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### Credit subsidies

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## **Abstract**

Credit subsidies are an alternative to interest rate and credit policies when dealing with high and volatile credit spreads. In a model where credit spreads move in response to shocks to the net worth of financial intermediaries, credit subsidies are able to stabilize those spreads avoiding the transmission to the real economy. Interest rate policy can be a substitute for credit subsidies but is limited by the zero bound constraint. Credit subsidies overcome this constraint. They are superior to a policy of credit easing as long as the government is less efficient than financial intermediaries in providing credit.

Keywords: Credit subsidies; monetary policy; zero lower bound on nominal interest rates; banks; costly enforcement. JEL Codes: E31, E40, E44, E52, E58, E62, E63.

# 1 Introduction

The 2008-2009 financial crisis and the Great Recession have exposed the limitations of standard monetary policy as a tool for macroeconomic stabilization. Even if policy rates were cut down to near-zero levels, the costs of financing for firms and households were kept high by unusually high credit spreads. Since further cuts in the policy rate were prevented by the zero bound constraint, alternative tools were considered by central banks, including various forms of credit policies.

In order to contribute to the design of policies that may respond to disturbances in financial markets, we consider a broader set of instruments, both monetary and fiscal, and study optimal policy in models with costly financial intermediation. The main message of the paper is that credit subsidies stand out as the natural policy tool to address the inefficiencies associated with high and volatile credit spreads.

In the model we use, firms borrow from banks in order to pay wages. The banks are subject to an enforcement problem similar to the one in Gertler and Karadi (2011) or Gertler and Kiyotaki (2011). This generates an inefficiency in that loan rates include a spread over the borrowing rate of banks. Credit subsidies can deal directly with the inefficiency associated with those credit spreads. The benchmark model is a monetary model where, in addition to the distortion from credit spreads, there is also a monetary distortion. The natural tool to deal with it is the policy interest rate which is the source of the distortion. We consider restrictions to fiscal policy such as debt being nominal and noncontingent. Monetary policy is effective, as price level policy, in affecting the real value of outstanding nominal debt. It also affects the real value of internal funds. We assume away direct lump-sum taxes, so there is a need to finance government spending and initial government liabilities with distortionary taxation.

The benchmark model is a monetary economy but in order to stress the role of credit

subsidies we also consider a version of the model without outside money or monetary policy. In that model the only stabilization policy tools are credit subsidies. We show that credit subsidies should be used to fully insulate the economy from the effects of financial shocks. In reaction to a financial shock that changes the internal funds of the banks, and therefore moves spreads and loan rates, credit subsidies stabilize borrowing rates net of taxes. The distortions are made invariant to financial shocks. There is no need to finance subsidies with debt, either nominal or real. The subsidies are fully financed by a tax on distributed profits.

We compare credit subsidies to a form of credit easing. As an alternative to private intermediation, we allow for direct lending by the government, provided a resource cost is paid. We obtain that credit easing should never be used. Credit subsidies can deal with the spreads and do not use resources, as credit easing does. This is in contrast with results in Gertler and Karadi (2011), where direct lending can be desirable in reaction to a large tightening of banks' balance-sheet constraints.

In the monetary model, in addition to credit subsidies, nominal interest rates and price level policy are also effective policy tools. In the monetary model, the inefficiency from high lending rates is due to high spreads but also to high borrowing costs for the banks. Low policy rates reduce those costs and can therefore be useful in reducing lending rates, and improving efficiency. Policy rates can also compensate for the variability in spreads. At times when spreads are high, and therefore so are lending rates, low policy rates can be used to induce lower lending rates. There is one limitation, however, associated with the zero bound constraint on interest rates. When spreads are particularly high, the drop in the policy rate that would stabilize lending rates could require the interest rate to become negative.

Price level policy can also be used in response to shocks to stabilize the real value of internal funds of banks, helping in stabilizing spreads. Furthermore, with noncon-

tingent nominal debt, price level policy can induce real state-contingent debt, reducing the financing costs for the government.

Even if policy could include a combination of fiscal and monetary policy instruments, there is no need for active monetary policy, once credit subsidies are used. The nominal rate could be set at its zero bound and the price level could be stabilized in response to shocks, without this restricting the set of possible allocations and therefore also the optimal allocation. This also means that the zero bound constraint on interest rates is irrelevant once credit subsidies are used. Credit subsidies can be used instead of negative rates, achieving the desired smoothing of lending rates.

The features of the allocation which can be achieved through credit subsidies naturally depend on the other financing instruments available to the government. For the results we have described so far, we consider a tax on distributed profits that is used fully. If this tax is not used, the optimal policy does not fully stabilize wedges even in response to financial shocks. And it matters whether debt is state-contingent. The case without the profit tax is solved numerically for the case in which debt is nominal and noncontingent. Even if nominal debt is noncontingent, the outstanding debt in real terms can still be state-contingent because of ex-post changes in inflation. To understand the implications of limits to this policy tool, we also solve for optimal policy without allowing for instantaneous price adjustments in reaction to shocks.

The fact that households keep profits should be reason enough for wedges not to be smoothed. Indeed at the basis of the optimal tax smoothing result of Diamond and Mirrlees (1971) is the assumption that profits are fully taxed. In addition, the non-contingency of debt also induces optimal volatility of wedges. The reason is that a volatile price level can make debt state contingent but also affects profits of banks by changing the real value of internal funds. In our numerical simulations, in response to a negative temporary financial shock, the subsidy more than compensates for the high

spread, and there are permanent effects of the shock.

We also compute numerical solutions restricting policy so that the price level does not move on impact. In this case, it is not possible to affect real internal funds on impact, and debt cannot be made state-contingent. Again, lending rates are not fully stabilized, and there are permanent effects of the shock, but the long run effects are different. Now, in response to a negative financial shock, the spreads net of taxes increase in the long run.

Our case for credit subsidies is robust to, and actually strengthened by, changes in the source of monetary non-neutrality. The very simple monetary friction that we assume, implies that interest rate policy affects the same margin as the credit subsidy. This implies that the two policy instruments are close substitutes. In particular, interest rate policy could, in normal times away from the zero bound, dispense with the subsidies. We could have considered alternative models where the monetary non-neutrality would be due to sticky prices as in Woodford (2003) or sticky information as in Mankiw and Reis (2002). In those models, interest rate policy would be a poor substitute for credit subsidies, so that the relevance of credit subsidies would be stronger. Both credit subsidies and monetary policy should be used, aimed at different goals. Credit subsidies would be correcting the distortions due to the high spreads, and interest rate policy would be correcting the distortions associated with sticky prices or information, ensuring price or inflation stability.

The paper is related to a literature that analyses the effects of financial market shocks and the desirability of non-standard monetary policy responses, as in Curdia and Woodford, 2011, De Fiore and Tristani, 2018, Eggertsson and Krugman, 2012.<sup>1</sup> This literature explores various forms of direct lending by the central bank, but does not explicitly allow for tax instruments. Optimal tax policy when interest rates are

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<sup>1</sup>Another related literature studies the optimal combination of monetary and fiscal policy in reaction to financial, or other, shocks. See Prestipino (2014) and Bianchi and Mendoza (2013).

at the zero bound has been studied by Eggertsson and Woodford (2006), Correia, Farhi, Nicolini and Teles (2013) among others. These papers abstract from financial market frictions. The friction is sticky prices. Fiscal policy is necessary to overcome the distortions imposed by the interaction of sticky prices with the zero lower bound. Relative to Correia et al. (2013), this paper confirms the result that standard tax instruments can overcome the zero bound constraint on interest rates.

The paper is organized as follows: In section 2, we describe the benchmark monetary model. In section 3, we compute optimal fiscal and monetary policy with taxes on distributed profits, credit subsidies, and state-contingent debt. We obtain general results on the use of credit subsidies that fully stabilize credit spreads in response to financial shocks. Credit subsidies are necessary to deal with the zero bound on interest rates, but there are multiple implementations of policy. In particular, in response to relatively small shocks, interest rate policy may be sufficient. A non-monetary version of the model is analyzed to emphasize the role of credit subsidies relative to monetary policy (section 3.2). We also show that credit easing should not be used, not in the steady state, nor in response to shocks (section 3.3). In section 4, the optimal response to shocks with and without credit subsidies is computed numerically in environments with further restrictions on policy. Section 5 discusses alternative sources of monetary non-neutrality. Section 6 contains concluding remarks.<sup>2</sup>

## 2 The model

The main feature of the model is that financial intermediation must be performed by banks that face an enforcement problem. A representative firm needs to borrow to pay for wages. A continuum of banks make those loans and borrow from the household.

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<sup>2</sup>An online appendix provides analytical expressions for the coefficients of the leverage function, a characterization of the equilibrium, and proofs for Propositions 1 and 3.

There is a large household that includes workers and bankers that share consumption. The preferences of the household are over consumption and labor. The production technology uses labor only and is linear. The household must pay for consumption with money. Bankers can appropriate a fraction of the assets of the bank, so they must be induced not to do it. In equilibrium there are going to be bank profits that are accumulated as internal funds. The government consumes, raises taxes and pays for subsidies on credit, issues money and debt.

**The household** The household is composed of workers and bankers. With probability  $1 - \theta$ , bankers exit and become workers. They are replaced by workers that become new bankers, keeping the fractions of bankers and workers constant, respectively  $f$  and  $1 - f$ . Bankers and workers share consumption.

The uncertainty in period  $t \geq 0$  is described the history of the realizations of a random variable up to period  $t$ .  $s^t \in \Gamma^t$ . For simplicity, we index by  $t$  the variables that are functions of  $s^t$ .

The household has preferences over consumption and labor,  $E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, N_t)$ , with the usual properties. The household starts period  $t$  with nominal wealth  $\mathbb{W}_t$ . At the beginning of period  $t$ , in an assets market, the household purchases  $E_t Q_{t,t+1} B_{t,t+1}$  in one-period state-contingent nominal claims.  $Q_{t,t+1}$  is the price in period  $t$  of a unit of money in period  $t+1$ , in some state, normalized by the probability of occurrence of the state. The household also purchases noncontingent public debt  $B_t^h$ , and deposits  $D_t$ , as well as money  $M_t$ . In the beginning of the following period the nominal wealth  $\mathbb{W}_{t+1}$  includes the state-contingent bonds  $B_{t,t+1}$ , the gross return on noncontingent public debt  $R_t B_t^h$  and on deposits  $R_t D_t$ , money  $M_t$ , the wage income  $W_t N_t$ , the dividends received from the banks  $(1 - \tau^\pi) \Pi_t^b$  net of a constant tax rate  $\tau^\pi$ .

The household pays for consumption expenditures,  $P_t C_t$ , in the goods market at



the end of the period with money  $M_t$ , satisfying the cash-in-advance constraint

$$P_t C_t \leq M_t. \quad (1)$$

The flow of funds constraints are therefore  $E_t Q_{t,t+1} B_{t,t+1} + B_t^h + D_t + M_t \leq \mathbb{W}_t$ , and  $\mathbb{W}_{t+1} = B_{t,t+1} + R_t B_t^h + R_t D_t + M_t - P_t C_t + W_t N_t + (1 - \tau^\pi) \Pi_t^b$ .

The single budget constraint of the households can be written as

$$E_0 \sum_{t=0}^{\infty} Q_t P_t C_t \leq E_0 \sum_{t=0}^{\infty} \frac{Q_t}{R_t} [W_t N_t + (1 - \tau^\pi) \Pi_t^b] + (1 - l_0) \mathbb{W}_0. \quad (2)$$

This is derived imposing a no-Ponzi games condition, the cash-in-advance constraint, (1), the arbitrage condition between contingent and noncontingent bonds,  $1 = R_t E_t Q_{t,t+1}$ , and  $Q_{t+1} = Q_t Q_{t,t+1}$ , with  $Q_0 = 1$ , that defines the price  $Q_{t+1}$  of one unit of money at the assets market at  $t + 1$ , in units of money at  $t = 0$ .  $l_0$  is a tax on initial wealth. The budget constraint is written under the assumption that  $R_t \geq 1$ . This is the zero bound on interest rates which is an equilibrium restriction.<sup>3</sup>

The first order conditions of the households problem include

$$-\frac{u_C(t)}{u_N(t)} = \frac{R_t P_t}{W_t}, \quad (3)$$

so that the nominal interest rate  $R_t - 1$  raises the cost of consumption for the household.

**Firms** A representative firm is endowed with a technology that transforms  $N_t$  units of labor into  $Y_t = A_t N_t$  units of output. The firm is required to hold enough funds in advance to pay the wage bill. More precisely, the firm borrows in the beginning of period  $t$  funds  $S_t$ , at gross interest rate  $R_t^l$ , receiving a credit subsidy  $\tau_t^l$ , on the gross

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<sup>3</sup>If it were not satisfied, the households would borrow an arbitrarily large amount and hold cash, making arbitrarily large profits.

interest. The funds are held as interest bearing assets to pay for the wage bill,  $B_t^f = S_t$ . Because the firm can hold the borrowed funds as remunerated assets, at gross interest rate  $R_t$ , the borrowing constraint is

$$\frac{W_t N_t}{R_t} \leq S_t. \quad (4)$$

The profits of the firm in each period  $t$  are  $\Pi_t^f = P_t Y_t - W_t N_t - [R_t^l (1 - \tau_t^l) - R_t] S_t$ . Using the borrowing constraint (4), profit maximization implies

$$P_t A_t = \frac{R_t^l (1 - \tau_t^l)}{R_t} W_t \text{ and } A_t N_t = R_t^l (1 - \tau_t^l) \frac{S_t}{P_t}. \quad (5)$$

It is also an equilibrium restriction on the subsidy that  $R_t^l (1 - \tau_t^l) \geq R_t$ . Otherwise firms could make arbitrarily large profits borrowing at  $R_t^l (1 - \tau_t^l)$  and holding government debt that pays  $R_t$ . This is an upper bound constraint on the credit subsidy, similar in substance to the zero bound constraint on interest rates.

**Banks** Each bank  $j$  channels funds from depositors to the firm. Because of costly enforcement, banks must have rents that are accumulated as internal funds,  $Z_{j,t}$ . This implies that there are going to be positive spreads and that internal funds will have high rates of return. There must be exit of bankers, so that internal funds can remain scarce.

The bank borrows  $D_{j,t}$  from the households and lends  $S_{j,t}^b$ . The balance sheet of a bank is such that  $S_{j,t}^b = D_{j,t} + Z_{j,t}$ . The equilibrium return on the internal funds is higher than the alternative return  $R_t$ , so profits are kept in the bank as internal funds until exit. The net worth of the bank evolves according to  $Z_{j,t} = R_{t-1}^l S_{j,t-1}^b - R_{t-1} D_{j,t-1}$ . Combining the two conditions, the balance sheet and the evolution of internal funds, it follows that  $Z_{j,t} = (R_{t-1}^l - R_{t-1}) S_{j,t-1}^b + R_{t-1} Z_{j,t-1}$ .

Bankers exit in the assets market at  $t$  with the accumulated funds  $Z_{j,t}$ . The value of a surviving bank at the assets market, before taxes, in period  $t$ , is  $V_{j,t} = E_t \sum_{s=0}^{\infty} (1 - \theta) \theta^s Q_{t,t+1+s} Z_{j,t+1+s}$

Bankers can appropriate a fraction  $\lambda$  of assets  $S_{j,t}^b$ , in the assets market at time  $t$ . The incentive constraint is thus

$$V_{j,t} \geq \lambda S_{j,t}^b. \quad (6)$$

Unless this condition is verified, banks won't be able to attract deposits. We assume that the same tax on distributed profits  $\tau^\pi$  is applied to the assets that the bankers may appropriate,  $\lambda S_{j,t}^b$ . This means that bankers can run away with part of their debt, but they cannot avoid paying taxes.<sup>4</sup>

As shown in the Appendix, the solution of this problem is such that loans are  $S_{j,t}^b = \phi_t Z_{j,t}$ , where  $\phi_t$  is defined as the ratio of assets to internal funds, also referred to as leverage ratio, and given by

$$\phi_t = \frac{\eta_t}{\lambda - v_t}, \quad (7)$$

for  $v_t = (1 - \theta) \left( \frac{R_t^l}{R_t} - 1 \right) + E_t Q_{t,t+1} \theta \frac{\phi_{t+1}}{\phi_t} [(R_t^l - R_t) \phi_t + R_t] v_{t+1}$  and  $\eta_t = (1 - \theta) + \theta E_t Q_{t,t+1} [(R_t^l - R_t) \phi_t + R_t] \eta_{t+1}$ . Notice that the growth rates of internal funds and loans and the leverage ratio are the same across banks. This makes it straightforward to aggregate across banks.

The total internal funds of bankers  $Z_t$  are the sum of the funds of surviving bankers  $Z_{et}$  and entering bankers  $Z_{nt}$ . Since a fraction  $\theta$  of bankers survive,  $Z_{et} = \theta [(R_{t-1}^l - R_{t-1}) \phi_{t-1} + R_{t-1}] Z_{t-1}$ . The remaining fraction,  $1 - \theta$ , exit and transfer back the internal funds to the households, net of  $\frac{\omega_t}{1-\theta}$  of those funds that are trans-

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<sup>4</sup>Otherwise, the profit tax would require a spread because of tax evasion, that we want to abstract from.

ferred to the entering bankers, so that  $Z_{nt} = \omega_t [(R_{t-1}^l - R_{t-1}) \phi_{t-1} + R_{t-1}] Z_{t-1}$ .

We can then write  $Z_t = Z_{et} + Z_{nt}$  as

$$Z_t = (\theta + \omega_t) [(R_{t-1}^l - R_{t-1}) \phi_{t-1} + R_{t-1}] Z_{t-1}. \quad (8)$$

Aggregate dividends transferred by exiting banks to the household in the assets market at  $t \geq 1$ , net of the transfers to entering banks, are

$$\Pi_{t-1}^b = (1 - \theta - \omega_t) [(R_{t-1}^l - R_{t-1}) \phi_{t-1} + R_{t-1}] Z_{t-1}. \quad (9)$$

These profits are indexed by  $t - 1$ , for  $t \geq 1$ , because they correspond to the borrowing and lending of banks between periods  $t - 1$  and  $t$ .

We consider a shock  $\omega_t$  to internal funds. This is a shock to the distribution of funds between households and banks. It affects the severity of the financial friction, by changing the availability of funds to bankers.

**The government** The government spends  $G_t$ , gives credit subsidies  $\tau_t^l$ , taxes distributed profits,  $\tau^\pi \Pi_t^b$ . The policy rate is  $R_t$ . Given nominal liabilities  $-\mathbb{W}_t^g$ , the government issues money  $M_t$ , issues noncontingent debt  $B_t$ , may also be able to issue contingent debt  $B_{t,t+1}$ , according to  $B_t + E_t Q_{t,t+1} B_{t,t+1} + M_t \geq -\mathbb{W}_t^g$ . Liabilities at the beginning of period  $t+1$ , for  $t \geq 1$  are  $-\mathbb{W}_{t+1}^g = R_t B_t + B_{t,t+1} + M_t + \tau_t^l R_t^l S_t + P_t G_t - \tau^\pi \Pi_t^b$ . At the beginning of period 1, the liabilities are  $-\mathbb{W}_1^g = R_0 B_0 + B_{0,1} + M_0 + \tau_0^l R_0^l S_0 + P_0 G_0 - \tau^\pi \Pi_0^b - l_0 \mathbb{W}_0$ .

The initial wealth of the government satisfies  $\mathbb{W}_0^g + \mathbb{W}_0 + R_{-1} Z_{-1} = 0$ .

**Market clearing** The market clearing condition in the goods market is

$$C_t + G_t = A_t N_t. \quad (10)$$

The market clearing condition for loans is  $S_t = S_t^b$ , and for noncontingent bonds is  $B_t^f + B_t^h = B_t$ .

**Equilibrium** An equilibrium in this economy is a sequence of allocations, prices and policies, that solves the problem of the household, the problem of banks with the incentive constraint holding with equality, the problem of the firms, and that satisfies the budget constraint of the government.

### 3 Policy with interest rates and credit subsidies

In order to understand the effect of financial shocks and policy in this economy, it is useful to use the marginal conditions of household and firm, (3) and (5), that imply

$$-\frac{u_C(t)}{u_N(t)} = \frac{R_t^l (1 - \tau_t^l)}{A_t}, \quad (11)$$

together with the resource constraints (10). The marginal condition (11) together with the resource constraint, (10), determine the allocation of consumption,  $C_t$ , and labor,  $N_t$ , for each state, as a function of the technology shock and of the wedge caused by the nominal lending rate,  $R_t^l$ , and the subsidy  $\tau_t^l$ . If different from one,  $R_t^l (1 - \tau_t^l)$  is the wedge relative to the first-best.<sup>5</sup>

The transmission of financial shocks to the allocation is through the effects on the lending rate,  $R_t^l$ , net of the subsidy. The lending rate can be decomposed into the policy rate and the interest rate spread. A negative financial shock that reduces the internal funds in the banks, increases the spread, and therefore the lending rate. Policy can respond by lowering the interest rate, which is passed through to the lending rate, or by increasing the subsidy. Because the assets are nominal, price level policy can

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<sup>5</sup>The first best allocation is the one that maximizes utility subject to the resource constraints alone.

also change the real value of internal funds. Fiscal and monetary policy is restricted in that there are no direct lump-sum taxes. Interest rate policy and credit subsidies are restricted by lower and upper bound constraints on the policy rate and the subsidies, respectively.

We first assume that distributed profits to households are fully taxed, but also consider the case where they are not taxed at all. In that case, that we solve numerically, we also impose restrictions on the state-contingency of debt.

### 3.1 Second best policy

We consider the limiting case where the tax on distributed profits approaches one,  $\tau^\pi = 1$ .<sup>6</sup> By substituting the prices and taxes from the marginal conditions of the household, the resulting budget constraint of the household can be written, with equality, as

$$E_0 \sum_{t=0}^{\infty} \beta^t [u_C(t) C_t + u_N(t) N_t] = u_C(0) (1 - l_0) \frac{\mathbb{W}_0}{P_0} = \overline{\mathbb{W}}_0, \quad (12)$$

When the government can issue state-contingent debt, this budget constraint is also the single intertemporal budget constraint of the government.

We impose the restriction on the initial confiscation that  $u_C(0) (1 - l_0) \frac{\mathbb{W}_0}{P_0} = \overline{\mathbb{W}}_0$  for a given exogenous  $\overline{\mathbb{W}}_0$ . This is the restriction on the initial confiscation in Armenter (2008) and Chari, Nicolini and Teles (2016). This assumption allows us to study the optimal policy problem abstracting from indirect ways of confiscating the initial wealth, through valuation effects.

The following proposition characterizes the implementable set.

**Proposition 1** (Implementable set) The set of implementable allocations  $\{C_t, N_t\}$  is characterized by the implementability condition (12), the resource constraints (10)

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<sup>6</sup>The tax on distributed profits is lump-sum since, by assumption, it does not affect the enforcement constraint. It follows that, if necessary, they should be fully taxed.

and the restriction that wedges must be nonnegative,  $-\frac{u_C(t)}{u_N(t)} \geq \frac{1}{A_t}$ .

The formal proof is in an online appendix. The implementability conditions are built from the equilibrium conditions so they are necessary conditions. But they are also sufficient, since all other conditions can be satisfied by other equilibrium variables. In order to show this, we take a generic, feasible allocation for consumption and labor and show that, together with the other variables, it satisfies all the other equilibrium conditions. There are multiple implementations of each allocation in the implementable set, so it is sufficient to do the demonstration for a particular one.<sup>7</sup> The particular implementation is the one in which the price level does not change contemporaneously in response to shocks. There are also degrees of freedom in the setting of the interest rate. We set it at the zero bound.

Let  $\varphi$  be the multiplier of the implementability condition. Then the optimal wedges  $-\frac{u_C(t)A_t}{u_N(t)}$  must satisfy

$$-\frac{u_C(t)A_t}{u_N(t)} = \frac{1 + \varphi [1 + \sigma_t^n - \sigma_t^{nc}]}{1 + \varphi [1 - \sigma_t - \sigma_t^{cn}]}, \quad t \geq 0, \quad (13)$$

where

$$\sigma_t = -\frac{u_{cc,t}c_t}{u_{c,t}}, \quad \sigma_t^n = \frac{u_{nn,t}n_t}{u_{n,t}}, \quad \sigma_t^{nc} = -\frac{u_{nc,t}c_t}{u_{n,t}}, \quad \sigma_t^{cn} = -\frac{u_{cn,t}n_t}{u_{c,t}}.$$

It follows that the optimal wedges in response to financial shocks are constant.

**Proposition 2** (Optimal wedges with financial shocks) The optimal wedges  $-\frac{u_C(t)A_t}{u_N(t)}$  are invariant to financial shocks.

The proof is straightforward. Consider only shocks to  $\omega_t$ . Let the optimal wedges be constant. Then the optimal allocation for consumption and labor must satisfy (13) with a constant wedge and the resource constraints (10). It follows that the allocation is constant and therefore the elasticities  $\sigma_t$ ,  $\sigma_t^n$ ,  $\sigma_t^{nc}$ ,  $\sigma_t^{cn}$  are all constant, which confirms

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<sup>7</sup>We thank Joao Sousa, that first suggested the possibility of multiple implementations in the price level.

the guess of a constant optimal wedge.

The optimal wedges would be constant also in response to other shocks if preferences were separable and constant elasticity  $\sigma_t = \sigma$  and  $\sigma_t^n = \sigma^n$ .

Condition (13) also characterizes the first best. The first best is the solution if the multiplier of the implementability condition is zero,  $\varphi = 0$ . This would be the case, if government assets and revenues from the profit tax were enough to pay for a full correction of the credit distortion together with government consumption.

We have assumed that the government can issue state-contingent debt, but state-contingent debt may not be necessary to implement the optimal solution. As we will see in the numerical solutions, the profit tax is all that is needed to finance the subsidies in response to shocks. But in any case, with noncontingent nominal debt, volatility of the price level may ensure the state-contingency of real debt.

In the particular implementation that we look at, policy affects allocations through credit subsidies but not through the policy rate that is set at the zero bound. We could alternatively have considered an implementation that also used the policy rate in response to shocks.

Credit subsidies and interest rate policy are both restricted, by the zero bound restriction on the policy rate and by an upper-bound restriction on the credit subsidy. In isolation each of the restrictions would limit the use of policy, unless the other policy was at the bound. If the wedge was to be reduced using only interest rate policy, the zero bound constraint could be binding, and similarly if only subsidies were used to smooth wedges, the upper bound on the subsidy could also be binding. Used jointly, neither interest rate policy nor credit subsidies are restricted by their lower and upper bounds. A proposition of irrelevance of the zero bound follows.

**Proposition 3** (Irrelevance of the zero bound) When credit subsidies are used, the zero bound on the nominal interest rate is irrelevant for the implementation of



allocations.

A formal proof is in an online appendix.

Fiscal policies can therefore overcome the nonnegativity constraint on the nominal interest rate. Allocations can be achieved which, without time varying credit subsidies, would only be feasible if interest rates could be negative. By setting the policy rate to zero, we also guarantee that the upper bound constraint on the credit subsidy is never binding.<sup>8</sup>

That both credit subsidies and policy rates are complementary policy tools is specially true in the implementation of the first best. The only way to implement the first best is to set both the credit subsidy and the policy rate at the, respective, upper and lower bound.

Policy also affects allocations through price level policy in response to shocks. Except for the shock  $\omega_t$ , internal funds of bankers are predetermined. This is so, even if the timing of transactions is such that financial assets can be adjusted contemporaneously in response to shocks. The reason is that it is optimal for the banks to accumulate all profits as internal funds. Movements in the price level affect the real value of internal funds. They also affect the real value of government liabilities which may be relevant in the absence of state-contingent debt.

As an illustration, it is useful to think of the consequences of a negative financial shock  $\omega_t$ , under the implementation of optimal policy with state-contingent debt and a predetermined price level. Because the price level does not move on impact, the real value of internal funds falls by the full amount of the shock. As a result, leverage and the spread have to go up. Once at the zero bound, it is not possible to further cut interest rates to counteract the effect of the spread on allocations. The subsidy, instead, can be adjusted for that purpose. State-contingent debt is used to finance the

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<sup>8</sup>This is ensured by the nonnegativity of the wedges.

subsidy.<sup>9</sup>

One way to interpret the monetary model in this paper is that there is a cash in advance constraint on households, that must hold outside money in order to make consumption purchases, and there is an inside money constraint on firms, that must hold funds in advance in order to make payments to workers. The cost of the cash in advance constraint on households is the rate of return on deposits that they forego. The cost of the funds that the firms must hold is the spread between the lending rate by banks (net of the credit subsidy) and the policy rate. The joint cost is the lending rate net of the subsidy. Setting that cost to zero would allow to achieve the first best and would amount to setting the price of outside money for households and inside money for firms equal to zero, which is an application of the Friedman rule. Notice that this result hinges on the assumption that financial intermediation is costless in terms of resources. With a positive intermediation cost, the optimal lending rate would have to include that cost.

### **3.2 Credit subsidies in a real economy**

In order to further understand the role of credit subsidies, and the relation to monetary policy, it is useful to consider the cashless limit of the economy with financial frictions but without outside money. In that economy, there is still a potential distortion due to the credit friction, that has to be dealt with using credit subsidies alone.

The economy in this section has the same features as the economy above, except that there is no outside money, not even as unit of account. The cash-in-advance constraint on the households, (1), is not imposed. The role of money as unit of account is also

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<sup>9</sup>Another implementation will have the price level adjust on impact in response to shocks. As a result, the dynamics of the financial variables and the credit subsidies would be different. In particular, in response to an i.i.d. shock to the value of internal funds, an adjustment in the price level on impact would be sufficient to completely neutralize all other effects of the shock on the equilibrium.

eliminated, by imposing that the price level is always equal to one,  $P_t = 1$ . In the resulting real economy, firms must still hold financial assets in advance of production. They borrow from banks, so that the cost of holding those assets is a real credit spread.

Since the price level is set equal to one at all times, the wage,  $W_t$ , is now a real wage, in units of goods, and the prices of state-contingent assets,  $Q_{t,t+1}$ , and interest rates,  $R_t$  and  $R_t^l$ , and asset levels,  $S_t$  and  $Z_t$ , are now also in units of the good. Similarly bank profits,  $\Pi_t^b$ , are also in real units.

The flow of funds constraints of the household are as in the monetary economy with  $M_t = 0$  and  $P_t = 1$ . The problems of the firms and the banks are unchanged. The constraints of the government are also the same except for the issuance of money.

In this cashless economy, the intratemporal marginal choices for the household are not distorted by the nominal interest rate, so that instead of (3), the marginal condition is now  $-\frac{u_C(t)}{u_N(t)} = \frac{1}{W_t}$ . The wedge in the consumption/leisure margin is the credit spread, net of the subsidy

$$-\frac{u_C(t)}{u_N(t)} = \frac{R_t^l (1 - \tau_t^l)}{A_t}, \quad (14)$$

with  $R_t^l (1 - \tau_t^l) \geq R_t$ . The credit subsidy is the effective way of dealing with the wedge. There is no role for a policy interest rate in directly affecting the wedge.

The set of allocations that can be implemented in this real economy is characterized by the same implementability condition, (12) and the resource constraints (10). In this sense, the two economies are equivalent.

**Proposition 4** (Credit subsidies in a real economy) In the real economy with full taxation of profits and state-contingent debt, the set of implementable allocations using credit subsidies is the same as the one in the monetary economy of proposition 1 that uses both credit subsidies and monetary policy.

Without lump-sum taxes, credit subsidies can be adjusted in response to shocks,

smoothing wedges across states, according to the same second best principles of taxation as in the monetary economy. The only difference is that the policy tools here are credit subsidies alone.

### 3.3 Credit easing

We now consider the possibility of the government lending directly to the firms. We introduce credit easing exactly as in Gertler and Karadi (2011). The government can directly provide intermediation  $S_t^g$  to non-financial firms at the lending rate  $R_t^l$ . In its intermediation activity the government is not subject to the incentive constraint, but has to pay an intermediation cost  $c$  per unit of real lending. The real resource cost is  $c \frac{S_t^g}{P_t}$ .

Government intermediation can be written as a fraction of total intermediation  $S_t^g = \psi_t S_t$ .<sup>10</sup> The government flow of funds constraints have to be modified to include credit easing as  $B_t + E_t Q_{t,t+1} B_{t,t+1} + M_t - \psi_t S_t \geq -\mathbb{W}_t^g$  and  $-\mathbb{W}_{t+1}^g = R_t B_t + B_{t,t+1} + M_t + \tau_t^l R_t^l S_t + (c - R_t^l) \psi_t S_t + P_t G_t - \tau^\pi \Pi_t^b$ . The resource constraint becomes

$$C_t + G_t + c \psi_t \frac{S_t}{P_t} = A_t N_t \quad (15)$$

and the market clearing condition for loans is now  $S_t = S_t^b + \psi_t S_t$ .

One could think that the desirability of the two unconventional measures, credit subsidies and credit easing, would depend on the magnitude of the resource cost  $c$ , in the case of credit easing, relative to the deadweight cost of the financing of the credit subsidies. That is not the case in the economy with a tax on profits and with state-contingent debt. With state-contingent debt there are no financing costs of credit subsidies in response to shocks. The subsidy in one state is financed by the tax in

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<sup>10</sup>Gertler and Karadi (2011) assume that policy is a simple rule for  $\psi_t$  as a function of credit spreads.

another state. Only the resource cost matters and credit easing should never be used in response to shocks.

But credit easing should also not be used in the steady state. The reason is that the distortion created by the enforcement problem generates its own lump-sum tax revenues that can be used to subsidize spreads eliminating the distortion. Again, there are no financing costs of credit subsidies, while credit easing always has a resource cost. Unless the cost is zero, there is no role for credit easing. Formally this is stated in the following proposition.

**Proposition 5** (Credit easing) In the economy with full taxation of profits and state-contingent debt, credit easing will never be used.

The proof is straightforward. The implementability conditions in the case with credit easing includes (12) and  $-\frac{u_C(t)}{u_N(t)} \geq \frac{1}{A_t}$  which are common to the case without credit easing. The resource constraint, instead, has an additional term  $c\frac{S_t^g}{P_t}$ , and there are other restrictions to the implementable set. If  $S_t^g$  was not zero, the optimal solution would achieve lower welfare than the one obtained with  $S_t^g = 0$ , which is the second best of the economy with credit subsidies only.

If profits cannot be taxed and if there is no state-contingent debt there is again a role for credit easing that we will analyze numerically.

## 4 The role of credit subsidies with further restrictions on policy: a numerical illustration

In this section we consider further restrictions on policy. In particular, we assume that distributed profits are not taxed, and government debt is nominal and noncontingent. We also consider the policy restriction that the price level may not move on impact. We provide a numerical illustration of the properties of credit subsidies in reaction to

adverse financial shocks. Throughout the section we focus on the scenario in which the interest rate is kept constant at the zero bound. This can only strengthen the case for credit subsidies.

Before we restrict policy, we compute the optimal response to a financial shock with the full set of instruments as in the previous section and with no instruments at all.

All results in this section are numerical. We assume as functional form for utility  $u(C_t, N_t) = \log C_t - \frac{\chi}{1+\varphi} N_t^{1+\varphi}$  and use standard values for utility parameters,  $\beta = 0.99$  and  $\varphi = 0$ . Concerning the financial sector parameters, we set  $\lambda = 0.35$  for the fraction of funds that can be diverted from the bank and then set the bankers survival probability,  $\theta$ , and the proportional transfer to entering bankers,  $\omega$ , so as to obtain in steady state an annualised lending spread equal to 1% and a leverage of 4. These targets require  $\theta = 0.9653$  and  $\omega = 0.0149$ , which are close to the parameter assumptions in Gertler and Karadi (2011). Government consumption is set to zero in the numerical analysis.

We study optimal policy under commitment, assuming that the economy starts from an arbitrary level of government liabilities including money and government bonds..

## 4.1 The benefits of credit subsidies

Figures 1 and 2 illustrate the benefits of credit subsidies with full taxation of distributed profits, as in section 3. Figure 1 shows the optimal policy responses to an exogenous fall in banks' internal funds for two different levels of initial liabilities of the government.<sup>11</sup> The policy instruments include in addition to the full tax on distributed profits, credit subsidies and price level policy. We set the nominal rate to zero but that is not restrictive. It simply selects a particular implementation. The only difference from the

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<sup>11</sup>More specifically, the figure shows impulse responses based on a log-linearization of the joint system of constraints and first order conditions of the Ramsey planner.

theoretical exercise in section 3 is that debt is nominal and noncontingent.

Credit subsidies are used to fully stabilize spreads. In response to the negative shock to internal funds, leverage goes up and so do lending rates. The subsidy fully compensates for the increase in the spread. Consumption does not move. There is no need for price level movements because there are no financing needs. The financial shock is such that fewer funds are transferred from the household to the bank. With the profit tax, the government can tax the extra funds, and use them to finance the subsidy. This provides extra profits to the banks that are able to recover the internal funds in one period. There is no role for interest rate policy that can be set at the zero bound, leaving full room for credit subsidy policy. The initial level of liabilities of the government affects the average steady state distortion. The larger the liabilities, the lower the steady state subsidy. But the optimal response of the subsidy to shocks is independent of the initial liabilities. Any increase in lending spreads is met by a one-to-one increase in the subsidy, so as to keep the distortion unchanged at its steady state level. Because there are no financing needs, the policy response with noncontingent debt coincides with the one in the theoretical exercises in section 3 with state-contingent debt.

Figure 2 compares optimal credit subsidy policy with the case with no policy reaction. The credit subsidy  $\tau^l$  is kept constant at its optimal steady state level, the nominal rate is set at zero and the price level cannot move on impact.<sup>12</sup> The response of the economy is to raise spreads persistently allowing for a slow build up of internal funds. There is no policy to deal with the distortionary effects of high and persistent spreads, and the economy experiences a prolonged downturn.

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<sup>12</sup>Since the price level cannot move on impact, the system of equations is non-recursive, so we solve it using a deterministic nonlinear method. More specifically, we solve all equations for all variables at all points in time between  $t = 1$  and  $t = T$ , for given state variables at  $t = 0$  and jump variables at  $t = T + 1$ . The horizon  $T$  is sufficiently long to ensure that at  $t = T$  the system settles back to the steady state.

## 4.2 Further restrictions on policy

Figure 3 compares policy with and without the profit tax. The other instruments are the ones considered before, credit subsidies, noncontingent nominal debt and price level policy. If distributed profits are not taxed, it is no longer optimal to fully stabilize spreads in response to financial shocks. The financing of the subsidies is with noncontingent nominal debt that is made contingent in real terms through volatility of the price level. But the price level also changes the real value of internal funds which has an effect on profits. Profits retained by the household are the reason for the deviation from full smoothing of wedges. The financing of the subsidies matters and it interacts with the motive to deviate from full stabilization of spreads. Optimal policy is such that the credit subsidy more than compensates for the higher spread resulting from a negative financial shock. The effects of the shock are permanent because of restrictions on the state-contingency of debt.<sup>13</sup>

The idea that the real value of the whole stock of government liabilities may be adjusted through instantaneous changes in the price level is not very plausible. In practice, the price level may change more slowly. For this reason, the results in Figure 3 should be interpreted as an illustration for an admittedly polar case.

Figure 4 shows the opposite polar case in which we restrict policy not to induce surprise movements in the price level (the dashed lines in the figure), so that real debt must be noncontingent.<sup>14</sup> As in figure 3, the economy does not return to the original steady state. The economy settles on a new steady state, where the higher debt is financed through a slightly lower level of the subsidy. Output falls permanently.

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<sup>13</sup>These results are consistent with those in Barro (1979) and Aiyagari et al (2002) where, in the absence of state-contingent debt, innovations in fiscal conditions are spread out over time and the optimal tax rate follows essentially a random walk.

<sup>14</sup>As in the case of Figure 2, this implies that the system of equations becomes non-recursive, so we solve it using a deterministic nonlinear method. For government debt, we also need to ensure a terminal condition. We do so by requiring that the evolution of government debt between  $t = T$  and  $t = T + 1$  must also satisfy the government budget constraint.



To summarize, we have shown that credit subsidies improve allocations substantially in reaction to adverse financial shocks. More specifically, they avoid a prolonged adjustment process in lending rates, banks' leverage and credit creation. Even if they can generate permanent implications for government debt, they significantly reduce the amplitude of the inefficient downturn after the financial shock.

## 5 Monetary non-neutralities

The model without outside money or monetary policy studied above, in Section 3, makes apparent the usefulness of credit subsidies as a policy tool. In a monetary model, the benefits of credit subsidies will depend on the precise source of monetary non-neutrality.

The monetary friction we assume in this paper is the most unfavorable to credit subsidies. If it was not for the upper and lower bounds on credit subsidies and interest rates, respectively, the two instruments would be fully equivalent. In models with other forms of monetary non-neutrality, such as sticky prices or sticky information, the two policy instruments would be complementary, because they would address different distortions.

The cashless model of Section 3, where the price level is constant at all times, provides the intuition for the results which would arise in a version of the model with sticky prices. To see this, take the cashless model and add sticky prices. The particular form of sticky prices is not very important, but Calvo (1983) is a good benchmark. In that economy, provided there are no other conflicting distortions, it is always optimal to eliminate the distortion from sticky prices by ensuring price stability. To ensure price stability the nominal rate would have to be set equal to the real. Being used fully for this purpose, the policy rate could not be used as a policy tool for any other

purpose. Credit subsidies would then be the remaining policy tool to deal with the inefficiency from the financial friction associated with high and volatile spreads, just as in the real model. In this benchmark case with lump sum taxes, credit subsidies and the nominal interest rate would jointly implement the first best allocation.

Similarly, in a model with sticky information such as Mankiw and Reis (2002) in which it is desirable that inflation be stable, the nominal interest rate will also be restricted in how to contribute to the attenuation of the distortions from the financial shocks. It can help reduce the average distortion, but not the one due to volatile spreads.

In the cashless model there are no costs of positive and volatile policy interest rates, but those costs would be present in a model with a money demand distortion. In particular, in our model, if the borrowing rate is high and volatile so will the lending rate. In a model with both sticky prices and monetary frictions, monetary policy would face a trade-off, unless other fiscal instruments were used.

The main conclusion from this discussion, is that credit subsidies are an effective instrument to deal with distortions associated with high and volatile spreads. Depending on the source of non-neutrality in a monetary model, and on other available fiscal instruments, monetary policy can be an imperfect substitute or a complementary policy tool aimed at other distortions such as price dispersion due to sticky prices or information.

## 6 Concluding remarks

Credit subsidies can be used to shield the economy from the adverse consequences of financial shocks on credit spreads. This is the main message of the paper.

We have analyzed optimal monetary and fiscal policy in a monetary model in which

financial intermediation is costly because of an enforcement problem, as in Gertler and Karadi (2011). This gives rise to high and volatile credit spreads that should be corrected by policy. The policy interest rate can partially address the inefficiency from the volatility in spreads, but is restricted by the zero bound constraint. The constraint may be binding, especially in response to a severe financial shock. Credit subsidies can overcome the zero bound constraint and be an effective tool to deal with the distortions associated with high and volatile spreads.

Credit subsidies implement a second best allocation in which wedges, and therefore also allocations, are fully stabilized in response to financial shocks. In that benchmark case with full taxation of distributed profits there are no financing costs of credit subsidies.

Full smoothing of wedges and no financing costs of credit subsidies are no longer part of the optimal policy solution if the household is able to keep distributed profits. In that case, in response to a financial shock, there are permanent effects on taxes, government debt, and output, which are particularly costly in the event of a large shock.

While credit subsidies, or interest rate policy, aim at minimizing the costs of ensuring the private incentives to the financial intermediaries, credit easing by central banks directly overcomes the need for those incentives, presumably at a cost in terms of resources. In our benchmark case with a full tax on distributed profits, credit subsidies are always preferable to central bank lending.

The production structure of the model is very simple, with a technology that uses labor only. If the model had capital, and financial intermediation was necessary to facilitate investment, then credit spreads would also distort the accumulation of capital. Credit subsidies would have a more relevant role in that economy.

The model is a simple model with a single incentive problem and with full infor-

mation on banks conditions. The implementation of the optimal credit subsidies could be a challenge in actual economies with multiple inefficiencies and heterogeneity and private information in types and actions. That could be particularly hard if credit subsidies were to treat different banks differently, depending on their exposure to the incentive problem. There would be room for misrepresentation. If all financial intermediaries are treated alike, then the difficulties in using credit subsidies are the same difficulties in using interest rate policy to affect loan rates. Either instrument will be set incorrectly with incomplete information.

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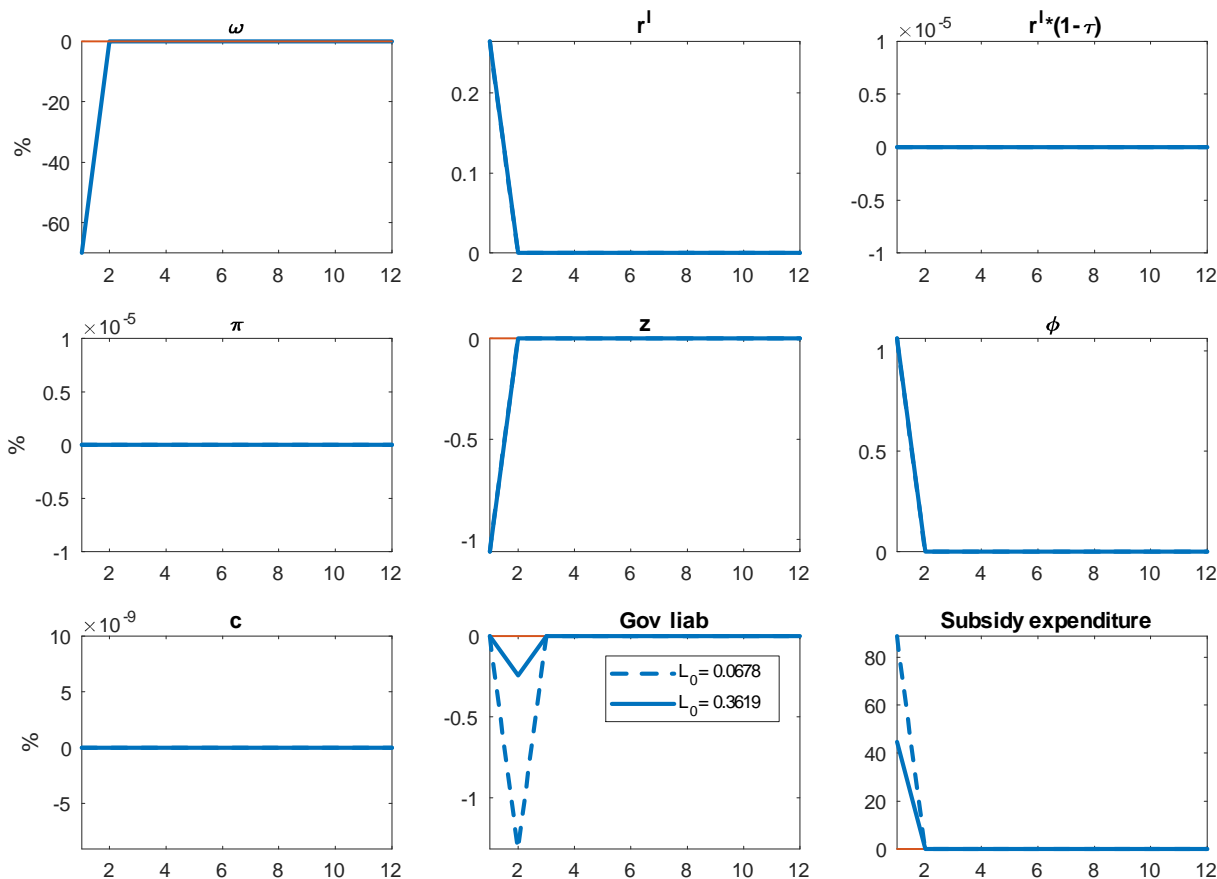


Figure 1: Credit subsidies after a financial shock for different gov't liabilities

Note: 100 x log-deviations from initial steady state. The financial shock is an i.i.d. reduction in  $\omega$ , which causes an annualised increase by 1 percentage point in credit spreads (25 basis points on a quarterly basis). The graph plots impulse responses for two levels of outstanding government liabilities  $L_0$ . For high levels of outstanding government liabilities, the credit subsidy becomes negative in steady state (it is a tax). In this case, for comparability, the impulse response of the government expenditure for the subsidy (bottom right panel) is shown with a minus sign. Full (100 percent) taxation of banks' profits is assumed in both scenarios.

Legend: " $\omega$ ": banks' start-up funds as a fraction of net worth; " $r^l$ ": lending rate; " $\tau$ ": fiscal subsidy; " $\pi$ ": inflation; " $z$ ": real value of banks' net worth; " $\phi$ ": leverage ratio; " $c$ ": consumption; "Gov liab": total outstanding government liabilities (in real terms); "Subsidy expenditure": government expenses to finance the credit subsidy.

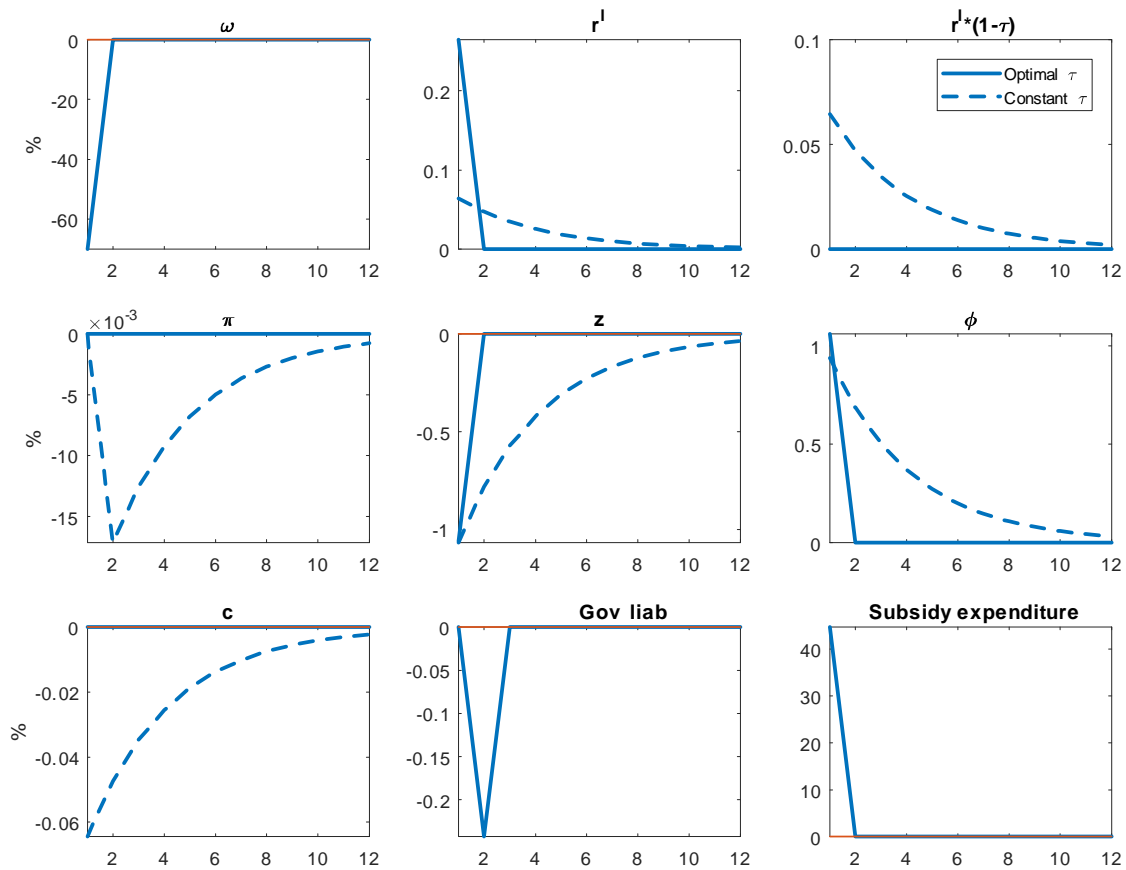


Figure 2. Optimal vs. constant credit subsidies after a financial shock

Note: see Figure 1. Initial government liabilities  $L_0 = 0.3611$  in the scenario with optimal credit subsidies. When subsidies are constant, government liabilities and the cost of the subsidy are not shown, because lump sum taxes are assumed. Full (100 percent) taxation of banks' profits is assumed in both scenarios.

Legend: see Figure 1.



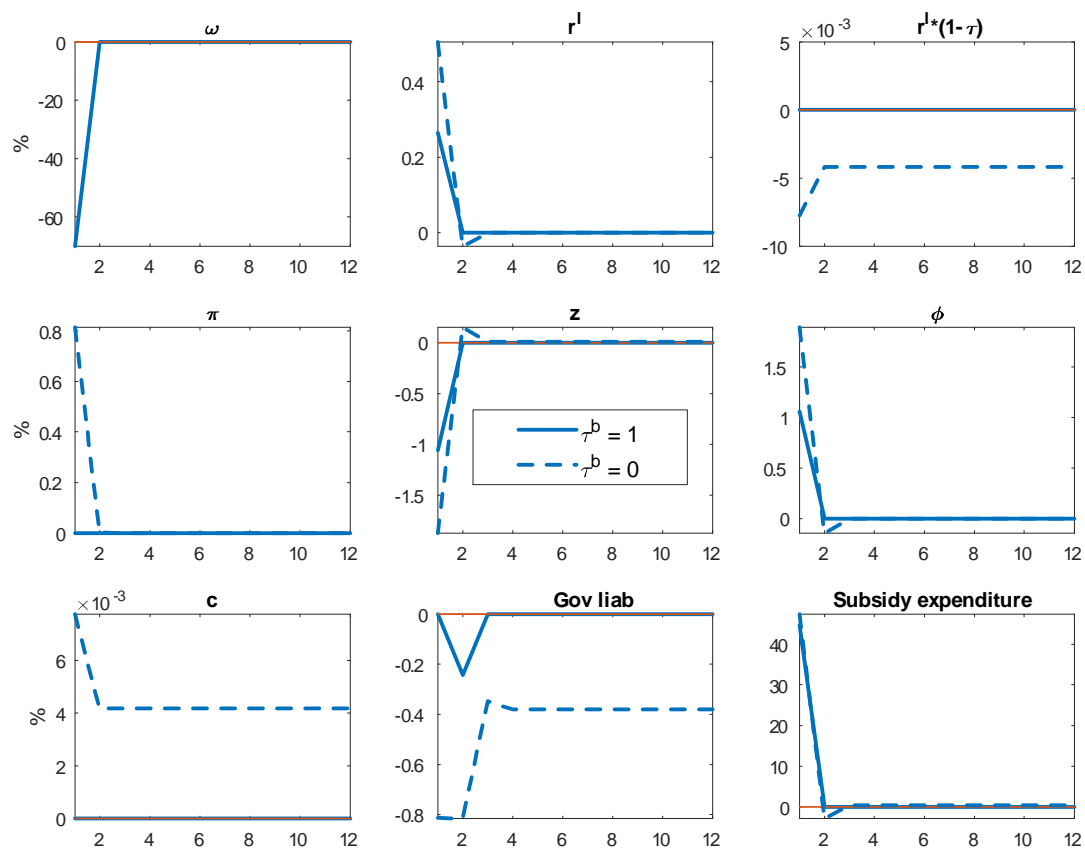


Figure 3: Credit subsidies after a financial shock with and without profit taxation

Note: see Figure 1. The figure compares the case with full (100 percent) profit taxation ( $\tau^b = 1$ ) to the case when profits are not taxed ( $\tau^b = 0$ ).

Legend: see Figure 1.

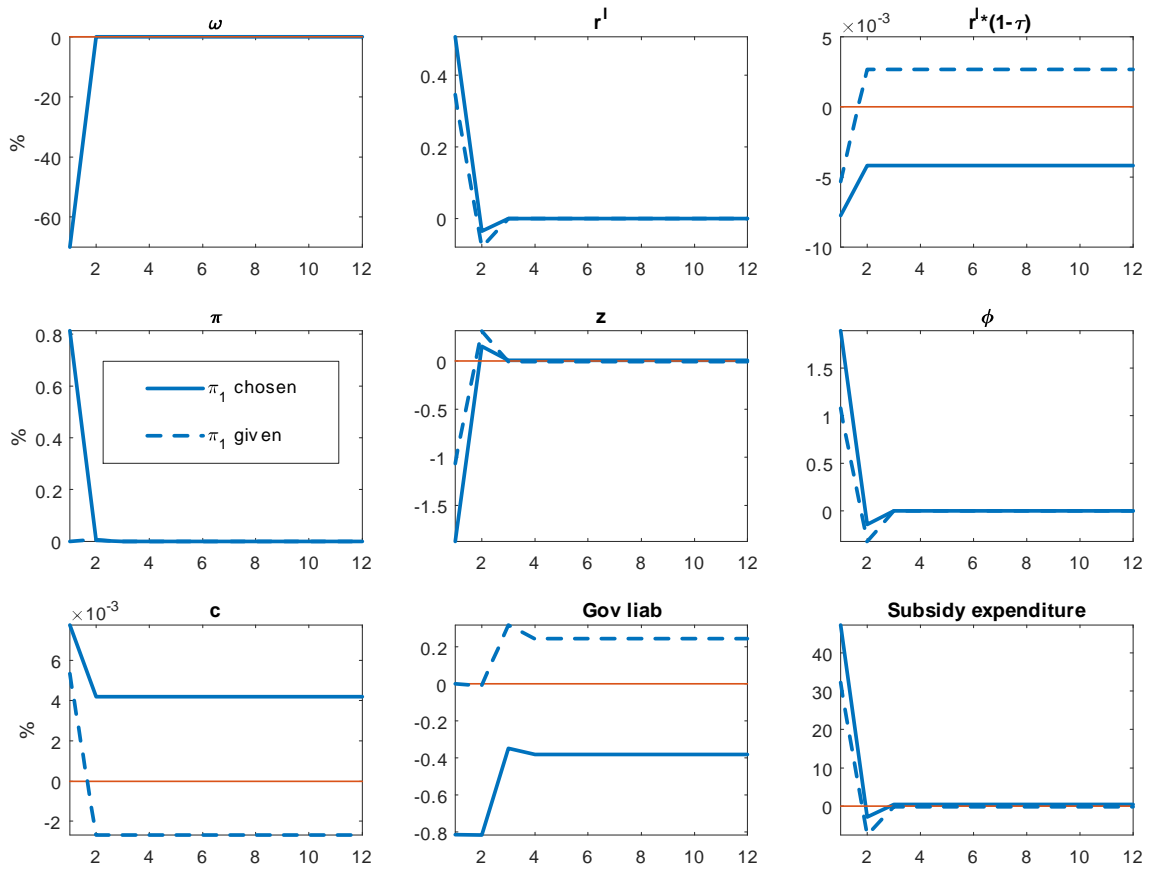


Figure 4: Credit subsidies after a financial shock: chosen vs. given initial  $\pi$

Note: see Figure 1. The figure compares the case when the Ramsey planner chooses the initial price level  $\pi_1$  to the case when the planner must take the initial price level as given. Legend: see Figure 1.

# A Online appendix

## A.1 Expressions for $v_t$ and $\eta_t$

In the beginning of period  $t$ , the net worth is  $Z_{j,t} = [(R_{t-1}^l - R_{t-1}) \phi_{t-1} + R_{t-1}] Z_{j,t-1}$ . The net worth going into period  $t + 1$ , giving rise to the profits in  $t$  is  $Z_{j,t+1} = [(R_t^l - R_t) \phi_t + R_t] Z_{j,t}$ . The value of a surviving bank, before taxes, in the beginning of period  $t$ , is given by:

$$V_{j,t} = \sum_{s=0}^{\infty} (1 - \theta) \theta^s Q_{t,t+1+s} Z_{j,t+1+s}$$

and can be written as

$$V_{j,t} = (1 - \theta) E_t Q_{t,t+1} Z_{j,t+1} + \theta E_t Q_{t,t+1} V_{j,t+1}$$

The conjecture for the value function is  $V_{j,t}(S_{j,t}^b, Z_{j,t}) = v_t S_{j,t}^b + \eta_t Z_{j,t}$ . Imposing that the incentive constraint is satisfied with equality gives

$$v_t S_{j,t}^b + \eta_t Z_{j,t} = \lambda S_{j,t}^b,$$

or

$$S_{j,t}^b = \frac{\eta_t}{\lambda - v_t} Z_{j,t} \equiv \phi_t Z_{j,t}.$$

From

$$V_{j,t}(S_{j,t}^b, Z_{j,t}) = (1 - \theta) E_t (1 - \tau_t^\pi) Q_{t,t+1} Z_{j,t+1} + \theta E_t Q_{t,t+1} V_{j,t+1}(S_{j,t+1}^b, Z_{j,t+1}),$$

$$Z_{j,t+1} = (R_t^l - R_t) S_{j,t}^b + R_t Z_{j,t},$$

and

$$S_{j,t}^b = \phi_t Z_{j,t},$$

we have

$$\begin{aligned} v_t S_{j,t}^b + \eta_t Z_{j,t} &= (1 - \theta) E_t Q_{t,t+1} [(R_t^l - R_t) \phi_t + R_t] Z_{j,t} + \\ &\quad \theta E_t Q_{t,t+1} [(R_t^l - R_t) \phi_t + R_t] \left[ v_{t+1} \frac{\phi_{t+1}}{\phi_t} S_{j,t}^b + \eta_{t+1} Z_{j,t} \right] \end{aligned}$$

It follows that

$$v_t = (1 - \theta) \frac{(R_t^l - R_t)}{R_t} + \theta [(R_t^l - R_t) \phi_t + R_t] E_t Q_{t,t+1} \frac{\phi_{t+1}}{\phi_t} v_{t+1}$$

and

$$\eta_t = (1 - \theta) + \theta [(R_t^l - R_t) \phi_t + R_t] E_t Q_{t,t+1} \eta_{t+1}$$

## A.2 Equilibrium

An equilibrium in the monetary economy of section 2, without credit easing, for  $\{C_t, N_t\}$ ,  $\{\tau_t^l, \tau^\pi, R_t, Q_{t,t+1}, P_t, W_t, R_t^l\}$  and  $\{\phi_t, \eta_t, v_t, S_t, Z_t\}$  is characterized by the intratemporal marginal condition for the households, (3), the intertemporal conditions,

$$\frac{u_C(t)}{P_t} = R_t E_t \frac{\beta u_C(t+1)}{P_{t+1}}, \quad (16)$$

$$Q_{t,t+1} = \frac{\beta u_C(t+1) P_t}{u_C(t) P_{t+1}}, \quad (17)$$

the firms marginal condition in (5), (7), (8), together with

$$S_t = \phi_t Z_t, \quad (18)$$

$$R_t^l (1 - \tau_t^l) \geq R_t \geq 1, \quad (19)$$

$$v_t = (1 - \theta) \left( \frac{R_t^l}{R_t} - 1 \right) + \theta R_t E_t Q_{t,t+1} \frac{\phi_{t+1}}{\phi_t} \left[ \left( \frac{R_t^l}{R_t} - 1 \right) \phi_t + 1 \right] v_{t+1}, \quad (20)$$

$$\eta_t = (1 - \theta) + \theta R_t E_t Q_{t,t+1} \left[ \left( \frac{R_t^l}{R_t} - 1 \right) \phi_t + 1 \right] \eta_{t+1}. \quad (21)$$

The household budget constraint (2), the condition for bank profits, (9), and the resource constraints, (10) must also be satisfied. The budget constraints of the government, state by state, are implied by these conditions because there is state-contingent debt and because of Walras law.

## A.3 Proof of Proposition 1

The implementability conditions in proposition 1 are built from the equilibrium conditions so they are necessary conditions. To see this notice that (2) with equality, and with  $\tau^\pi = 1$ , can be written as

$$E_0 \sum_{t=0}^{\infty} Q_t P_t C_t = E_0 \sum_{t=0}^{\infty} Q_t P_t \frac{W_t}{R_t P_t} N_t + (1 - l_0) \mathbb{W}_0$$

which, using the conditions for the households (3) and (17) can be written as

$$E_0 \sum_{t=0}^{\infty} \beta^t [u_C(t) C_t + u_N(t) N_t] = u_C(0) (1 - l_0) \frac{\mathbb{W}_0}{P_0} = \bar{\mathbb{W}}_0.$$

From (3) and (5) we have that

$$-\frac{u_C(t)}{u_N(t)} = \frac{R_t^l (1 - \tau_t^l)}{A_t}. \quad (22)$$

Because of the lower bound and upper bound restrictions on the interest rate and the subsidy, (19), it follows that

$$-\frac{u_C(t)}{u_N(t)} \geq \frac{1}{A_t}. \quad (23)$$

In order to show that the implementability and feasibility conditions (12), (23) and (10) are sufficient, we now take a generic allocation for consumption and labor,  $\{C_t, N_t\}$  restricted by (12), (23) and (10) and show that all the other conditions are satisfied by other equilibrium variables. There are multiple implementations of each allocation, so it is sufficient to do the demonstration for a particular one. The particular implementation is the one in which the price level does not change contemporaneously in response to shocks. It is predetermined. The policy rate is also set at the zero bound.

Conditions (7), (8), together with

$$S_t = \phi_t Z_t, \quad (24)$$

$$R_t^l (1 - \tau_t^l) \geq R_t \geq 1, \quad (25)$$

$$v_t = (1 - \theta) \left( \frac{R_t^l}{R_t} - 1 \right) + \theta R_t E_t Q_{t,t+1} \frac{\phi_{t+1}}{\phi_t} \left[ \left( \frac{R_t^l}{R_t} - 1 \right) \phi_t + 1 \right] v_{t+1}, \quad (26)$$

$$\eta_t = (1 - \theta) + \theta R_t E_t Q_{t,t+1} \left[ \left( \frac{R_t^l}{R_t} - 1 \right) \phi_t + 1 \right] \eta_{t+1}, \quad (27)$$

must be satisfied. The household budget constraint (2), the condition for bank profits, (9), and the resource constraints, (10) must also be satisfied. The budget constraints of the government, state by state, are implied by these conditions because there is state-contingent debt and because of Walras law.

The household intratemporal condition (3) is satisfied by  $W_t$ , the intertemporal conditions (16) and (17) are satisfied by the predetermined price level  $P_{t+1}$  and  $Q_{t,t+1}$ , respectively. The firm conditions (5) are satisfied by  $S_t$  and  $\tau_t^l$ . Since the wedge must be positive from (23) and  $R_t = 1$ ,  $\tau_t^l$  also satisfies

$$R_t^l (1 - \tau_t^l) \geq R_t \geq 1. \quad (28)$$

The leverage condition (7) is satisfied by  $\eta_t$ , the accumulation condition (8) is satisfied by  $Z_t$

$$S_t = \phi_t Z_t, \quad (29)$$

is satisfied by  $\phi_t$ ,

$$v_t = (1 - \theta) R_t E_t Q_{t,t+1} \left( \frac{R_t^l}{R_t} - 1 \right) + \theta R_t E_t Q_{t,t+1} \frac{\phi_{t+1}}{\phi_t} \left[ \left( \frac{R_t^l}{R_t} - 1 \right) \phi_t + 1 \right] v_{t+1}, \quad (30)$$

$$\eta_t = (1 - \theta) R_t E_t Q_{t,t+1} + \theta R_t E_t Q_{t,t+1} \left[ \left( \frac{R_t^l}{R_t} - 1 \right) \phi_t + 1 \right] \eta_{t+1}, \quad (31)$$

are satisfied by  $R_t^l$  and  $v_t$ . The budget constraints of the government, state by state, are implied by these conditions because there is state-contingent debt and because of Walras law. ■

## A.4 Proof of Proposition 3

For a moment we abstract from the zero bound constraint on the nominal interest rates. With negative interest rates, the household could borrow and hold cash, and make arbitrarily large profits. Banks could also do the same arbitrage. We need to assume that the household and banks are prevented from exploiting these profit opportunities. Subject to those restrictions, there is an equilibrium with negative rates, with associated (lower) lending rates. The overall set of feasible equilibria is larger than in the case where the nominal interest rate is restricted to be positive. The extended set of equilibria can always be equivalently implemented with a zero policy rate and with credit subsidies. Equivalence here means that the alternative implementation produces the same wedges and raises the same tax revenues. This means that the zero bound constraint on interest rates is made irrelevant when credit subsidies are used, which is the content of the following proposition.

Let  $\{C_t, N_t\}$  and  $\left\{\phi_t, \frac{R_t^l}{R_t}, \eta_t, \nu_t, \frac{S_t}{P_t}, \frac{Z_t}{P_t}\right\}$  be an equilibrium allocation in which the nominal interest rate is allowed to be negative. Suppose now that whenever  $R_t < 1$ , the path for the nominal interest rate is modified to  $\tilde{R}_t = 1$ . The equilibrium allocation will remain unchanged provided there are appropriate changes in  $\tau_t^l, R_t^l, Q_{t,t+1}$  and in the growth rate of nominal variables  $S_t, Z_t, P_t$ . More precisely in the equilibrium with nominal interest rate given by  $\tilde{R}_t = 1$  these variables (also denoted with a tilde) will have to be adjusted so as to respect the following conditions:

$$R_t^l (1 - \tau_t^l) = \tilde{R}_t^l (1 - \tilde{\tau}_t^l), \quad t \geq 0, \quad (32)$$

so that the wedges between marginal rate of substitution and marginal rate of transformation is unchanged;

$$\frac{R_t^l}{R_t} = \frac{\tilde{R}_t^l}{\tilde{R}_t}, \quad t \geq 0, \quad (33)$$

so that the lending spreads are unchanged; and

$$\tilde{Q}_{t,t+1} \tilde{R}_t = Q_{t,t+1} R_t, \quad t \geq 0, \quad (34)$$

and

$$\tilde{R}_t \frac{\tilde{P}_t}{\tilde{P}_{t+1}} = R_t \frac{P_t}{P_{t+1}}, \quad t \geq 0,$$

so that the growth rates of the nominal variables are adjusted by the change in the nominal rates.

With an appropriate adjustment in the initial levy  $l_0$ , the change from the original path  $R_t$  to the modified path  $\tilde{R}_t$  is also revenue neutral for the government. Since  $Z_0$  is predetermined, the initial price level,  $P_0$ , and nominal loans,  $S_0$ , must be the same in the two cases. However, because  $R_0$  affects the value of the initial wealth in (12), the movement to  $\tilde{R}_0$  can produce effects on the initial wealth. These effects can be neutralized by an adjustment in the initial levy.

## A.5 The nonmonetary economy

The economy has the same features as the monetary economy, except that there is no outside money, not even as unit of account. The cash-in-advance constraint on the households, (1), is not imposed. The role of money as unit of account is also eliminated, by imposing that the price level is always equal to one,  $P_t = 1$ . In the resulting real economy, firms must still hold financial assets in advance of production. They borrow from banks, so that the cost of holding those assets is a real credit spread.

Since the price level is set equal to one at all times, the wage,  $W_t$ , is now a real wage, in units of goods, and the prices of state-contingent assets,  $Q_{t,t+1}$ , and interest rates,  $R_t$  and  $R_t^l$ , and asset levels,  $S_t$  and  $Z_t$ , are now also in units of the good. Similarly bank profits,  $\Pi_t^b$  are also in real units.

The flow of funds constraints of the household are as in the monetary model with  $M_t = 0$  and  $P_t = 1$ . The single budget constraint is now

$$E_0 \sum_{t=0}^{\infty} Q_{t+1} C_t \leq E_0 \sum_{t=0}^{\infty} Q_{t+1} W_t N_t + E_0 \sum_{t=0}^{\infty} Q_{t+1} (1 - \tau^\pi) \Pi_t^b + (1 - l_0) W_0. \quad (35)$$

The intratemporal marginal choices for the household are not distorted by the nominal interest rate, so that instead of (3), the marginal condition is now

$$-\frac{u_C(t)}{u_N(t)} = \frac{1}{W_t}. \quad (36)$$

The intertemporal marginal conditions (16) and (17) become

$$u_C(t) = \beta E_t [R_{t+1} u_C(t+1)], \quad (37)$$

$$Q_{t+1,t+2} = \frac{\beta u_C(t+1)}{u_C(t)}. \quad (38)$$

Notice that the intertemporal prices that are relevant for the decisions between period  $t$  and  $t+1$  are prices between the asset market in  $t+1$  and  $t+2$ . This is a feature of Lucas timing, that payments are done in the asset market the period after.

In the cashless version of the model, the problems of the firms and the banks are unchanged. The constraints of the government are also the same except for the issuance of money.

The equilibrium conditions for the variables  $\{C_t, N_t\}$ ,  $\{\tau_t^l, R_t, Q_{t,t+1}, W_t, R_t^l\}$ ,  $\{\phi_t, \eta_t, v_t\}$ , and  $\{S_t, Z_t\}$  are (37) and (38), together with

$$-\frac{u_C(t)}{u_N(t)} = \frac{\frac{R_t^l}{R_t} (1 - \tau_t^l)}{A_t}, \quad (39)$$

$$\frac{R_t^l}{R_t} (1 - \tau_t^l) W_t = A_t, \quad (40)$$

$$A_t N_t = \frac{R_t^l}{R_t} (1 - \tau_t^l) S_t, \quad (41)$$

$$R_t^l (1 - \tau_t^l) \geq R_t, \quad (42)$$

as well as the constraints (7), (8), (10), (18), (20), and (21), which are common to the monetary and the real economy.



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