms' innovation decisions in industry equilibrium while accounting for the level and time-variation of discount rates—two determinants of R&D investments that are usually overlooked in macroeconomic studies, and on which we intend to elaborate further in future research.

A Appendix

A.1 Methodological Details on Figure 1 and Table 1

The sample period throughout is 1982 to 2017, and we use multiple datasets to conduct our tests. We download the measure of the aggregate risk premium from Professor Erik Loualiche's website, whose construction is described in Haddad, Loualiche, and Plosser (2017). We use WRDS Compustat yearly data for the firm-level analyses on R&D investment. To study variation in the number of firms (i.e., the extensive innovation margin), we use the surveys on industrial R&D from the National Science Foundation (NSF) and the historical series reported by Business Dynamics Statistics (BDS) project from the US Census Bureau. We also use the aggregate time series of R&D over sales from the NSF. Unlike WRDS Compustat, the NSF and BDS datasets cover public and private firms, and have been widely used in the literature—i.e., Barlevy (2007) uses NSF data to study R&D cyclicality.

The evidence in Table 1 aligns well with the predictions of the model. Namely, the top panel of Table 1 reports firm-level panel regressions using Compustat data. To proxy for the firm's intensive margin, we use both R&D-to-assets and R&D-to-sales as our dependent variable. We use firm fixed effects and cluster standard errors at the firm level. The controls included in the second and fourth columns are Average Q, Asset Tangibility, Book Leverage, Market Value, Return on Assets, and the Kaplan-Zingales Index. The choice of controls (as well as the construction of such controls) follows Fang, Tian, and Tice (2014). Notably, irrespective of the dependent variable or controls used, we find that higher discount rates are unambiguously related to more innovation along the intensive margin—i.e., active firms invest more in innovation. To further strengthen this pattern, the first regression in the bottom panel of Table 1—using aggregate data from NSF—suggests that this conclusion is robust to considering both public and private firms.

The tests reported in the bottom panel of Table 1 study how discount rates relate to the extensive margin. In these regressions, the dependent variable is the number of firms—more specifically, NSF reports innovating firms only, whereas BDS covers the entire universe of firms (innovating or not) in each industry. When using BDS data, for consistency in the comparison with the firm-level evidence in Figure 1, we restrict our analysis to the same set of industries covered in the firm-level Compustat tests on R&D. Results remain qualitatively unchanged if we remove this restriction.

Using data aggregated at the national level, we first observe that the number of innovating firms in the US—as reported by NSF—is negatively related to discount rates. We obtain the same qualitative inference if we instead consider BDS data at the aggregate level. These findings align with our model's prediction that higher discount rates erode innovation in the extensive margin. Last, using BDS data, we verify that the result also holds at the industry level. We define industries at the 4-digit NAICS code level, and use industry fixed effects throughout and cluster standard errors at the industry level. Overall, the evidence suggests that a higher aggregate discount rate discourages entry (and, thus, innovation) by new firms—effectively acting as an entry barrier.

A.2 Proofs of the results in Section 2

Using Girsanov theorem, the risk-neutral dynamics of the cash flows of the initiator, exploiters, and entrants satisfy:

$$\begin{aligned} dC_t =& Y_t \left(p_t - 1 - \sigma\rho\eta\right) M_t dt - \Phi(z_t, q_t, M_t)dt + \sigma Y_t M_t dB_t^U, \\ dC_t^X =& Y_{Xt} \left(p_{Xt} - 1 - \eta\rho\sigma_X\right) M_{Xt} dt + \sigma_X Y_{Xt} M_{Xt} dB_t^X \\ dC_t^W =& \left[-\left(\frac{1}{2}\zeta_v v_t^2 + \frac{1}{2}\zeta_h h_t^2 + \eta\rho_W\sigma_W \right) dt + \sigma_W dB_t^W \right] M_t q_t, \end{aligned}$$

where B_t^k, $k = U, X, W$, are the Brownian motion describing the initiator, exploiter, and entrant shocks under the risk-neutral measure.

Substituting $U(q_t, M_t) = q_t M_t u$, into equation (9) gives the scaled HJB of the initiator:

$$ru = \max_{z,y} \; y^{1-\beta}\Gamma - y - \sigma\eta\rho y - \frac{z^2}{2}\zeta + \phi z \left(\lambda\varphi - 1\right) u - \Psi_v u(1-\alpha) - \Psi_h u\omega\delta \tag{36}$$

and, plugging in this equation the optimal z, y, and p gives the valuation equation of the initiator:

$$\frac{\phi^2}{2\zeta}\left(\lambda\varphi - 1\right)^2 u^2 - (r + \Psi_v(1-\alpha) + \Psi_h\omega\delta)u + \Upsilon(\eta) = 0. \tag{37}$$

Moreover, the scaled HJB of the exploiter satisfies:

$$rx = \max_{y_X \geq 0} \quad \frac{\omega}{1-\omega\delta} y_X \left(\Gamma y_X^{-\beta} - 1 - \eta\rho\sigma_X \right) - \left(\phi z + \Psi_v\right)\left(1 - \alpha_X\right) x - \Psi_h\omega\delta x. \tag{38}$$

Substituting equation (14) into (38) gives the valuation equation for the exploiter:

$$rx = \frac{\beta\omega}{1-\omega\delta}\left(\frac{1-\beta}{1+\sigma_X\rho\eta}\right)^{\frac{1}{\beta}-1}\Gamma^{\frac{1}{\beta}} - (\phi z + \Psi_v)x\,(1-\alpha_X) - \Psi_h\omega\delta x. \tag{39}$$

Finally, the scaled HJB of the entrant satisfies:

$$\begin{aligned} rw = \max_{v,h} &- \eta\rho_W\sigma_W - \frac{\zeta_v}{2}v^2 - \frac{\zeta_h}{2}h^2 + \phi_v v\,[\Lambda u - w] + \phi_h h\,[\omega x - w] \\ &+ \phi z\,(\lambda\varphi - 1)\,(w-\kappa) + \Psi_v^-\,(\Lambda - 1)\,(w-\kappa) - \Psi_h^-(w-\kappa)\omega\delta. \end{aligned} \tag{40}$$

Substituting equations (16) and (17) back into the HJB equation (40) gives:

$$\begin{aligned} rw = &- \eta\rho_W\sigma_W + \frac{\phi_v^2}{2\zeta_v}\left[\Lambda u - w\right]^2 + \frac{\phi_h^2}{2\zeta_h}\left[\omega x - w\right]^2 + \phi z\,(\lambda\varphi - 1)\,(w-\kappa) \\ &+ \Psi_v^-\,(\Lambda - 1)\,(w-\kappa) - \Psi_h^-\omega(w-\kappa) \end{aligned} \tag{41}$$

Using the free-entry condition $w = \kappa$, the above equation boils down to

$$rw = - \eta\rho_W\sigma_W + \frac{\phi_v^2}{2\zeta_v}\left[\Lambda u - w\right]^2 + \frac{\phi_h^2}{2\zeta_h}\left[\omega x - w\right]^2 \tag{42}$$

and becomes a function of u and x. In turn, the value of the initiator u (equation (37)) is a function of Ψ_v and Ψ_h, which are themselves endogenous functions of μ, u, and x. Similarly, the valuation equation of the exploiter x (equation (38)) depends on z, Ψ_v, and Ψ_h. As a result, we solve the system of equations (37), (38), and (42) to get the endogenous quantities μ, u, and x, which in turn we substitute into equations (11), (16), and (17) to get the optimal innovation rates z, v, and h. Finally, using the expressions for v and h, together with μ, we pin down Ψ_v and Ψ_h.

A.2.1 Proof of Proposition 1

The expression for $z(\eta)$ with exogenous industry dynamics follows by substituting equation (21)—solved for a given (exogenous) Ψ_v and Ψ_h—into equation (11). Notably, $z(\eta)$ is a function of η through the function $\Upsilon(\eta)$, defined in Section 2.1, which decreases with η as

$$\Upsilon'(\eta) = -\rho\sigma\left(\frac{1-\beta}{1+\eta\rho\sigma}\right)^{\frac{1}{\beta}}\Gamma^{\frac{1}{\beta}} < 0.$$

Thus, $z(\eta)$ decreases with η too, and the result follows.

A.2.2 Proof of Proposition 2

In the case with vertical innovation only, the value of the initiator satisfies:

$$rU(q,M) = \max_{z,Y} MY(p-1-\sigma\eta\rho) - \zeta\frac{z^2}{2}qM + \phi z\left[U(\lambda q,\varphi M) - U(q,M)\right] + \Psi_v\left[\alpha U(q,M) - U(q,M)\right] \tag{43}$$

where the expressions for the optimal z, y, and p are given by equations (11) and (10).

The value of an entrant, denoted by $W(q,M)$, satisfies:

$$rW(q,M) = \max_{v\geq 0} -qM\left(\eta\rho_W\sigma_W + \frac{\zeta_v}{2}v^2\right) + \phi_v v\left[U(\Lambda q,M) - W(q,M)\right] \tag{44}$$
$$+ \phi z\left[W(\lambda q,\varphi M) - W(q,M) - K(\lambda\varphi - 1)\right] + \Psi_v^-\left[W(\Lambda q,M) - W(q,M) - K(\Lambda - 1)\right]$$

where the terms admit a similar interpretation to equation (15). Using the scaling property and differentiating with respect to v gives the optimal innovation rate in equation (16). Plugging the optimal v back into the HJB gives:

$$rw = -\eta\rho_W\sigma_W + \frac{\phi_v^2}{2\zeta_v}\left[\Lambda u - w\right]^2 + \left[\phi z\left(\varphi\lambda - 1\right) + \Psi_v^-\left(\Lambda - 1\right)\right](w-\kappa). \tag{45}$$

Using the free-entry condition $w = \kappa$, we solve the above equation with respect to u, which then gives equation (24). By substituting u into (11) and (16), we obtain $z(\eta)$ and $v(\eta)$ as reported in Proposition 2, which are straightforward to show to be increasing with η.

We now prove the sensitivity of $\Psi_v(\eta)$ and $\mu(\eta)$ with respect to η. Scaling equation (43) by q and M, substituting the optimal y and z, and solving with respect to $\Psi_v(\eta)$ gives:

$$\Psi_v(\eta) = \frac{1}{1-\alpha}\left(\frac{\phi^2}{2\zeta}u(\eta)\left(\lambda\varphi - 1\right)^2 - r + \frac{\Upsilon(\eta)}{u(\eta)}\right)$$

Differentiating with respect to η gives

$$\Psi_v'(\eta) = \frac{\Upsilon'(\eta)}{u(\eta)(1-\alpha)} - \left[\Upsilon(\eta) - \frac{\phi^2(\lambda\varphi-1)^2u^2(\eta)}{2\zeta}\right]\frac{u'(\eta)}{u^2(\eta)(1-\alpha)}. \tag{46}$$

The first term is negative as $1-\alpha > 0$, and $\Upsilon'(\eta) < 0$ as shown in Appendix A.2.1. The second

term is also negative, as $u'(\eta) > 0$ (as is straightforward from equation (24)) and the term in square brackets is positive when we consider values of η that rule out the degenerate case in which the initiator always makes losses in expectation—i.e., we consider values of η such that $y(p-1-\eta\sigma\rho) - \frac{\zeta}{2}\zeta^2 > 0$, as explained in Section 1. Indeed

$$y(p-1-\eta\sigma\rho) - \frac{\zeta}{2}z^2 = \Upsilon(\eta) - \frac{\phi^2(\lambda\varphi-1)^2u^2(\eta)}{2\zeta} > 0$$

is the term in brackets in (46). Thus, Ψ_v decreases with η, as stated in Proposition 2.

As the last step, we differentiate $\mu(\eta) = \frac{\Psi_v(\eta)}{\phi_v v(\eta)}$ with respect to η, that gives

$$\mu'(\eta) = \frac{\Psi_v'(\eta)}{\phi_v v(\eta)} - \frac{\Psi_v(\eta)v'(\eta)}{\phi_v v^2(\eta)}. \tag{47}$$

The first term is negative as $\Psi_v'(\eta) < 0$, as shown above. The second term is also negative, as $\Psi_v(\eta) > 0$ and $v'(\eta) > 0$. The claim in Proposition 2 then follows.

A.2.3 Proof of Proposition 3

Assuming that entrants only innovate horizontally, their value satifies:

$$\begin{aligned} rW(q,M) = \max_{v,h} \quad & -qM\left(\eta\rho_W\sigma_W + \frac{\zeta_h}{2}h^2\right) + \phi_h h\left[X(q,\omega M) - W(q,M)\right] \\ & + \phi z\left[W(\lambda q,\varphi M) - W(q,M) - K(\lambda\varphi-1)\right] + \Psi_h^-\left[W(q,M(1-\omega\delta)) - W(q) + K\omega\delta\right] \end{aligned} \tag{48}$$

where the terms admit a similar interpretation to equation (15). Using the same scaling property used in the full case and differentiating with respect to h, we obtain the optimal innovation rate reported in equation (17). Plugging this expression back into the HJB and imposing $w = \kappa$ gives:

$$rw = -\eta\rho_W\sigma_W + \frac{\phi_h^2}{2\zeta_h}\left[\omega x - w\right]^2. \tag{49}$$

Solving this equation with respect to x gives equation (27). By substituting x into (17) then gives the expression for $h(\eta)$ reported in Proposition 3.

In this case, differently from the full case, the initiator is not subject to creative destruction Ψ_v. Thus, the value of the initiator satisfies:

$$\frac{\phi^2}{2\zeta}\left(\lambda\varphi-1\right)^2u^2 - (r+\Psi_h\omega\delta)u + \Upsilon(\eta) = 0. \tag{50}$$

In this case, the exploiters face the threat of exit only due to the initiator's breakthroughs (i.e., they are not subject to creative destruction due to the entrants' innovations). Thus, the value of the exploiters satisfies the following equation:

$$rx = \frac{\beta\omega}{1-\omega\delta}\left(\frac{1-\beta}{1+\sigma_X\rho\eta}\right)^{\frac{1}{\beta}-1}\Gamma^{\frac{1}{\beta}} - \phi z\left(1-\alpha_X\right)x - \Psi_h\omega\delta x. \tag{51}$$

Now, recall that $\Psi_h = \mu\phi_v h$, where h satisfies the equation reported in Proposition 3. As a result, we can find v and μ by solving the system of equations (50)-(51) and, thus, the optimal innovation rate of initiators z as well as the rate of horizontal displacement Ψ_h.

A.2.4 Proof of the results in Section 2.3

In scaled terms, equation (29) becomes

$$ru = \max_{z,y,s} y(p-1-\sigma\eta\rho) - \frac{\zeta^2}{2}z - \zeta_s s + \phi z\left[\lambda\varphi - 1\right]u + \Psi_v\left[-\frac{(1-\alpha)\,u}{1+s} + \frac{s}{1+s}\left(\Lambda u - u - \Lambda a\right)\right]. \tag{52}$$

Differentiating this equation with respect to z and y, we get the same expression for the optimal innovation rate and production rate that we get in the baseline model. Additionally, differentiating the above equation and solving for s gives equation (30).

In turn, the entrants' HJB equation satisfies:

$$\begin{aligned} rW(q,M) = \max_{v} \quad & -qM\left(\eta\rho_W\sigma_W + \frac{\zeta_v}{2}v^2\right) + \phi_v v\left[\frac{\left(U(\Lambda q,M) - W(q,M) - G\right)}{1+s} + \frac{s\left(A - W(q,M)\right)}{1+s}\right] \\ & + \phi z\left[W(\lambda q,\varphi M) - W(q,M) - K(\lambda\varphi - 1)\right] + \Psi_v^{-}\left[W(\Lambda q,M) - W(q,M) - K(\Lambda - 1)\right], \end{aligned}$$

where the second term on the right-hand side specifies the outcome of a breakthrough for the entrant. With probability $1/(1+s)$, the entrant takes over the initiator's market position by paying the setup cost G_t. Conversely, with probability $s/(1+s)$, the entrant is acquired and receives the acquisition cost A_t. Exploiting the scaling property gives:

$$\begin{aligned} rw = \max_{v} \quad & -\left(\eta\rho_W\sigma_W + \frac{\zeta_v}{2}v^2\right) + \phi_v v\left[\frac{1}{1+s}\left(\Lambda u - w - \Lambda g\right) + \frac{s}{1+s}\left(\Lambda a - w\right)\right] \\ & + \phi z\left[\lambda\varphi w - w - \kappa(\lambda\varphi - 1)\right] + \Psi_v^{-}\left[\Lambda w - w - \kappa(\Lambda - 1)\right]. \end{aligned} \tag{53}$$

Maximizing with respect to v gives the optimal innovation rate in equation (31).

The initiator and the target entrant negotiate over the acquisition cost. The solution of their Nash bargaining solves:

$$\arg\max \left(\Lambda u - \alpha u - \Lambda a\right)^{b} \left(\Lambda a - \Lambda u + g\Lambda\right)^{1-b}. \tag{54}$$

The first term is the incremental value to the initiator stemming from the acquisition as opposed to being kicked out of the industry. The second term is the incremental value for the entrant stemming from being acquired as opposed to becoming initiator. Solving (54) gives the acquisition cost reported in equation (32). Note that the acquisition adds value to both parties if the setup cost g is sufficiently large, and the liquidation cost sufficiently low, i.e., $\frac{u}{\Lambda}\alpha < g$.

In the numerical implementation of this extension, we use our baseline parameterization with the only change that we assume that $\alpha = 0$. Additionally, we assume that the search cost is $\zeta_s = 0.001$ (meaning that searching for a target is much cheaper than innovating), the bargaining power of the initator is $b = 0.4$, and the setup cost $g = 0.65$.

A.3 Proof of the results in Section 3

A.3.1 Derivation of firm values and optimal investment rates

In the two-state model, all value functions and the endogenous quantities are a function of (η_j, η_{j-})—i.e., the market risk prices in the two states. For the ease of exposition throughout this appendix, we omit these arguments. Consider first the value of the initiator. Following standard arguments, the initiator's scaled HJB equation in each state j satisfies:

$$ru_j = \max_{z_j, y_j} \Gamma_j y_j^{1-\beta} - y_j - \frac{z_j^2}{2}\zeta - \sigma\eta_j\rho y_j + \phi z_j\left[\lambda - 1\right]u_j - \Psi_{vj}u_j(1-\alpha) - \Psi_{hj}u_j\omega\delta + \pi_j\left[u_{j-} - u_j\right] \tag{55}$$

where $\pi_j = \tilde{\pi}_j e^{\theta_j}$ is the transition intensity under the risk-neutral measure. The last term on the right-hand side captures the effect of a state switch, in which case firm value goes from u_j to u_{j-}. Differentiating the above equation with respect to y_j gives:

$$y_j = \left(\frac{\Gamma_j(1-\beta)}{1+\sigma\eta_j\rho}\right)^{\frac{1}{\beta}} \quad \Rightarrow \quad p_j = \Gamma_j y_j^{-\beta} = \frac{1+\sigma\eta_j\rho}{1-\beta}.$$

Similarly, differentiating equation (55) with respect to z_j gives the optimal innovation rate:

$$z_j = \frac{\phi}{\zeta}(\lambda\varphi - 1)\, u_j.$$

Plugging back the expressions for z_j and y_j into equation (55) gives the value of the initiator.

Consider now the dynamics of the exploiters. Following arguments similar to those in Section 2, their scaled value satisfies the following equation:

$$rx_j = \max_{y_{Xj}\geq 0} \frac{\omega}{1-\omega\delta} y_{Xj}\left(p_{Xj} - 1 - \eta_j\rho\sigma_X\right) - \left(\phi z_j + \Psi_{vj}\right)(1-\alpha_X)\, x_j - \omega\delta\Psi_{hj}x_j + \pi_j\left[x_{j-} - x_j\right],$$

where the last term on the right-hand side captures the effect of a state switch, in which case firm value goes from x_j to x_{j-}. Maximizing with respect to y_{Xj} gives:

$$y_{Xj} = \left(\frac{\Gamma_j(1-\beta)}{1+\eta_j\rho\sigma_X}\right)^{\frac{1}{\beta}}.$$

Finally, the scaled entrant value satisfy the following equation:

$$\begin{aligned} rw_j = \max_{v_j,h_j} &-\eta_j\rho_W\sigma_W - \frac{\zeta_v}{2}v_j^2 - \frac{\zeta_h}{2}h_j^2 + \phi_v v_j\left[\Lambda u_j - w_j\right] + \phi_h h_j\left[\omega x_j - w_j\right] \\ &+ \phi z_j(\lambda\varphi - 1)(w_j - \kappa) + \Psi_{vj}^{-}(\Lambda - 1)(w - \kappa) - \Psi_{hj}^{-}(w_j - \kappa)\omega\delta + \pi_j\left[w_{j-} - w_j\right]. \end{aligned} \tag{56}$$

The last term on the right-hand side captures the effect of a state switch. In each state, the optimal rate of vertical and horizontal innovation respectively satisfy:

$$v_j = \frac{\phi_v}{\zeta_v}\left[\Lambda u_j - w_j\right], \quad \text{and} \quad h_j = \frac{\phi_h}{\zeta_h}\left[\omega x_j - w_j\right]. \tag{57}$$

In each state, the rate of creative destruction and the rate of horizontal displacement satisfy $\Psi_{vj} = \mu_j\phi_v v_j$ and $\Psi_{hj} = \mu_j\phi_h h_j$, and the free-entry condition $w_j = \kappa$ holds.

A.3.2 Proof of Proposition 4

Vertical innovation Following steps as in Appendix A.2.2, the initiator value in each j satisfies:

$$u_j = \frac{\kappa}{\Lambda} + \frac{1}{\Lambda}\sqrt{\frac{2\zeta_v(r\kappa + \eta_j\rho_W\sigma_W)}{\phi_v^2}}. \tag{58}$$

As $\eta_B > \eta_G$, the initiator value is greater in $j = B$. Moreover, using equation (58) gives

$$z_j = \frac{\phi(\lambda - 1)}{\zeta} \left[\frac{\kappa}{\Lambda} + \frac{1}{\Lambda} \sqrt{\frac{2\zeta_v(r\kappa + \eta_j \rho_W \sigma_W)}{\phi_v^2}} \right] \tag{59}$$

as well as the optimal (vertical) innovation rate of active entrants:

$$v_j = \frac{\phi_v}{\zeta_v} \sqrt{\frac{2\zeta_v(r\kappa + \eta_j \rho_W \sigma_W)}{\phi_v^2}} = \sqrt{\frac{2(r\kappa + \eta_j \rho_W \sigma_W)}{\zeta_v}}. \tag{60}$$

Hence, the first part of the claim in Proposition 4 follows.

Consider now the the rate of creative destruction. In the two states, it satisfies:

$$\Psi_{vj}(\eta_j, \eta_{j-}) = \frac{1}{1-\alpha} \left(\frac{\phi^2}{2\zeta} u_j \left(\lambda\varphi - 1\right)^2 - r + \frac{\Upsilon(\eta_j)}{u_j} + \pi_j \frac{(u_{j-} - u_j)}{u_j} \right). \tag{61}$$

Let us start by considering the case in which Γ_j does not vary across states—i.e., $\Gamma_G = \Gamma_B \equiv \Gamma$. Now, express $\eta_B = \eta_G + \Delta$, with $\Delta \geq 0$. If $\Delta = 0$, $\eta_B = \eta_G$, and we are back to the one-state case, meaning that $\Psi_{vB} = \Psi_{vG}$—basically, there is no variation across the two states. Conversely, when $\Delta > 0$, $\Psi_{vB} \neq \Psi_{vG}$. To study the relative magnitude of creative destruction in the two states, we next define the function $F(\Delta) = \Psi_{vB}(\Delta) - \Psi_{vG}(\Delta)$ for $\Delta \geq 0$. As just discussed, $F(0) = 0$ holds. Using equation (61), we study $F'(\Delta)$. Let us also express u_B and u_G as a function of Δ. By calculations, we find that

$$\begin{aligned} F'(\Delta) = &- \frac{\rho\sigma}{(1-\alpha)u_B} \left(\frac{\Gamma(1-\beta)}{1 + (\eta_G + \Delta)\rho\sigma} \right)^{\frac{1}{\beta}} - \frac{u_B'}{(1-\alpha)} \left(\frac{\pi_G}{u_G} + \frac{\pi_B u_G}{u_B^2} \right) \\ &- \left[\beta \left(\frac{(1-\beta)}{1 + (\eta_G + \Delta)\rho\sigma} \right)^{\frac{1}{\beta} - 1} \Gamma^{\frac{1}{\beta}} - \frac{\phi^2 (\lambda\varphi - 1)^2 u_B^2}{2\zeta} \right] \frac{u_B'(\Delta)}{(1-\alpha)u_B^2} \end{aligned}$$

with $u_B'(\Delta) = \frac{\zeta_v \rho_W \sigma_W}{\Lambda \phi_v \sqrt{2\zeta_v [r\kappa + (\eta_G + \Delta)\rho_W \sigma_W]}} > 0$. The term $\left[\beta \left(\frac{(1-\beta)}{1 + (\eta_G + \Delta)\rho\sigma} \right)^{\frac{1}{\beta} - 1} \Gamma^{\frac{1}{\beta}} - \frac{\phi^2 (\lambda\varphi - 1)^2 u_B^2}{2\zeta} \right]$ is positive under our assumption that the initiator's expected net cash flow is positive. $F'(\Delta)$ is then negative. Thus, the function F is zero at $\Delta = 0$ and decreases for $\Delta > 0$, so that $\Psi_{vB}(\Delta) < \Psi_{vG}(\Delta)$ if $\eta_B > \eta_G$. That is, creative destruction is procyclical.

Consider now the case $\Gamma_G > \Gamma_B$. If $\Delta = 0$, then $u_B = u_G$, $z_B = z_G$, and $v_B = v_G$, as these quantities do not depend on Γ_j (see equations (58), (59), and (60)). Consider again the function $F(\Delta)$ defined above. Let us first evaluate this function for $\Delta = 0$. Using equation (61), we have

that $F(0) = \frac{1}{(1-\alpha)u_B(0)}\beta\left(\frac{(1-\beta)}{1+\eta_G\rho\sigma}\right)^{\frac{1}{\beta}-1}\left(\Gamma_B^{\frac{1}{\beta}} - \Gamma_G^{\frac{1}{\beta}}\right)$, where we have used that $u_B(0) = u_G(0)$. As $\Gamma_G > \Gamma_B$ by assumption, then $F(0) < 0$. As $F'(\Delta) < 0$ following the steps above, then $\Psi_{vB}(\Delta) < \Psi_{vG}(\Delta)$ for the case $\Gamma_G > \Gamma_B$ too.

Recall that $\Psi_{vj} = \mu_{vj}\phi_v v_j$. As shown above, $\Psi_{vB} - \Psi_{vG} < 0$ and $v_B > v_G$. Thus, for $\Psi_{vB} - \Psi_{vG} = \phi_v\left[\mu_B v_B - \mu_{vG} v_G\right] < 0$ to hold, it must be that $\mu_B < \mu_G$. Thus, the mass of active entrants is also procyclical. Moreover, using the expression for v_j gives:

$$\Psi_{vB} - \Psi_{vG} = \phi_v\left[\mu_B\sqrt{\frac{2(r\kappa + (\eta_G + \Delta)\rho_W\sigma_W)}{\zeta_v}} - \mu_G\sqrt{\frac{2(r\kappa + \eta_G\rho_W\sigma_W)}{\zeta_v}}\right] \tag{62}$$

The first square root is greater than the second, so $\sqrt{\frac{2(r\kappa+(\eta_G+\Delta)\rho_W\sigma_W)}{\zeta_v}} = A\sqrt{\frac{2(r\kappa+\eta_G\rho_W\sigma_W)}{\zeta_v}}$, with $A > 1$. Given $\mu_B < \mu_G$, we express $\mu_G = B\mu_B$ with $B > 1$. Hence

$$\Psi_{vB} - \Psi_{vG} = \phi_v\mu_B\sqrt{\frac{2(r\kappa + \eta_G\rho_W\sigma_W)}{\zeta_v}}\left[A - B\right]. \tag{63}$$

As $\Psi_{vB} - \Psi_{vG} < 0$, then it must be that $A < B$, meaning that the mass of entrants μ_j (the extensive margin) varies more than v_j (the intensive margin) for a given variation in Δ.

Horizontal innovation Following steps similar to those in Appendix A.2.3, we solve for the exploiter value:

$$x_j = \frac{\kappa}{\omega} + \frac{1}{\omega}\sqrt{\frac{2\zeta_h(r\kappa + \eta_j\rho_W\sigma_W)}{\phi_h^2}}. \tag{64}$$

Using this expression into h_j gives

$$h_j = \sqrt{\frac{2(r\kappa + \eta_j\rho_W\sigma_W)}{\zeta_h}}. \tag{65}$$

Because $\eta_B > \eta_G$, then $h_B > h_G$. The claims in Proposition 4 then follow.

A.4 Proof of the results in Section 4

A.4.1 Proof of Proposition 7

The derivative of the risk premium of the initiator, $\mathcal{R}_{U,j}$, with respect to ϕ_v is given by:

$$\frac{\partial \mathcal{R}_{U,j}}{\partial \phi_v} = \frac{\eta_j \Lambda \rho \sigma y_j \sqrt{2\zeta_v(\eta_j \rho_W \sigma_W + \kappa r)}}{\left(\kappa\phi_v + \sqrt{2\zeta_v(\eta_j \rho_W \sigma_W + \kappa r)}\right)^2} + \frac{\kappa\sqrt{2\zeta_v}\left(\sqrt{(\eta_{-j}\rho_W\sigma_W + \kappa r)} - \sqrt{(\eta_j \rho_W \sigma_W + \kappa r)}\right)}{\left(e^{\theta_j} - 1\right)^{-1} \tilde{\pi}_j^{-1} \left(\kappa\phi_v + \sqrt{2\zeta_v(\eta_j \rho_W \sigma_W + \kappa r)}\right)^2},$$

which is positive, so $\mathcal{R}_{U,j}$ increases with ϕ_h.[36]

Next we calculate the derivative of $\mathcal{R}_{U,j}$ with respect to Λ, to obtain:

$$\frac{\partial \mathcal{R}_{U,j}}{\partial \Lambda} = \frac{\eta_j \phi_v \rho \sigma y_j}{\left(\kappa\phi_v + \sqrt{2\zeta_v(\eta_j \rho_W \sigma_W + \kappa r)}\right)},$$

which is strictly positive for any parameter value. It follows that $\mathcal{R}_{U,j}$ is increasing in Λ.

Consider next the risk premium of the exploiter, $\mathcal{R}_{X,j}$ defined in equation (35). Using the expression for x_j in equation (64), we obtain:

$$\frac{\partial \mathcal{R}_{X,j}}{\partial \phi_h} = \frac{\sqrt{2}\kappa\tilde{\pi}_j \left(\sqrt{\zeta_h(\eta_j \rho_W \sigma_W + \kappa r)} - \sqrt{\zeta_h(\eta_{-j}\rho_W\sigma_W + \kappa r)}\right)}{\left(1 - e^{\theta_j}\right)^{-1} \left(\kappa\phi_h + \sqrt{2}\sqrt{\zeta_h(\eta_j \rho_W \sigma_W + \kappa r)}\right)^2} + \frac{\eta_j \sqrt{2\zeta_h(r\kappa + \eta_j \rho_W \sigma_W)}}{(\rho_X \sigma_X y_{Xj})^{-1} x_j^2 (1 - \delta\omega)\phi_h^2},$$

which is strictly positive for any parameter value, given $\eta_B > \eta_G$, $e^{\theta_G} - 1 > 0$ and $e^{\theta_B} - 1 < 0$. It follows that $\mathcal{R}_{X,j}$ is increasing in ϕ_h as stated in Proposition 7.

We next consider the derivative of $\mathcal{R}_{X,j}$ with respect to ω:

$$\frac{\partial \mathcal{R}_{U,j}}{\partial \omega} = \frac{\eta_j \rho \sigma_X y_{X,j} \left(\kappa\phi_h + \sqrt{2\zeta_h(\eta_j \rho_W \sigma_W + \kappa r)}\right)}{x_j^2 \omega (1 - \omega\delta)^2 \phi_h} + \frac{\eta_j \rho \sigma_X y_{X,j}}{x_j (1 - \omega\delta)},$$

which is strictly positive. Similarly, the derivative of $\mathcal{R}_{X,j}$ with respect to δ equals:

$$\frac{\partial \mathcal{R}_{U,j}}{\partial \delta} = \frac{\eta_j \rho \sigma_X \omega^3 y_{X,j} \phi_h}{(1 - \delta\omega)^2 \left(\kappa\phi_h + \sqrt{2\zeta_h(\eta_j \rho_W \sigma_W + \kappa r)}\right)},$$

which is strictly positive, proving that $\mathcal{R}_{X,j}$ is increasing in δ. Proposition 7 then follows.

[36] In $j = G$, $\eta_B > \eta_G$ and $e^{\theta_G} - 1 > 0$. In $j = B$, $\eta_B > \eta_G$ and $e^{\theta_B} - 1 < 0$.

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Table 1: Motivating Evidence on R&D, Number of Firms and Discount Rates. The top panel shows Compustat firm-level panel tests with R&D to assets or R&D to sales as dependent variables. The bottom panel uses data from the National Science Foundation (NSF) and Business Dynamics Statistics (BDS), covering both public and private firms. NSF surveys innovating firms only, whereas BDS covers all firms. The period is 1982 to 2017. All coefficients are standardized. Standard errors are reported in parentheses. Details on dataset construction and controls are provided in Appendix A.1. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Compustat Data

	R&D to Assets		R&D to Sales	
	No Controls	Controls	No Controls	Controls
Premium	0.015***	0.007***	0.008***	0.005*
	(0.00)	(0.00)	(0.00)	(0.00)
Adjusted R^2	0.586	0.641	0.468	0.528
N	151,933	110,032	145,229	106,955
Fixed Effects	Y	Y	Y	Y
Level	Firm	Firm	Firm	Firm

NSF and BDS Data

	R&D to Sales	No. of Firms		
		Innovating	Total	Total
Premium	0.290*	-0.352*	-0.555***	-0.033***
	(0.14)	(0.16)	(0.14)	(0.01)
Adjusted R^2	0.082	0.086	0.288	0.962
N	36	36	36	8,424
Fixed Effects	N	N	N	Y
Level	Aggregate	Aggregate	Aggregate	Industry
Source	NSF	NSF	BDS	BDS

Table 2: Baseline parameters.

Parameter	Description	Value
r	Risk-free rate	0.01
η	Market price of risk (one state)	0.30
ϕ	Poisson coefficient (initiator)	1.00
ϕ_v	Poisson coefficient (entrant, vertical)	1.00
ϕ_h	Poisson coefficient (entrant, horizontal)	1.00
ζ	R&D cost coefficient (initiator)	0.60
ζ_v	R&D cost coefficient (entrant, vertical)	6.00
ζ_h	R&D cost coefficient (entrant, horizontal)	0.08
λ	Vertical jump (initiator)	1.055
Λ	Vertical jump (entrants)	1.12
φ	Horizontal jump (initiator)	1.14
ω	Horizontal jump (entrants)	0.25
δ	Obsolescence due to horizontal innovations	0.20
β	Inverse of price elasticity of demand	0.13
Γ	Demand shift parameter	1.00
α	Recovery in liquidation (initiator)	0.60
α_X	Recovery in liquidation (exploiter)	0.85
σ	Coefficient of cash flow volatility (initiator)	0.80
σ_X	Coefficient of cash flow volatility (exploiter)	0.70
σ_W	Cash flow volatility (entrant)	0.20
ρ	Correlation with aggregate shocks (initiator and exploiter)	0.55
ρ_W	Correlation with aggregate shocks (entrant)	0.20
κ	Entry cost	0.015

Table 3: Innovation in the vertical and horizontal dimension. This table reports the model endogenous quantities for the corner case in which entrants invest in vertical innovation only, for the corner case in which entrants invest in horizontal innovation only, and in the full case featuring both vertical and horizontal innovation. The top panel illustrates the case in which $\omega = 0.25$ (as in the baseline parameterization), whereas the bottom panel illustrates the case in which ω is higher and equal to 0.45.

	Vertical only	Horizontal only	Full case (both)
$\omega = 0.25$ (Baseline)			
z	0.120	–	0.116
v	0.064	–	0.062
h	–	–	0.141
μ	2.559	–	2.112
Ψ_v	0.163	–	0.130
Ψ_h	–	–	0.297
$\omega = 0.45$			
z	0.120	0.248	0.071
v	0.064	–	0.037
h	–	0.551	0.451
μ	2.559	0.918	1.881
Ψ_v	0.163	–	0.069
Ψ_h	–	0.506	0.849

Table 4: EQUILIBRIUM QUANTITIES. This table reports the quantities of interest for the case in which the market price of risk varies over time (first to third column) as well as when assuming that there is just one state of the economy in which the market price of risk is fixed at its two-state average. In the top panel, we assume that only the market price of risk η_j varies across the different states; in the middle panel, we assume that η_j and the demand shift parameter Γ_j vary; in the bottom panel, we assume that η_j and the financing cost κ_{Fj} vary.

	State G	State B	Average (2 states)	One-state
Varying η_j only				
z	0.112	0.128	0.116	0.116
v	0.060	0.068	0.062	0.062
h	0.134	0.151	0.138	0.141
μ	2.873	0.170	2.260	2.107
Ψ_v	0.171	0.012	0.135	0.130
Ψ_h	0.385	0.026	0.304	0.296
$\mathcal{I}$	0.283	0.140	0.251	0.246
Varying η_j and Γ_j				
z	0.112	0.128	0.115	0.115
v	0.059	0.068	0.061	0.061
h	0.147	0.164	0.151	0.154
μ	3.224	0.160	2.529	2.341
Ψ_v	0.191	0.011	0.150	0.143
Ψ_h	0.475	0.026	0.373	0.360
$\mathcal{I}$	0.302	0.139	0.265	0.258
Varying η_j and κ_{Fj}				
z	0.112	0.130	0.116	0.116
v	0.059	0.069	0.062	0.062
h	0.133	0.145	0.136	0.139
μ	2.946	0.027	2.284	2.109
Ψ_v	0.175	0.002	0.136	0.130
Ψ_h	0.392	0.004	0.304	0.294
$\mathcal{I}$	0.287	0.132	0.252	0.246

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